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Some New Properties of Q* Compact Space

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Abstract
In this paper, we study the properties of Q* compact space and generate new results in the space. In particular, we investigate the Q*-compactness of topological spaces with separable, Q*-metrizable, Q*-Hausdorff, homeomorphic, connected and finite intersection properties.

Keywords: Q* compact space, homeomorphic, bitopological spaces, finite sub cover, dense subset.

1. Introduction

Borel proved, in his 1894 Ph.D. thesis, that a countable covering of a closed interval by open intervals has a finite subcover. It turns out that Borel approach was similar to the approach used by Heine in 1872 to prove that a continuous function on a closed interval was uniformly continuous which was first proved, but unpublished for 60 years, by Dirichlet in 1852. Lebesgue (1898) and Cousins (1895) removed countable from the hypothesis of Borel's result. Thus, we have the generalized theorem, which is now called the Heine-Borel theorem. For references see Borel's work of 1919.

Murugalingam and Lalitha (2010) introduced the concept of Q* sets. They further studied the properties of Q* closed and Q* open sets in affine space in (2011). Padma and Udayakumar (2015a) and (2015b) introduced the concept of Q*O compact spaces and obtained some interesting results by applying the results to pairwise SC compact spaces (Padma *et al*., 2013). Some important results on bitopological spaces are obtained in Kannan (2009), Padma & Udayakumar (2012a), Padma and Udayakumar (2012b) and Sharma (1990). Let (X,τ) be a topological space. A subset S in X is Q* closed in (X,τ) if S is closed and $\text{Int}(S)=\phi$.

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Its compliment S' is therefore Q* open (Sidney, 2011; Tom *et. al.*, 2007). If every open cover of X has a finite sub cover then X is called a compact space. (X,τ) which is separable if it has a countable dense subset. Let X be a set and $\Im$ a family of subsets of X. Then $\Im$ is said to have finite intersection property if for any finite number $F_1, F_2, ..., F_n$ of members of $\Im$, $F_1 \cap ... \cap F_n \neq \phi$, Sidney (2011).

2. Preliminaries

In this Section, we give some useful definitions to the proof of our main results:

Definition 2.1 (Tom *et al.*, 2007)**:** A subset B of a topological space (X,τ) is said to be compact if every open covering of B has a finite subcovering. If the compact subset B equals X, then (X,τ) is said to be a compact space.

Definition 2.2 (Tom *et al.*, 2007)**:** Let (X,τ) be a topological space. Then, it is said to be connected if the only clopen subsets of X are X and ϕ.

Definition 2.3 (Tom *et al.*, 2007): A subset A of a topological space (X,τ) is said to be Q*O - compact space if every $\tau - Q^*$ open cover of X has a finite sub cover.

Definition 2.4 (Tom *et al.*, 2007)**:** Let (X,τ) be a topological space. Then it is said to be Q*-connected if the only Q* clopen subsets of X are X and ϕ.

3. Generalization of Q*O Compact space

We state our main results in this section:

Theorem 3.1: The closed interval [0, 1] is Q*-compact.

Proof: Let G_α, $\alpha \in \Lambda$ be any open covering of [0, 1]. Then for each $x \in [0,1]$, there is a G_α such that $x \in G_\alpha$. As G_α is open in x, there exist an interval U_x, open in [0, 1] such that $x \in U_x \subseteq G_\alpha$. Now, we define S of [0, 1] as:

$$S = \{z : [0,z] \, can\, be\, covered\, by\, a\;\; finite\, number\, of\;\; the\, sets\, U_x\}.$$

Then, $[z \in S \Rightarrow [0,z] \subseteq U_{x_1} \cup U_{x_2} \cup ... \cup U_{x_n}$ for some $x_1, x_2, ..., x_n$]. Let $x \in S$ and $y \in U_x$. Then as U_x is an interval containing x and y, $[x,y] \subseteq U_x$. Here we are assuming without loss of generality that $x \leq y$. Therefore, $[0,y] \subseteq U_{x_1} \cup U_{x_2} \cup ... \cup U_{x_n} \cup U_x$ and hence $y \in S$. For each $x \in [0,1], U_x \cap S = U_x$ or ϕ. This implies that $S = \bigcup_{x \in S} U_x$ and $[0,1] | S = \bigcup_{x \notin S} U_x$. Thus, we have that S is open in [0, 1] and S is closed in [0, 1]. But [0, 1] is connected. Therefore

$S=[0,1]$ or ϕ. However, $0\in S$ and $S=[0,1]$ that is, [0, 1] can be covered by a finite number of U_x. So that $[0,1]\subseteq U_{x_1}\cup U_{x_2}\cup\ldots\cup U_{x_m}$. But each U_x is contained in $G_\alpha,\alpha\in\Lambda$. Hence, $[0,1]\subseteq G_{\alpha_1}\cup G_{\alpha_2}\cup\ldots\cup G_{\alpha_m}$ which implies that [0, 1] is Q*-compact.

Example 3.1: If $(X,\tau)=G_\alpha$ and $A=(0,\infty)$ then A is not Q*-compact.

Proof: For any integer α , Let G_α be any interval $(0,\alpha)$. Then clearly, $A\subseteq\bigcup_{\alpha=1}^{\infty}G_\alpha$. But there do not exist $\alpha_1,\alpha_2,\ldots,\alpha_n$ such that $A\subseteq(0,\alpha_1)\cup(0,\alpha_2)\cup\ldots\cup(0,\alpha_n)$. Therefore, A is not Q*-compact.

Corollary 3.1: For a and b in Gά with $a<b$, [*a*, *b*] is Q* compact, while (*a*, *b*) is not Q*-compact.

Proof: The space [*a*, *b*] is homeomorphic to Q*- compact [0, 1] and by Padma (2015[b]), Q*-compact. The space (a,b) is homeomorphic to $(0,\infty)$. If (a,b) were Q*-compact, then $(0,\infty)$ would be Q*-compact, but by example 3.1 $(0,\infty)$ is not Q*-compact. Hence (a,b) is not Q*-compact.

Example 3.2: Suppose $X=\{e,f,g,h\},\tau=\{\phi,X,\{e,f,g\},\{e,f,h\}\}$, Let $S=\{e,g,h\}$. Now $A\subset\{a,d\}\cup\{b,c\}$. By definition, S is compact set. But, S is not a Q*O - compact set because S is not Q* closed since its complement $\{b\}$ is not Q* open.

Remark: Every Q*O - compact space is compact, but the converse is obviously not necessarily true from example 3.2.

Theorem 3.2: A subset S of Gά is Q*-compact if and only if S is closed and bounded.

Proof: Suppose that S is Q*-compact. To see that S is bounded, we let $I_n=(-n,n)$

Then $\bigcup_{n=1}^{\infty}I_n=G_\alpha$, therefore, S is covered by the collection of $\{I_n\}$. Since S is Q*-compact, finitely many will suffice $S\subseteq\left(I_{n_1}\cup\ldots\cup I_{n_k}\right)=I_m$, where $m=max\{n_1,\ldots,n_k\}$. Therefore, $|x|\leq m$ for all $x\in S$ and S is bounded. Now, we will show that S is closed.

Suppose not, then, there exists a point $p\in cl(S)|$ S; for each n , define the neighborhood around p of radius $1/n$, $N_n=N\left(p,1/n\right)$. Take the complement of the closure of $N_nU_n=R\,|\,cl(N_n)$ is open, since its complement is closed and we have $\bigcup_{n=1}^{\infty}\cup_n=$ $G_\alpha|\bigcap_{n=1}^{\infty}cl(N_n)=G_\alpha\{p\}\supseteq S$.

Therefore, $\{U_n\}$ is an open cover of S. Since S is Q*-compact, there is a finite subcover $U_{n_1},...,U_{n_k}$ for S.

Furthermore, they are constructed as $U_i \subseteq U_j$ if $i \leq j$. It follows that $S \subseteq U_m$ where $m = max\{n_1,...,n_k\}$. But, $S \cap N(p,1/m) = \phi$ which contradicts the choice of $p \in cl(S)|$ S.

Conversely, we need to show that if S is closed and bounded, then S is Q*-compact. Let $\Im$ be an open cover for S. For each $x \in$ Gά, define the set $S_x = S \cap (-\infty, x]$, and let $B = \{x : S_x \, is\, covered\, by\, a\; finite\, subcover\, of\; \Im\}$.

Since S is closed and bounded, by hypothesis, S has both a maximum and a minimum. Let $d = minS$. Then $S_x = \{d\}$ and this is certainly covered by a finite subcover of $\Im$. Therefore, $d \in B$ and B is nonempty. If we can show that B is not bounded above, then it will contain a number p greater than max S. But then, $S_p = S$ so we can conclude that S is covered by a finite subcover, and is therefore Q*-compact. Toward this end, suppose that B is bounded above and let $m = supB$. We shall first show that $m \in S$ and then $m \notin S$ both lead to contradictions.

If $m \in S$, then since $\Im$ is an open cover of S, there exists F_0 in $\Im$ such that $m \in F_0$. Since F_0 is open, there exists an interval $[x_1, x_2]$ in F_0 such that $x_1 < m < x_2$. Since $x_1 < m$ and $m = supB$, there exists $F_1,...,F_k$ in $\Im$ that cover S_{x_1}. But then $F_0, F_1,...,F_k$ cover S_{x_2}, so that $x_2 \in B$. But this contradicts $m = supB$. If $m \notin S$, then since S is closed there exists ò>0 such that $N(m,ò) \cap S = \phi$. But then $S_{m-ò} = S_{m+ò}$. Since $m -ò \in B$ we have $m + ò \in B$, which again contradicts $m = supB$.

Therefore, either way, if B is bounded above, we get a contradiction. We conclude that *B* is not bounded above, and *S* must be Q*-compact.

Theorem 3.3: Let (X,τ) be a Q*-compact metrizable space. Then, (X,τ) is separable.

Proof: Let d be a metric space on X which induces the topology τ. For each positive integer n, let S_n be the family of all open balls having centres in X and radius $\frac{1}{n}$. Then, S_n is an open covering of X and so there is a finite subcovering $\mu_n = \{U_{n_1}, U_{n_2},...,U_{n_k}\}$, for some

$k \in \aleph$. Let y_{n_j} be the centre of U_{n_j}, j=1,…k' and $Y_n = \{y_{n_1}, y_{n_2}, ..., y_{n_k}\}$. Put $Y = \bigcup_{n=1}^{\infty} Y_n$. Then, Y is a countable subset of X. We now show that Y is dense in (X,τ). If V is any non-empty open set in (X,τ), then for any $v \in V$, V contains an open ball, B, of radius $\frac{1}{n}$, about v, for some $n \in \aleph$. As μ_n is an open cover of X, $v \in U_{n_j}$, for some j. Thus, $d\left(v, y_{n_j}\right) < \frac{1}{n}$ and so $y_{n_j} \in B \subseteq V$. Hence, $V \cap Y \neq \phi$, and so Y is dense in X.

Theorem 3.4: Let (X,τ) be a topological space. Then (X,τ) is Q*-compact if and only if every family $\Im$ of closed subsets of X with the finite intersection property satisfies $\bigcap_{F \in \Im} F \neq \phi$.

Proof: Assume that every family $\Im$ of closed subsets of X with the finite intersection property satisfies $\bigcap_{F \in \Im} F \neq \phi$. Let μ be any open covering of X. Put $\Im$ equal to the family of complements of members of μ. So each $F \in \Im$ is closed in (X,τ). as μ is an open covering in X, $\bigcap_{F \in \Im} F \neq \phi$. By the assumption, then $\Im$ does not have finite intersection property. So for some $F_1, F_2, ..., F_n$ in $\Im$, $F_1 \cap F_2 \cap ... \cap F_n \neq \phi$. Thus $U_1 \cup U_2 \cup ... \cup U_n = X$, where $U_i = X \mid F_i$, $i = 1, ..., n$. So μ has a finite subcovering. Hence, (X,τ) is Q*-compact. The converse statement is proved similarly.

Theorem 3.5: Let f be a continuous mapping of a Q*-compact metric space (X, d) onto a Q*-Hausdorff space (Y,τ_1). Then (Y,τ_1) is Q*-compact and metrizable.

Proof: Since every Q*-continuous image of a compact space is compact (Padma 2015[b]), the space (Y,τ_1) is certainly compact and metrizable. As the map f is surjective, we can define the metric d_1 on Y as follows: $d(y_1, y_2) = \inf\{d(a,b): a \in f^{-1}\{y_1\}$ and $a \in f^{-1}\{y_1\}$ for all y_1 and y_2 in Y. We need to show that d_1 is indeed a metric. Since $\{y_1\}$ and $\{y_2\}$ are closed in the Q*-Hausdorff space (Y,τ_1), $f^{-1}\{y_1\}$ and $f^{-1}\{y_2\}$ are Q*- compact. So the product $f^{-1}\{y_1\} \times f^{-1}\{y_2\}$, which is a subspace of the product space $(X,\tau) \times (X,\tau)$, is Q*-compact, where τ is the topology induced by the metric d. Observing that $d : (X,\tau) \to G_\alpha$ is a continuous mapping, then $d\left(f^{-1}\{y_1\}\times f^{-1}\{y_2\}\right)$ has a least element. So there exists an element $x_1 \in f_{-1}\{y_1\}$ and an element $x_2 \in f_{-1}\{y_2\}$ such that:

$d(x_1,x_2)=inf\,\{d(a,b): a\in f^{-1}\{y_1\}, b\in f^{-1}\{y_2\}\}=d_1(y_1,y_2)$.

Clearly, if $y_1 \neq y_2$, then $f^{-1}\{y_1\}\cap f^{-1}\{y_2\}=\phi$. Thus, $x_1\neq x_2$ and hence $d(x_1,x_2)>0$; that is $d_1(y_1,y_2)>0$. It is easily verified that d_1 has the other properties required of a metric and so a metric on Y. Let τ_2 be the topology induced on Y by d_1, we have to show that $\tau_1=\tau_2$. Firstly, by the definition of d_1, $f:(X,\tau)\to(X,\tau_2)$ is certainly continuous. Observe that for a subset C of Y, C is a closed subset of (Y,τ_1) implies $f^{-1}(C)$ is a closed subset of (X,τ). Hence $f^{-1}(C)$ is a Q*-compact subset of (X,τ) which implies that $f\left(f^{-1}(C)\right)$ is a Q*-compact subset of (X,τ_2). Hence $\tau_1\subseteq\tau_2$. Similarly, we have $\tau_2\subseteq\tau_1$ and thus $\tau_1=\tau_2$.

Theorem 3.6: An infinite subset of a Q*-compact space must have a limit point.

Proof: Let *X* be a Q*-compact space with an infinite subset *S* which has no limit point. Our interest is to show that *S* is finite. We can find an open cover of *X* consisting of open neighborhoods *T*(*x*) such that $T(x)\cap S=\begin{cases}\emptyset \; if\; x\notin S\\ \{x\} if\; x\epsilon S\end{cases}$ each of these *T*(*x*) exist since otherwise *x* is a limit point of *S*. The open cover *T*(*x*) must admit a finite subcover and since we defined *T*(*x*) to contain no more than one point of *S*, *S* must be finite.

Theorem 3.7: Let (X,τ) be a Q*-compact space and $f:(X,\tau)\to G_\alpha$ a continuous mapping. Then, $f(X)$ has a greatest element and a least element.

Proof: Since f is continuous and $f(X)$ is Q*-compact, therefore, $f(X)$ is a closed bounded subset of G_α. Since $f(X)$ is closed and $f(X)$ is a subset of G_α, it is obvious that if p is the supremum of $f(X)$ then $p\in f(X)$. That is, we need to show that the supremum of $f(X)$ is contained in $f(X)$. Suppose $p\,\epsilon\, G_\alpha \left| f(x)\right.$. As $G_\alpha \left| f(x)\right.$ is open, there exist real numbers a and b with $a<b$ such that $p\in(a,b)\subseteq\mathbb{R}\,|\,f(X)$. As p is the least upper bound for $f(X)$ and $a<p$, it is clear that there exists an $x\in f(X)$. Also, $x<p<b$ and so $p\,\epsilon\,(a,b)\subseteq G_\alpha \left| f(x)\right.$. But this is a contradiction, since $x\in f(X)$. Hence, our

supposition is false and $p \in f(X)$. Thus, $f(X)$ has a greatest element p. Similarly, it can be shown that $f(X)$ has a least element.

Theorem 3.8: If $(X_1,\tau_1),(X_2,\tau_2),...,(X_n,\tau_n)$ are Q*- compact spaces, then $\prod_{i=1}^{n}(X_i,\tau_i)$ is a Q*- compact space.

Proof: The first part of this proof is to show that the product of any two Q*- compact topological spaces is Q*- compact. Suppose (X_1,τ_1) and (X_1,τ_1) are Q*- compact then $(X_1,\tau_1)\times(X_2,\tau_2)$ is also Q*- compact Padma (2015) [8].Then by induction, we can say that: $(X_1,\tau_1)\times(X_2,\tau_2)\times(X_3,\tau_3)=[(X_1,\tau_1)\times(X_2,\tau_2)]\times(X_3,\tau_3)$ is also Q*- compact since it is also a product of two Q*- compact spaces. Conclusively, suppose that the product of any two N Q*- compact spaces is Q*- compact. Consider the product $(X_1,\tau_1)\times(X_2,\tau_2)\times...\times(X_{N+1},\tau_{N+1})$ of Q*- compact spaces (X_i,τ_i), i = 1, ..., N+1. Then, $(X_1,\tau_1)\times...\times(X_N,\tau_N)\times(X_{N+1},\tau_{N+1})\cong[(X_1,\tau_1)\times...\times(X_N,\tau_N)]\times(X_{N+1},\tau_{N+1})$.

By inductive hypothesis $(X_1,\tau_1)\times...\times(X_N,\tau_N)$ is Q*- compact, so the right-hand side is the product of two Q*- compact spaces and thus is Q*- compact. Therefore, the left-hand side is also Q*- compact.

Theorem 3.9: Let $\{(X_i,\tau_i): i\in I\}$ be any family of topological spaces. Then, $\prod_{i\in I}(X_i,\tau_i)$ is Q*-compact if and only if each (X_i,τ_i) is Q*-compact.

Proof: We shall use Theorem 3.6 to show that $(X,\tau)=\prod_{i\in I}(X_i,\tau_i)$ is Q*-compact. Let $\Im$ be any family of closed subsets of X with the finite intersection property. We have to prove that $\bigcap_{F\in\Im}F=\phi$. By Sydney (2011), there is a maximal family Ξ of (not necessarily closed) subsets of (X,τ) that contains $\Im$ and has the finite intersection property. We shall prove that $\bigcap_{H\in\Xi}\bar{H}=\phi$, from which follows the required result $\bigcap_{F\in\Im}F=\phi$, since each $F\in Im$ is closed. Since Ξ is maximal with the property that it contains $\Im$ and has a finite intersection property, if $H_1,H_2,...,H_n\in\Xi$, for any $n\in\aleph$, then the set $H'=H_1\cap H_2\cap...\cap H_n\in\Xi$. Suppose this was not the case, then the set $\Xi'=H\cap\{\Xi'\}$ would properly contain Ξ and also have the property that it contains $\Im$ and has the finite intersection property. This is a contradiction to Ξ being maximal. So $\Xi'=H$ or $H'=H_1\cap H_2\cap...\cap H_n\in\Xi$. Let S be any subset of X that intersects non-trivially every member of Ξ, we claim that $\Xi \cap \{S\}$ has the finite intersection property. To see this, let $H_1^{'}, H_2^{'}, ..., \mathrm{H}_m^{'}$ be members of Ξ', we shall show that $S \cap H_1^{'}, H_2^{'} \cap ... \cap H_m^{'} \neq \phi$. But, $H_1^{'}, H_2^{'} \cap ... \cap H_m^{'} \in \Xi$. By assumption $S \cap (H_1^{'}, H_2^{'} \cap ... \cap H_m^{'}) \neq \phi$. Hence, $\Xi \cup \{S\}$ has the finite intersection property which contains $\mathfrak{I}$.

Again, using the fact that Ξ is maximal with the property that it contains $\mathfrak{I}$ and has the finite intersection property, we see that $S \in \mathfrak{I}$. Fix $i \in I$ and let $p_i : \prod_{i\in I}(X_i, \tau_i) \to (X_i, \tau_i)$ be the projection mapping, then the family $\{p_i(H) : H \in \Xi\}$ has a finite intersection property. Therefore, the family $\{\overline{p_i(H)} : H \in \Xi\}$ has a finite intersection property. As (X_i, τ_i) is Q*-compact, $\bigcap_{H\in\Xi} \overline{p_i(H)} \neq \phi$. Let $x_i \in \bigcap_{H\in\Xi} \overline{p_i(H)}$, then for each $i \in I$, we can find a point $x_i \in \bigcap_{H\in\Xi} \overline{p_i(H)}$. Put $x \in \prod_{i\in I} x_i \in X$, we shall prove that $x \in \bigcap_{h\in\Xi} \overline{p_i(H)}$. Let T be any open set containing x. Then, T contains a basic open set about x for the form $\bigcap_{i\in J} p_i^{-1}(U_i)$, where $U_i \in \tau_i$, $x_i \in U_i$ and J is a finite subset of I. As $x_i \in \overline{p_i(H)}$, $U_i \cap p_i(H) \neq \phi$, for all $H \in \Xi$. Thus, $p_i^{-1}(U_i) \cap H \neq \phi$ for all $H \in \Xi$. By the observation above, this implies that $p_i^{-1}(U_i) \in \Xi$, for all $i \in J$. As Ξ has the intersection property, $\bigcap_{i\in J} p_i^{-1}(U_i) \cap H \neq \phi$ for all $H \in \Xi$. Hence, $x \in \bigcap_{H\in\Xi} \bar{H}$, as required. Conversely, if $\prod_{i\in I}(X_i, \tau_i)$ is Q*-compact, by Corollary 3.1 and Theorem 3.6 each (X_i, τ_i) is Q*-compact.

Theorem 3.10: If X is not Q*-compact, then X is homeomorphic to an open dense set in χ.(where χ is not too larger than X).

Proof: Suppose we ensure that χ is not too large, that is, not too much larger than X. First, we shall show that X is homeomorphic to the set $\{X\} \subset \chi$. Construct a function that sends each point of X to the corresponding point in $\{X\}$. This function is obviously one-to-one and onto and it is continuous (and so is it has inverse) because the open sets in $\{X\}$ are exactly the open sets in X. The set $\{X\}$ is open in χ because it does not contain ∞ and it is open in X. To show that $\{X\}$ is dense, we can simply show that it is not closed or that ∞ is not open. (If that is the case, then $\{X\}$ is not its own closure and the only other option is that its closure is χ). If ∞ is open, then its complement, $\{X\}$, must be compact. But this would imply that X is Q*-compact, contradicting our earlier assumption. So ∞ cannot be open, meaning $\{X\}$ must be dense.

Theorem 3.11: If none of the components of X is Q*-compact, then χ is connected.

Proof: Assume that χ is not connected. That is, there is some set U in χ that is open and closed, but is not ϕ or χ. It's complement, V, is also open and closed without being ϕ or χ. Either U or V contains ∞; take the one that does not, and call it W. W is Q*-compact because its complement is open and contains ∞. First let us consider the case that X is connected. We have already established that W is not ϕ. It cannot be all of X either, because W is Q*-compact and X is not. W is open in X because it is open in χ and does not contain ∞. It is closed in X because its complement (either $U \cap X$ or $V \cap X$) is open in X. Hence, W is open, closed, not ϕ, and not X, which implies that X is not connected. This contradicts the assumption, and χ must be connected.

Remark: Suppose X is not connected? In this case, we look at the connected components of X. Any open set including ∞ must also contain points in each of the components of X because the complement of the open set is Q*-compact, and if the complement included an entire connected component, then that component would need to be Q*-compact, but it is not. So W contains some points in each of the components. But this would imply that the connected components are not connected, which is a contradiction. Again, χ must be connected.

4. Results and Discussion

Some new properties of Q*-compactness of topological spaces with separable, Q*-metrizable, Q*-Hausdorff, homeomorphic, connected and finite intersection properties were critically reviewed, generated and evaluated.

5. Conclusion

The results in this article are a refinement, extension and generalization of the work in Padma (2015) and Padma and Udayakumar (2015).

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References

Borel, E. (1919): Sur la classification des ensembles de mesure nulle. *Bull. Soc. Math.* France. **47**, 97–125.

Kannan, K. (2009): Contribution to the study of some generalized closed sets in bitopological spaces. *Ph.D Thesis*.

Murugalingam, M. and Laliltha, N. (2010): Q star sets. *Bulletin of Pure and Applied Sciences*, **29E**(2), 369 - 376.

Murugalingam, M. and Laliltha, N. (2011): Properties of Q* sets. *Bulletin of Pure and Applied Sciences*, **3E**(2), 267 - 277.

Padma, P. and Udayakumar, S. (2012a): Pairwise Q* Separation Axioms in Bitopological Spaces. *International Journal of Mathematical Achieve*, **3**(12), 4959 - 4971.

Padma, P. and Udayakumar, S. (2012b): $\tau_1\tau_2$ - Q* continuous Maps in Bitopological Spaces *Asian Journal of Current Engineering and Mathematics*, **1(**4), 227-229.

Padma, P., Chandrasekhararao, K. and Udayakumar, S. (2013): Pairwise SC Compact Spaces. *International Journal of Analysis and Applications*, **2**(2), 162 - 172.

Padma, P. (2015): On Q*O Compact Spaces. *Journal of Progressive Research in Mathematics* (JPRM), **1**(1), 13-17.

Padma, P. and Udayakumar, S. (2015): On Q*s - regular spaces, *Journal of Progressive Research in Mathematics* (JPRM), **1(**1), 27-33.

Sidney A. M. (2011): Topology without Tears. *University of Ballarat Press Australia* **2**, 155-167

Tom B., Chris B. and Michael B. (2007): Introduction to Topology. *Renzo's Math* **470**, Winter, 24-55.

Sharma, V. K. (1990): A study of some separation and covering axioms in topological and bitopological spaces. *Ph. D. Thesis*, Meerut University.