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## **Alternative Ratio - Regression Type Estimator in Simple Random Sampling**

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### **Abstract**

In this study, an alternative ratio-regression type estimator was proposed which was compared with Kadilar and Cingi (2004), and Ekaete *et al.* (2014) estimators. This proposed estimator combined classical ratio estimator with our usual regression estimator. The same data set used by Kadilar and Cingi (2004) and Ekaete *et al.* (2014) were also used to determine the efficiency of this alternative ratio-regression type estimator. The finding

was that, when  $\alpha = 1$ , Ekaete *et al.* (2014) was found to be better but at  $\alpha \leq \sqrt{\frac{(\frac{\bar{X}}{\bar{X} + \rho})^2 c_y^2 (1 - \rho^2)}{(c_y^2 + c_x^2 - 2\rho c_y c_x)}}$ , this

alternative ratio-regression type estimator was said to be more efficient than both Kadilar and Cingi (2004), and Ekaete *et al.* (2014) estimators based on their estimated mean square error (mse).

**Keywords:** Alternative ratio regression, Combined classical ratio, estimator, bias, mean square error.

### **1. Introduction**

Let  $N$  and  $n$  be the population and sample sizes respectively,  $\bar{X}$  and  $\bar{Y}$  be the population means for the auxiliary variable ( $X$ ) and the variable of interest ( $Y$ ),  $\bar{x}$  and  $\bar{y}$  be the sample means based on the sample drawn. Then classically (Cochran, 1977 and Okafor, 2002)

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}} \bar{X}, \quad (1)$$

$$bias(\bar{y}_r) = \left( \frac{1-f}{n} \right) \bar{Y} [c_x^2 - \rho c_x c_y] \quad (2)$$

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and

$$mse(\bar{y}_r) = \left( \frac{1-f}{n} \right) \bar{Y}^2 [c_x^2 + c_y^2 - 2\rho c_x c_y]. \quad (3)$$

The literature on survey sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ratio method of estimation is a good example in this context. If the correlation between the study variable  $y$  and the auxiliary variable  $x$  is positive (high), the ratio method of estimation is quite effective.

In sample surveys, supplementary information is often used for increasing the precision of estimators (Adewara, 2006; Ogunyinka and Sodipo, 2013; Onyeka, 2012 and Pandey *et al.*, 2011).

Many authors including: Adewara (2015), Solanki *et al.* (2012) and Subramani and Kumarapandiyan (2012) have used auxiliary information for improved estimation of population mean of study variable  $y$ .

## 2. Materials and Methods

### On the Kadilar and Cingi (2004) Estimator

Kadilar and Cingi (2004) proposed an estimator,  $\bar{y}_{kc} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X}$  and gave its bias and mean square error (mse) respectively as:

$$bias(\bar{y}_{kc}) = \left( \frac{1-f}{n} \right) \bar{Y} c_x^2 \quad (4)$$

and

$$mse(\bar{y}_{kc}) = \left( \frac{1-f}{n} \right) \bar{Y}^2 [c_x^2 + c_y^2 (1 - \rho^2)], \quad (5)$$

where  $f = \frac{n}{N}$ .

### On Ekaete *et al.* (2014) Estimator

Ekaete *et al.* (2014) suggested an estimator

$$\bar{y}_{ek} = \frac{\alpha \bar{y} (\bar{X} + \rho)}{\bar{x} + \rho} + \frac{\beta [\bar{y} + b(\bar{X} - \bar{x})] \bar{X}}{\bar{x}} \quad (6)$$

with an

$$mse(\bar{y}_{eh}) = \left(\frac{1-f}{n}\right)\bar{Y}^2[c^2_y(1-\rho^2)] . \quad (7)$$

### On the proposed alternative ratio-regression type estimator

The classical ratio estimator used in this study is defined as  $\bar{y}_r^* = \alpha\left(\frac{\bar{y}}{\bar{x}}\right)(\bar{X} + \rho)$  while our traditional regression estimator used is defined as:  $\bar{y}_{reg} = \beta[\bar{y} + b(\bar{X} - \bar{x})]$  such that  $\alpha + \beta = 1$ . Their biases and mean square errors are given respectively as:

$$bias(\bar{y}_r) = \left(\frac{1-f}{n}\right)\bar{Y}[c^2_x - \rho c_x c_y], \quad (8)$$

$$mse(\bar{y}_r) = \left(\frac{1-f}{n}\right)\bar{Y}^2[c^2_x + c^2_y - 2\rho c_x c_y], \quad (9)$$

$$bias(\bar{y}_{reg}) = 0, \quad (10)$$

and

$$mse(\bar{y}_{reg}) = \left(\frac{1-f}{n}\right)\bar{Y}^2[c^2_y(1-\rho^2)]. \quad (11)$$

The alternative ratio-regression type estimator is given as:

$$\bar{y}_{aa^*} = \alpha\left(\frac{\bar{y}}{\bar{x}}\right)(\bar{X} + \rho) + \beta[\bar{y} + b(\bar{X} - \bar{x})], \quad (12)$$

linear combination of classical ratio estimator,  $\bar{y}_r^*$ , and the traditional regression estimator,  $\bar{y}_{reg}$ , where  $\bar{x} = \bar{X}(1 + \Delta_{\bar{x}})$ ,  $\bar{y} = \bar{Y}(1 + \Delta_{\bar{y}})$ ,  $\Delta_{\bar{y}} = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ ,  $\Delta_{\bar{x}} = \frac{\bar{x} - \bar{X}}{\bar{X}}$  such that  $|\Delta_{\bar{y}}| < 1$  and  $|\Delta_{\bar{x}}| < 1$ .

The bias and mean square error of this alternative ratio-regression type estimator,  $\bar{y}_{aa^*}$  are respectively given as:-

$$bias(\bar{y}_{aa^*}) = \left(\frac{1-f}{n}\right)\left(\frac{\bar{X} + \rho}{\bar{X}}\right)\alpha\bar{Y}(c^2_x - \rho c_x c_y) \quad (13)$$

and

$$mse(\bar{y}_{aa^*}) = \left(\frac{1-f}{n}\right)\left(\frac{\bar{X} + \rho}{\bar{X}}\right)^2\alpha^2\bar{Y}^2[c^2_x + c^2_y - 2\rho c_x c_y], \quad (14)$$

provided

$$\frac{\bar{X} + \rho}{\bar{X}} \geq 1. \quad (15)$$

The proof of (13) and (14) are as shown below:

### Derivation of Bias and Mean Square Error of $\bar{y}_{aa}$ .

Let

$$\begin{aligned} \bar{y}_{aa^*} &= \alpha\left(\frac{\bar{y}}{\bar{x}}\right)(\bar{X} + \rho) + \beta[\bar{y} + b(\bar{X} - \bar{x})], \text{ where, } \bar{x} = \bar{X}(1 + \Delta_{\bar{x}}), \bar{y} = \bar{Y}(1 + \Delta_{\bar{y}}), \\ \Delta_{\bar{y}} &= \frac{\bar{y} - \bar{Y}}{\bar{Y}}, \Delta_{\bar{x}} = \frac{\bar{x} - \bar{X}}{\bar{X}} \text{ such that } |\Delta_{\bar{y}}| < 1 \text{ and } |\Delta_{\bar{x}}| < 1 \text{ as earlier defined.} \end{aligned}$$

Therefore, using power series expansion

$$\begin{aligned} \bar{y}_{aa^*} &= \alpha\left(\frac{\bar{y}}{\bar{x}}\right)(\bar{X} + \rho) + \beta[\bar{y} + b(\bar{X} - \bar{x})] = \alpha\left(\frac{\bar{y}}{\bar{x}}\right)(\bar{X} + \rho) + (1 - \alpha)[\bar{y} + b(\bar{X} - \bar{x})], \\ &= \alpha\left(\frac{\bar{Y}(1 + \Delta_{\bar{y}})}{\bar{X}(1 + \Delta_{\bar{x}})}\right)(\bar{X} + \rho) + (1 - \alpha)[\bar{Y}(1 + \Delta_{\bar{y}}) + b(\bar{X} - \bar{X}(1 + \Delta_{\bar{x}}))], \\ &= \alpha\left(\frac{\bar{Y}}{\bar{X}}(1 + \Delta_{\bar{y}})\right)(1 + \Delta_{\bar{x}})^{-1}(\bar{X} + \rho) + \bar{Y}(1 + \Delta_{\bar{y}}) + b\bar{X} - b\bar{X} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} + \\ &\quad ab\bar{X} - ab\bar{X} - ab\Delta_{\bar{x}} \\ &= \alpha\left(\frac{\bar{Y}}{\bar{X}}\right)(\bar{X} + \rho)(1 + \Delta_{\bar{y}})(1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}}, \\ &= \alpha\bar{Y}\left(\frac{\bar{X} + \rho}{\bar{X}}\right)(1 + \Delta_{\bar{y}})(1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}}, \\ &= \alpha\bar{Y}k(1 + \Delta_{\bar{y}})(1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}}, \end{aligned}$$

$$\text{where } k = \left(\frac{\bar{X} + \rho}{\bar{X}}\right),$$

$$= \alpha\bar{Y}k(1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2 + \Delta_{\bar{y}} - \Delta_{\bar{x}}\Delta_{\bar{y}} + \Delta_{\bar{x}}^2\Delta_{\bar{y}}) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}},$$

$$\begin{aligned} Bias(\bar{y}_{aa^*}) &= E[((\alpha\bar{Y}k(1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2 + \Delta_{\bar{y}} - \Delta_{\bar{x}}\Delta_{\bar{y}} + \Delta_{\bar{x}}^2\Delta_{\bar{y}})) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}}) - \bar{Y}], \end{aligned}$$

$$\begin{aligned} Bias(\bar{y}_{aa^*}) &= E[(\alpha\bar{Y}k - \alpha\bar{Y}k\Delta_{\bar{x}} + \alpha\bar{Y}k\Delta_{\bar{x}}^2 + \alpha\bar{Y}k\Delta_{\bar{y}} - \alpha\bar{Y}k\Delta_{\bar{x}}\Delta_{\bar{y}} + \alpha\bar{Y}k\Delta_{\bar{x}}^2\Delta_{\bar{y}} + \bar{Y} + \\ &\quad \bar{Y}\Delta_{\bar{y}} - b\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\Delta_{\bar{y}} - ab\Delta_{\bar{x}}) - \bar{Y}], \end{aligned}$$

$$Bias(\bar{y}_{aa^*}) = E(\alpha \bar{Y} k \Delta_{\bar{x}}^2 - \alpha \bar{Y} k \Delta_{\bar{x}} \Delta_{\bar{y}}).$$

Let,  $E(\Delta_{\bar{y}}) = E(\Delta_{\bar{x}}) = 0$ ,  $E(\Delta_{\bar{x}}^2) = \frac{S_x^2}{\bar{X}^2} = c_x^2$ ,  $E(\Delta_{\bar{y}}^2) = \frac{S_y^2}{\bar{Y}^2} = c_y^2$  and

$$E(\Delta_x \Delta_y) = \frac{S_{xy}}{\bar{X}\bar{Y}} = \rho c_x c_y. \text{ Then}$$

$$Bias(\bar{y}_{aa^*}) = \frac{(1-f)}{n} \alpha \bar{Y} k [c_x^2 - \rho c_x c_y],$$

$$bias(\bar{y}_{aa^*}) = \left( \frac{1-f}{n} \right) \left( \frac{\bar{X} + \rho}{\bar{X}} \right) \alpha \bar{Y} (c_x^2 - \rho c_x c_y) \quad (\text{as in eq. (13) above}),$$

$$Mse(\bar{y}_{aa^*}) = E[(\alpha \bar{Y} k ((1 - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2 + \Delta_{\bar{y}} - \Delta_{\bar{x}} \Delta_{\bar{y}} + \Delta_{\bar{x}}^2 \Delta_{\bar{y}}) + \bar{Y} + \bar{Y} \Delta_{\bar{y}} - b \Delta_{\bar{x}} - \alpha \bar{Y} - \alpha \Delta_{\bar{y}} - \alpha b \Delta_{\bar{x}}) - \bar{Y}]^2,$$

$$Mse(\bar{y}_{aa^*}) = E(\alpha \bar{Y} k (\Delta_{\bar{y}} - \Delta_{\bar{x}}))^2,$$

$$Mse(\bar{y}_{aa^*}) = \left( \frac{1-f}{n} \right) \left( \frac{\bar{X} + \rho}{\bar{X}} \right)^2 \alpha^2 \bar{Y}^2 [c_x^2 + c_y^2 - 2\rho c_x c_y], \quad (\text{as in eq. (14) above}).$$

## COMPARISON

### Efficiency of $\bar{y}_{aa^*}$ over $\bar{y}_{kc}$ .

The proposed alternative ratio-regression type estimator,  $\bar{y}_{aa^*}$ , is said to be better and more efficient than Kadilar and Singh (2004) estimator,  $\bar{y}_{kc}$ , whenever  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{kc})$ . That is,

$$\begin{aligned} \left( \frac{1-f}{n} \right) \left( \frac{\bar{X} + \rho}{\bar{X}} \right)^2 \alpha^2 \bar{Y}^2 [c_x^2 + c_y^2 - 2\rho c_x c_y] &< \left( \frac{1-f}{n} \right) \bar{Y}^2 [c_x^2 + c_y^2 (1 - \rho^2)] \text{ provided} \\ \alpha &\leq \sqrt{\frac{\left( \frac{\bar{X}}{\bar{X} + \rho} \right)^2 [c_x^2 + c_y^2 (1 - \rho^2)]}{(c_y^2 + c_x^2 - 2\rho c_x c_y)}}. \end{aligned} \quad (16)$$

### Efficiency of $\bar{y}_{aa^*}$ over $\bar{y}_{ek}$ .

The proposed alternative ratio-regression type estimator,  $\bar{y}_{aa^*}$ , is said to be better and more efficient than Ekaete *et al.* (2014) estimator,  $\bar{y}_{ek}$ , whenever  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{ek})$ . That is,

$$\left(\frac{1-f}{n}\right)\left(\frac{\bar{X}+\rho}{\bar{X}}\right)^2 \alpha^2 \bar{Y}^2 [c_x^2 + c_y^2 - 2\rho c_x c_y] < \left(\frac{1-f}{n}\right) \bar{Y}^2 [c_y^2 (1-\rho^2)] \text{ provided,}$$

$$\alpha \leq \sqrt{\frac{\left(\frac{\bar{X}}{\bar{X}+\rho}\right)^2 c_y^2 (1-\rho^2)}{(c_y^2 + c_x^2 - 2\rho c_y c_x)}}. \quad (17)$$

### 3. Results and Discussion

#### Data used

Four data sets used by Kadilar and Singh (2004), and Ekaete *et al.* (2014) are also used here to compare the efficiency of this alternative ratio-regression type estimator with Kadilar and Singh (2004), and Ekaete *et al.* (2014) estimators.

**Table 1: Data used**

**Population I:-** Kadilar and Cingi (2004).

$$c_x = 2, c_y = 15, \bar{X} = 25, \bar{Y} = 500, \rho = 0.90, n = 50, N = 200 \text{ and } \frac{\bar{X} + \rho}{\bar{X}} = 1.03.$$

**Population II:-** Kadilar and Cingi (2003).

$$c_x = 2.02, c_y = 4.18, \bar{X} = 24375.59, \bar{Y} = 1536.77, \rho = 0.82, n = 20, N = 106 \text{ and}$$

$$\frac{\bar{X} + \rho}{\bar{X}} = 1.00000336.$$

**Population III:-** Murthy (1967).

$$c_x = 0.7507, c_y = 0.3542, \bar{X} = 11.2624, \bar{Y} = 51.8264, \rho = 0.9413, n = 20, N = 80 \text{ and}$$

$$\frac{\bar{X} + \rho}{\bar{X}} = 1.08358.$$

**Population IV:-** Das (1988).

$$c_x = 1.6198, c_y = 1.4451, \bar{X} = 25.111, \bar{Y} = 39.068, \rho = 0.7213, n = 48, N = 278 \text{ and}$$

$$\frac{\bar{X} + \rho}{\bar{X}} = 1.02872446.$$

#### Results

The estimates obtained using the above data sets are shown in Table 2 below:

**Table 2:-** Bias and Mean Square Error obtained on  $\bar{y}_{aa^*}$ ,  $\bar{y}_{kc}$  and  $\bar{y}_{ek}$ 

Population	I	II	III	IV
bias( $\bar{y}_{kc}$ )	-167.512	254.3749	1.095255	1.766793
bias( $\bar{y}_{aa^*}$ )	-178.70	-177.2624	0.6597	0.6479
$\sqrt{\frac{(\frac{\bar{X}}{\bar{X} + \rho})^2 c_y^2 (1 - \rho^2)}{(c_y^2 + c_x^2 - 2\rho c_y c_x)}}$	0.4771	0.8619	0.2542	0.8435
mse( $\bar{y}_{kc}$ )	775,312.50	939,289.60	58.20311	95.38072
mse( $\bar{y}_{aa^*}$ ) when $\alpha = 1.0$	704,350.0	738,242.23	22.2844	37.1750
0.9	570,523.50	597,976.2063	18.0504	30.1118
0.8	450,784	<b>472,475</b>	14.2620	<b>23.7920\</b>
0.7	345,131.50	<b>361,738.6927</b>	10.9194	<b>18.2158</b>
0.6	253,566	<b>265,767.2028</b>	8.0224	<b>13,383</b>
0.5	176,087.50	<b>184,560.5575</b>	5.5711	<b>9.1750</b>
0.4	<b>112,696</b>	<b>118,118.7568</b>	3.5655	<b>5.9480</b>
0.3	<b>63,391.50</b>	<b>66,441.8007</b>	2.0056	<b>3.3458</b>
0.2	<b>28,174</b>	<b>29,529.6892</b>	<b>0.8914</b>	<b>1.4870</b>
0.1	<b>7,043.50</b>	<b>7,382.4223</b>	<b>0.2228</b>	<b>0.3718</b>
mse( $\bar{y}_{ek}$ )	160,312.50	548,359.8951	1.4348	26.3544
$(\frac{\bar{X} + \rho}{\bar{X}})$	1.07329	1.0000336	1.0836	1.0287
$(\frac{\bar{X} + \rho}{\bar{X}})^2$	1.060	1.00006728	1.1741889	1.05827402

## Discussion

From the estimates in the Table 2 above for:

(a). when  $\alpha = 1$ ,

- (i). For Population I,  $mse(\bar{y}_{kc}) = 775,312.50$ ,  $mse(\bar{y}_{aa^*}) = 704,350$  and  $mse(\bar{y}_{ek}) = 160,312.50$ . Hence,  $mse(\bar{y}_{ek}) < mse(\bar{y}_{aa^*}) < mse(\bar{y}_{kc})$ .
- (ii). For Population II,  $mse(\bar{y}_{kc}) = 939,289.60$ ,  $mse(\bar{y}_{aa^*}) = 738,242.23$  and  $mse(\bar{y}_{ek}) = 548,359.8951$ . Hence,  $mse(\bar{y}_{ek}) < mse(\bar{y}_{aa^*}) < mse(\bar{y}_{kc})$ .
- (iii). For Population III,  $mse(\bar{y}_{kc}) = 58.20311$ ,  $mse(\bar{y}_{aa^*}) = 22.2844$  and  $mse(\bar{y}_{ek}) = 1.4340$ . Hence,  $mse(\bar{y}_{ek}) < mse(\bar{y}_{aa^*}) < mse(\bar{y}_{kc})$ .
- (iv). For Population IV,  $mse(\bar{y}_{kc}) = 95.38072$ ,  $mse(\bar{y}_{aa^*}) = 37.1750$  and  $mse(\bar{y}_{ek}) = 26.3544$ . Hence,  $mse(\bar{y}_{ek}) < mse(\bar{y}_{aa^*}) < mse(\bar{y}_{kc})$ .

$$(b). \text{ When } \alpha \leq \sqrt{\frac{\left(\frac{\bar{X}}{\bar{X} + \rho}\right)^2 c_y^2 (1 - \rho^2)}{(c_y^2 + c_x^2 - 2\rho c_y c_x)}},$$

- (i). For Population I,  $\alpha \leq 0.4771$ ,  $mse(\bar{y}_{aa^*}) = 112,696$ ,  $mse(\bar{y}_{kc}) = 775,312.50$ , and  $mse(\bar{y}_{ek}) = 160,312.50$ . Hence,  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{ek}) < mse(\bar{y}_{kc})$ .
- (ii). For Population II,  $\alpha \leq 0.8619$ ,  $mse(\bar{y}_{aa^*}) = 472,475$ ,  $mse(\bar{y}_{kc}) = 939,289.60$ , and  $mse(\bar{y}_{ek}) = 548,359.8951$ . Hence,  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{ek}) < mse(\bar{y}_{kc})$ .
- (iii). For Population III,  $\alpha \leq 0.2542$ ,  $mse(\bar{y}_{aa^*}) = 0.8914$ ,  $mse(\bar{y}_{kc}) = 58.20311$ , and  $mse(\bar{y}_{ek}) = 1.4340$ . Hence,  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{ek}) < mse(\bar{y}_{kc})$ .
- (iv). For Population IV,  $\alpha \leq 0.8435$ ,  $mse(\bar{y}_{aa^*}) = 23.7920$ ,  $mse(\bar{y}_{kc}) = 95.38072$ , and  $mse(\bar{y}_{ek}) = 26.3544$ . Hence,  $mse(\bar{y}_{aa^*}) < mse(\bar{y}_{ek}) < mse(\bar{y}_{kc})$ .

#### 4. Conclusion

Therefore, from all the estimates obtained above, at  $\alpha = 1$ , Ekaete *et al.* (2014) was found to

be better but when  $\alpha \leq \sqrt{\frac{\left(\frac{\bar{X}}{\bar{X} + \rho}\right)^2 c_y^2 (1 - \rho^2)}{(c_y^2 + c_x^2 - 2\rho c_y c_x)}}$ , the alternative ratio-regression type

estimator was said to be more efficient than both Kadilar and Cingi (2004), and Ekaete *et al.* (2014) estimators based on their estimated mean square error (mse). Hence, the alternative ratio-regression type estimator,  $\bar{y}_{aa^*}$ , is recommended for usage in sample survey.

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