Ilorin Journal of Science
Volume 2, Number 1, 2015, pp. 48 – 67 (Printed in Nigeria)
ISSN: 2408 – 4840 © 2015 Faculty of Physical Sciences, University of Ilorin
https://doi.org/10.54908/iljs.2015.02.01.005

ILORIN
JOURNAL OF SCEINCE

ILJS-15-011

Effects of Partial Slippage and Couple stresses on Entropy Generation in a porous Channel filled with Highly Porous Medium

Gbadeyan[1]*, J.A., Yusuf[1], T. A., Dada[1], M. S. and Akinremi[1], J.O.

[1]Department of Mathematics, University of Ilorin, Ilorin, Nigeria.

Abstract

This work investigates the influence of slip condition and couple stresses on entropy generation rate in a steady flow of an incompressible viscous fluid through a porous channel occupied by a highly porous medium with suction/injection. It is assumed that the "no-slip" condition at one of the walls of the channel is no longer valid for both velocity and temperature. Also the porous medium is the non-Darcian type known as the Darcy-extended Brinkman-Forchheimer model. Semi-Analytical solutions of the dimensionless momentum and energy equations are obtained using Differential transform method (DTM). The approximate solutions for velocity and temperature are used to compute the Entropy generation rate, Bejan number and the Irreversibility distribution ratio in the flow field. The variation of the velocity and temperature fields are examined for various values of couple stresses parameter, slip parameter, Brinkman number, entropy generation number, Darcy number, Nusselt number, skin fiction and other parameters. It is found that each of these parameters has significant effect on the velocity, temperature and Entropy generation rate profiles.

Keywords: Couple stresses, Slip condition, Entropy generation rate, porous medium, Differential transform method.

1. Introduction

The theory of fluids for which the shear stress depends on the shear rate has many practical applications in industries and modern technology. For instance, they could be used to describe the rheological properties of complex fluid (Adesanya and Makinde, 2012). This fluid such as, paints, coal tar, and polymer solutions are known as Non-Newtonian. One type of such fluids which is of a particular interest is the couple stress fluid. Couple stress fluid theory may be defined as a direct extension of the classical theory of viscous Newtonian fluid

*Corresponding Author: Gbadeyan, J. A.
Email: j.agbadeyan@yahoo.com

for which the body couples and corresponding couple stresses in the in the fluids are continuously retained (Devakar *et al.*, 2014).

For instance (Devakar *et al.*, 2014) obtained exact solution of couette, poiseuille and generalized couette flows of an incompressible couple stresses fluid between parallel plates with slip boundary conditions. Also in (Adesanya and Makinde, 2012) the effect of radiative heat transfer to oscillatory magneto-hydrodynamic couple stress fluid flow through a porous channel was investigated. An analysis of an unsteady Non-Newtonian flow of an incompressible couple stress fluid was carried out in (Eldabe *et al.*, 2003). The flow of couple stress fluid between two parallel porous plates is also studied in (Mohammadyari *et al.*, 2014). Moreover, it was assumed for a long time in several works on both Newtonian and Non-Newtonian fluids that the velocity of the fluid close to the surface of the solid boundary assumes the velocity of the solid boundaries.

However it has been found that for many applications such assumptions no longer remain valid. For instance, the works of (Adesanya and Gbadeyan, 2011; Ellahi, 2009; Adesanya and Makinde, 2014; Eegunjobi and Makinde, 2012; Khalid and Vafia, 2014) to mention a few, showed the occurrence of slippage at the solid boundary and that such effects can no longer in general be discarded.

Entropy production occurs frequently in industrial and engineering flow systems. It deals with the consumption of power due to thermodynamics losses in such flow systems. To maximize power generation in such system one needs to minimize entropy production. A basic engineering problem during convective heat transfer in a fluid flow is that of having efficient energy utilization. Hence it becomes paramount to study entropy generation and the combined influence of slippage, porous medium, wall suction or injection and couple stresses on it. In a related works, (Eegunjobi and Makinde, 2012) studied combined effects of suction/injection and Navier slip on the entropy generation rate in a steady flow of an incompressible viscous fluid through a porous channel subjected to non-uniform temperature at the wall.

Some other interesting investigations on entropy generation include (Eegunjobi and Makinde, 2012; Srinivasacharya *et al.*, 2010; Ajibade *et al.*, 2011; Adesanya and Makinde, 2014; Adesanya and Makinde, 2015). Furthermore in fluid flow through porous channel and/or media is of great importance both in biophysical and technological flows Applications are found in food preservations, blood flow transpiration cooling, cosmetic industry, soil mechanic and haemodialysis. Several models involving both Darcian and non Darcian abound. For example, (Dada and Disu, 2014) investigated heat transfer with radiation and temperature dependent heat source in MHD free convective flow in a porous medium between two vertical wavy walls. The effect of slip condition on forced convection and entropy generation in a circular channel occupied by non Darcian highly porous medium was investigated in (Chauhan and Kumar 2009).

Other investigations can be found in (Eegunjobi and Makinde, 2012; Eegunjobi and Makinde 2012; Chinyoka and Makinde, 2013; Bejan, 1982; Bejan, 1995; Makinde and Eegunjobi, 2013). However, it seems that the literature lacks investigations that take into account the combined effects of temperature slip, couple stress, velocity slip, non-Darcy porous medium on entropy generation in a porous channel with suction/injection which this paper considered.

The aim of the present study is, therefore, to investigate the combined effects of couple stresses, non-Darcy porous parameters and slippage of velocity and temperature on the entropy generation rate of an incompressible fluid through a porous channel filled with highly porous medium having suction/injection. By employing the appropriate non-dimensional variables, the governing non-linear boundary-value problem is transformed into its dimensionless form. The resulting set of differential equations and the boundary conditions are then solved using a semi-analytic technique known as differential transform method DTM (Ayaz F, 2004; Mohammadyari, 2014). Results are presented graphically and discussed for the Nusselt number, Skin friction entropy generation rate, velocity distribution as well as temperature distribution.

The rest of the paper is organised in terms of four sections. Section two is devoted for presenting the formulation of the governing equations. This is followed by the solution of the governing equations in section three. The results are discussed in section four while conclusions are given in the last section.

2.0 Mathematical Model

Consider the steady flow of couple stress non Newtonian fluid flowing through a porous channel filled by a highly porous medium under the combined action of constant axial pressure gradient, with wall suction/injection and slip.

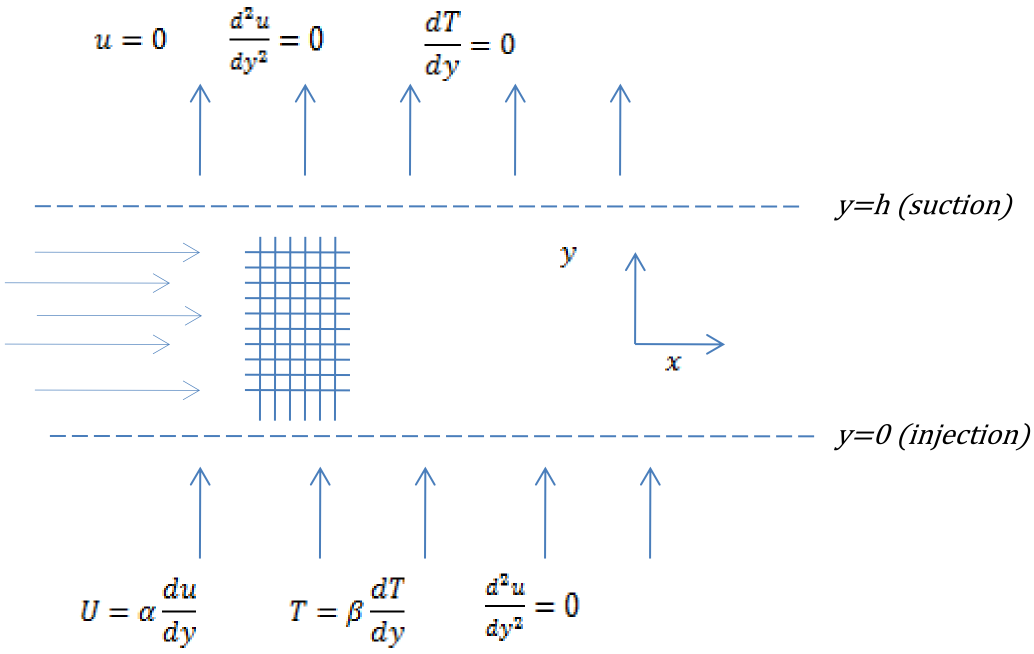

Fig.1 Schematic Diagram of the problem

The flow is subjected to injection of the fluid at the lower plate and sucked off at the upper plate with the same velocity. The wall plates are assumed to be subjected to the exchange heat in an asymmetrical manner with the ambient temperature. Under these assumptions the governing equations for the momentum and heat balance following (Chauhan and Kumar, 2009; Adesanya and Makinde, 2014) can be written as

$$\rho v_0 \frac{du'}{dy'} = -\frac{dp}{dx'} + \mu \frac{d^2u'}{dy^2} - \eta \frac{d^4u'}{dy'^4} - \frac{\mu}{k_0} u' - \frac{\rho b}{k_0} u'^2, \tag{1}$$

$$\rho C_p v_0 \frac{dT'}{dy'} = k \frac{d^2T'}{dy^2} + \mu \left(\frac{du'}{dy'}\right)^2 + \eta \left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\mu}{k_0} u'^2. \tag{2}$$

The entropy generation is also given as

$$E_G = \frac{k}{{T_0}^2}\left(\frac{dT'}{dy'}\right)^2 + \frac{\mu}{T_0}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_0}\left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\mu}{T_0 k_0} u'^2. \tag{3}$$

The corresponding boundary conditions are (Chauhan and Kumar, 2009)

$$u' = \alpha \frac{du'}{dy'}, \frac{d^2u'}{dy'^2} = 0, T = \beta \frac{dT'}{dy'} \quad on\ y = 0,$$

$$u' = 0 = \frac{d^2u'}{dy'^2}, \frac{dT'}{dy'} = 0 \quad on\ y = h. \tag{4}$$

In these equations u' is the axial velocity, μ is the dynamics viscosity, ρ is the fluid density, T is the fluid temperature, T_0 is the initial fluid temperature, T_f is the final fluid temperature, k is the thermal conductivity of the fluid, k_0 is the porous media permeability, ρ is the fluid density, C_p is the specific heat at constant pressure, v_0 is the constant velocity of fluid suction/injection η is the fluid particle size due to couple stress, k_0 is the porous permeability, b is the Forchheimer geometrical inertia parameter of the medium, α' *and* β' are the non-dimensionalized velocity slip and temperature slip coefficient respectively, E_G is the local volumetric entropy generation rate.

The following dimensionless variables are introduced for non-dimensionalizing the governing equations

$$y = \frac{y\prime}{h}\; u = \frac{u\prime}{v_0} \quad Da = \frac{k_0}{h^2} \quad G = \frac{h}{\mu v_0}\frac{dp}{dx\prime} \quad a^2 = \frac{\mu h^2}{\eta},$$
$$F = \frac{b}{h}, s = \frac{v_0 h}{v}\;, Pr = \frac{v\rho C_p}{\mu v_0}, Br = \frac{\mu {v_0}^2}{k(T_f - T_0)}, \alpha = \frac{\alpha\prime}{h},$$

$$\beta = \frac{\beta\prime}{h}, v = \frac{\mu}{\rho}, N_s = \frac{T_0^2 h^2 E_G}{K(T_f - T_0)^2}, \Omega = \frac{\mu v_0^2}{k(T_f - T_0)}\;. \tag{5}$$

Substituting (5) into equations (1) - (4), we obtained

$$s\frac{du}{dy} - G - \frac{d^2u}{dy^2} + \frac{1}{a^2}\frac{d^4u}{dy^4} + \frac{1}{Da}u + \frac{sF}{Da}u^2 = 0, \tag{6}$$

$$sPr\frac{d\theta}{dy} = \frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 + \frac{Br}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 + \frac{Br}{Da}u^2, \tag{7}$$

$$N_S = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 + \frac{Br}{\Omega a^2}\left(\frac{d^2u}{dy^2}\right)^2 + \frac{Br}{\Omega Da}u^2. \tag{8}$$

The corresponding boundary conditions are:

$$u = \alpha\frac{du}{dy}\,, \frac{d^2u}{dy^2} = 0\,, \theta = \beta\frac{d\theta}{dy} \qquad \text{on } y = 0,$$
$$u = 0 = \frac{d^2u}{dy^2}\,, \frac{d\theta}{dy} = 0 \qquad \text{on } y = 1\,. \tag{9}$$

In equations (6) – (9), u is the dimensionless velocity, s is the suction/injection parameter, θ is the dimensionless temperature, a^2 is the couple stress parameter, α and β are the dimensionalized velocity slip and temperature slip coefficient Pr is the prandtl number, Br is the Brinkman number, Ω is the parameter that measure the temperature difference between the two heat reservoirs, Da is the Darcy number and F is the Forchheimer inertia number, N_s is the dimensionless entropy generation rate and Be is the Bejan number.

Denoting N_{s1} and N_{s2} as the entropy generation due to heat transfer and entropy generation due viscous dissipation respectively, then

$N_s = N_{s1} + N_{s2}$,

where

$N_{s1} = \left(\frac{d\theta}{dy}\right)^2, \ N_{s2} = \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 + \frac{Br}{\Omega a^2}\left(\frac{d^2u}{dy^2}\right)^2 + \frac{Br}{\Omega Da}u^2.$

We also obtain the Bejan number Be, the ratio of N_{s1} to the total entropy generation rate, as $Be = \frac{N_{s2}}{N_s} = \frac{1}{1+\Phi}, \Phi = \frac{N_{s1}}{N_{s2}}.$

While the Skin friction and the Nusselt number are given as $Sf = \frac{du}{dy}$ and $Nu = \frac{d\theta}{dy}$.

It is stated, at this junction that the work of (Adesanya and Makinde, 2014) is completely recovered in the asymptotic limit as$k_0 \to \infty$. This implies that the work (Adesanya and Makinde, 2014) is extended in this paper by including the effect of porous medium. On the other hand, the boundary conditions hereby adopted equation (9) is similar to those in (Chauhan and Kumar, 2009).

3.0 Differential Transform Method

Taking the differential transform of (6)-(8) using the following table:

Table 1: Some operations of DTM

Original function	Transformed Function
$u(y) = g(y) \pm h(y)$	$U(k) = G(k) \pm H(k)$
$u(y) = \lambda g(k)$	$U(k) = \lambda G(k)$
$u(y) = g(k)h(y)$	$U(k) = \sum_{l=0}^{k} G(l)H(k-l)$
$u(y) = \frac{d^n g(k)}{dy^n})$	$U(k) = \frac{(k+n)!}{k!} G(k+n)$
$u(y) = y^n$	$U(k) = \delta(k+n) = \begin{cases} 0, & k \neq n \\ 1, & k = n \end{cases}$

We have

$U(k+4) = \frac{a^2}{(k+1)(k+2)(k+3)(k+4)}\Big\{G\delta(k) + (k+1)(k+2)U(k+2) - s(k+1)U(k+1) - \frac{1}{Da}U(k) - \frac{sF}{Da}\left(\sum_{r=0}^{k} U(r)U(k-r)\right)\Big\}$, (10)

$$\theta(k+2) = \frac{1}{(k+1)(k+2)}\Big\{sPr(k+1)\theta(k+1) - Br\left(\sum_{r=0}^{k}(r+1)U(r+1)U(k-r+1)\right) - \frac{Br}{a^2}\left(\sum_{r=0}^{k}(r+1)(r+2)U(r+2)(k-r+2)(k-r+1)U(k-r+2)\right) - \frac{Br}{Da}\left(\sum_{r=0}^{k} U(r)U(k-r)\right)\Big\}. \quad (11)$$

The transformed boundary conditions are

$$U(0) = \alpha.A\,, U(1) = A, U(2) = 0, U(3) = B, \theta(0) = \beta.C, \theta(1) = C, \quad (12)$$

$$U(1) = 0\,, \sum_{r=0}^{k} k(k-1)U(k) = 0\,, \sum_{r=0}^{k} k\theta(k) = 0, \quad (13)$$

where A, B, and C are constants. $U\,(k)\ and\ \theta(k)$ for $k = 0{,}1{,}2{,}3 \ldots 12$ values can now be evaluated in terms of $a = 1, F = 1, s = 1, G = 1, Da = 0.1, \alpha = 0.1, \beta = 0.1, Br = 1, Pr = 0.71, A, B\ and\ C$. Specifically these values were obtained by using mathematical software (MAPLE).

$$U(4) = \frac{a^2}{24}\left[G - sA - \frac{\alpha A}{Da} - \frac{sF\alpha^2A^2}{Da}\right],$$

$$U(5) = \frac{a^2}{120}\left[6B - \frac{A}{Da} - \frac{2sF\alpha A^2}{Da}\right],$$
$$U(6) = \frac{a^2}{360}\left[\frac{a^2}{2}\left(G - sA - \frac{\alpha A}{Da} - \frac{sF\alpha^2A^2}{Da}\right) - 3sB - \frac{sFA^2}{Da}\right], \quad (14)$$

and so on.

$$\theta(2) = \frac{1}{2}\left[sPrC - \frac{Br\alpha^2A^2}{Da}\right],$$
$$\theta(3) = \frac{1}{3}\left[\left(\frac{sPrC}{2} - \frac{Br\alpha^2A^2}{2Da}\right) - \frac{Br\alpha^2A^2}{Da}\right],$$
$$\theta(4) = \frac{sPr}{4}\left[\frac{sPr}{3}\left(\frac{sPr}{2}C - \frac{1}{2}\frac{Br\alpha^2A^2}{Da}\right) - \frac{1}{3}\frac{Br\alpha A^2}{Da}\right] - 3\frac{BrB^2}{a^2} - \frac{BrAB}{2} - \frac{BrA^2}{12Da},$$

and so on.

Substituting the $U(i) and\ \theta(i), i = 0, 1, 2, 3$ ...terms into the following inverse formulae

$$u(y) = \sum_{i=0}^{k} U(k)\ and\ \theta(y) = \sum_{i=0}^{k}\theta(i) \quad (15)$$

gives the finite series solution of the velocity and temperature distribution respectively.

$$u(y) = \alpha A + Ay + By^3 + \frac{a^2}{24}\left[G - sA - \frac{\alpha A}{Da} - \frac{sF\alpha^2A^2}{Da}\right]y^4 + \frac{a^2}{120}\left[6B - \frac{A}{Da} - \frac{2sF\alpha A^2}{Da}\right]y^5 + \cdots$$

$$\theta(y) = \beta C + Cy + By^2 + \frac{1}{3}\left[\left(\frac{sPrC}{2} - \frac{Br\alpha^2A^2}{2Da}\right) - \frac{Br\alpha^2A^2}{Da}\right]y^3 + \left[\frac{sPr}{4}\left(\frac{sPr}{3}\left(\frac{sPr}{2}C - \frac{1}{2}\frac{Br\alpha^2A^2}{Da}\right) - \frac{1}{3}\frac{Br\alpha A^2}{Da}\right) - 3\frac{BrB^2}{a^2} - \frac{BrAB}{2} - \frac{BrA^2}{12Da}\right]y^4 + \cdots$$

To obtain the constants $A, B\ and\ C$, the second boundary conditions, equation (13) are used [21] and taking $a = 1, F = 1, s = 1, G = 1, Da = 0.1, \alpha = 0.1, \beta = 0.1, Br = 1, Pr = 0.71$ as the default thermo physical parameters, three set of linear equations were obtained and solved simultaneously. The values of A, B and C were obtained as $A = 0.3097026195, B = -0.6928834300, C = 0.004210192473$. Therefore substituting the above defaults parameter and constants A, B and C into the above finite series solutions, the desired analytical solutions for velocity and temperature are obtained as follows:

$$u(y) = 0.003097026195 + 0.03097026195y - 0.06928834300y^3 + 0.03908181501y^4 - 0.006061258265y^5 + 0.001853486772y^6 + 0.0004995509058y^7 - 0.0001573871673y^8 + 0.00001542353062y^9 - 0.00001492352798y^{10} + 6.357748068 * 10^{-6}y^{11} - 2.010194184 * 10^{-6}y^{12} + \cdots, \tag{16}$$

$$\theta(y) = 0.0004210192473 + 0.004210192473y + 0.001446660472y^2 + 0.000226572699y^3 - 0.01412496029y^4 + 0.01722167610y^5 - 0.008681157272y^6 + 0.003891074507y^7 - 0.1994468458y^8 + 0.5875135754y^9 - 0.0001331545464y^{10} - 0.00001004213352y^{11} + 0.00001496100212y^{12} + \cdots \tag{17}$$

Equations (16) and (17) were then substituted into equation (8) to calculate the rate of entropy generation and we obtained

$$Ns = 0.001072798559 + 0.001942677127\,y + 0.1341607538y^6 - 0{:}3850237664\,y^3 + 0.3219697943\,y^4 + 0.1695567272\,y^2 - 0.06334359851\,y^7 + 0.02899447856\,y^8 + 0.007145604001\,y^{10} - 0.0000000002556061638\,y^{23} - 0.000003781637034\,y^{18} + 0.000001749358700\,y^{19} - 0.0000003728930123\,y^{20} - 0.00000004555514843\,y^{21} + 0.0000003381782831\,y^{22} - 0.004593137071\,y^{11} - 0.001466348060\,y^{13} + 0.0006285495479\,y^{14} - 0.0002151035418\,y^{15} + 0.00005161935258\,y^{16} - 0.000003348834325\,y^{17} + 4.040880657 * 10^{11}\,y^{24} - 0.01187162222\,y^9 - 0{:}2003548538\,y^5 + 0.002755923091\,y^{12} + \cdots \tag{18}$$

Table 2: $Convergence\ result\ for\ Pr = 0.71, Br = 1, a = 1, G = 1, Da = 0.1, \alpha = 0.1, \beta = 0.1, s = 1, y = 0.1$

n	u_n	$\sum_{n=0}^{m} u_n$	θ_n	$\sum_{n=0}^{m} \theta_n$
0	0.003097026194	0.003097026194	0.0004210192473	0.0004210192473
1	0.030970261940	0.006194052388	0.004210192473	0.0008420384916
2	0.00000000000	0.006194052388	0.001446660472	0.0008565050963
3	−0.06928834300	0.006124764045	0.000022657269	0.0008565277536
4	0.03908181502	0.006128672227	−0.01412496029	0.0008551152576
5	−0.006061258263	0.006128611614	0.01722167610	0.0008552874744
6	0.001853486772	0.006128613467	−0.008681157272	0.0008552787932
7	0.0004995509058	0.006128613517	0.003891074506	0.0008552791823
8	−0.000157387167	0.006128613515	−0.001994468458	0.0008552791624
9	0.00001542353062	0.006128613515	0.0005875135756	0.000855279163
10	−0.0000149235279	0.006128613515	−0.000133154546	0.000855279163
11	6.357748068 × 10 − 6	0.006128613515	−0.000010042133	0.000855279163
12	−2.01019418 × 10 − 6	0.006128613515	0.0000149610021	0.000855279163

Table 3: Comparison of the semi-analytical DTM Solution of the velocity u with previous corresponding ADM Solution [2], when G = 1, s = α = 0.1

y	u_{Exact}	u_{ADM}	u_{DTM}	$Absolute\ error$
0	0.0030316	0.0030316	0.0030316	$1.04453432448609 \times 10^{-10}$
0.1	0.00599037	0.00599037	0.00599037	$2.072959390947426 \times 10^{-10}$
0.2	0.00854451	0.00854451	0.00854451	$3.016574681019568 \times 10^{-10}$
0.3	0.0103768	0.0103768	0.0103768	$3.78986992994574 \times 10^{-10}$
0.4	0.0112628	0.0112628	0.0112628	$4.504766092200207 \times 10^{-10}$
0.5	0.0110683	0.0110683	0.0110683	$4.4712230966303394 \times 10^{-10}$
0.6	0.00974853	0.00974853	0.00974853	$4.196588371946364 \times 10^{-10}$
0.7	0.00734757	0.00734757	0.00734757	$3.387224424555857 \times 10^{-10}$
0.8	0.00399891	0.00399891	0.00399891	$1.959608300280968 \times 10^{-10}$
0.9	−0.0000730536	−0.0000730536	−0.0000730536	$1.18590777474465 \times 10^{-11}$
1	−0.00455071	−0.00455071	−0.00455071	$2.70713184399651 \times 10^{-10}$

4.0 Discussion of Results

In this work, the effects of slip conditions and couple stresses fluid on entropy generation rate in a steady flow of an incompressible viscous fluid through a porous channel occupied by a highly porous medium with suction/injection were studied. A semi-analytical solution is obtained for velocity profile, temperature profile and entropy generation. It is important to note that the fluid suction takes place at the upper wall while the fluid injection takes place at the lower wall simultaneously. Following (Chauhan and kumar 2009; Adesanya and Makinde, 2014) the default values for the thermo-physical parameter are $a = 1, F = 1, s = 1, G = 1, Da = 0.1, \alpha = 0.1, \beta = 0.1, \Omega = 1, Pr = 0.71$.

The DTM approximate solutions to the problem under consideration are in the convergent series of equations (16) and (17). It is confirmed in Table 1 above that the series solutions for the velocity and temperature profile are convergent and hence reliable. A comparison with previously obtained velocity result in (Adesanya and Makinde, 2014) is carried in Table 2 above. It is observed that the DTM results of the velocity agreed totally with the exact result in (Adesanya and Makinde, 2014). The observation confirms the uniqueness of the solution.

Fig. 2 shows the effect of couple stresses on the flow velocity. As observed from the graph, an increase in the inverse of the couple stress parameter'a', leads to an increase in the flow velocity at the lower channel. The velocity profile attains its maximum at the centre-line region and becoming zero at the suction wall.

Fig. 3 depicts the effect of suction/injection on the velocity flow. It is observed that an increase in suction/injection parameter $'s'$, the fluid injection into the channel through the lower wall of the channel increases, while the rate of fluid suction at the upper wall of the channel increases as well. This leads to a decrease in the fluid velocity at the lower channel wall and an increase in flow reversal at the upper wall region. In Fig. 4, the influence of slip parameter 'α' on velocity profile is illustrated. It is found that an increase in slip parameter at the lower wall causes an increase in the velocity at the wall, while the velocity increases slightly at the suction wall, which gives a clear effect on the fluid.

Fig. 5 depicts effect of Forchheimer number on velocity profile. It is observed that an increase in Forchheimer number has an increasing influence on the maximum value of the velocity. Fig. 6 depicts effect of the Darcy number $'Da'$ on velocity profile. It is seen that velocity profile increases by increasing the Darcy number $'Da'$. Effect of couple stress on temperature profile is shown in Fig. 7 it is found that an increase in couple stress inverse 'a' leads to an increase in the temperature profile across the channel. Fig. 8 depicts the effects of suction/injections on temperature profile. It is found that as the suction/injection parameter, s, increases the fluid temperature decreases. This shows that as less fluid is injected into the channel the fluid temperature increases.

Fig. 9 shows the effect of Prandtl number on fluid temperature. It is observed that when Prandtl number increases the fluid temperature decreases across the channel. The temperature profile when there is an increment in Brinkman number $'Br'$ due to viscous dissipation effects is shown in Fig. 10. The fluid temperature increases with an increase in Br with minimum value at the injection wall and maximum value at the suction wall. Fig. 11 describes the effect of slip parameter $'\beta'$ on temperature profile. It is seen that increasing the slip parameter $'\beta'$ enhances the fluid temperature. The effect of couple stress inverse $'a'$ on the entropy generation rate within the channel is depicted in Fig. 12.

It is found that an increase in couple stress inverse 'a' leads to a decrease in the entropy generation rate at the upper plate while the entropy generation rate is slightly decreases at the injection wall. Fig. 13 shows the effect of fluid suction/injection on the entropy generation rate. The result shows that as the suction/injection parameter, s, increases, the entropy generation rate slightly increase at the injection wall and increases throughout the channel. Fig. 14 deals with the effect of slip coefficient 'α' on entropy generation rate. It is found that when the slip coefficient α increases, the entropy generation rate increases throughout the channels. Figs. 15 and 16 show the variation of entropy generation rate with the Darcy number and Forchheimer parameter respectively. It is observed that an increase in Forchheimer number causes an increase in the entropy generation rate at the suction wall and a slight increase at the injection wall while an increase in Darcy number reduces the entropy generation rate.

The effects of various thermo-physical parameters on the Bejan number are depicted in Figs. 17 – 20. In Fig. 17, it is observed that increase in couple stress inverse $'a'$ decreases the Bejan number at the injection wall. Fig. 18 shows the effect of fluid suction/injection parameter on Bejan number. It is observed that as the suction/injection parameter increases, the Bejan number decreases at the injection wall. Fig. 19 demonstrates the effect of slip parameter α on the Bejan number. It is found that the Bejan number at the injection wall decreases while it increases slightly at the suction wall with an increase in $'\alpha'$. Furthermore it is observed in Fig. 20 that the Bejan number increases with increasing Darcy number parameter.The plot of the variation of the skin friction with respect to velocity slip coefficient α is depicted in Fig 21. It is observed that as the slip parameter increases the skin friction decreases. Figs. 22 and 23, depict, the plot of Nusselt number versus couple stress inverse $'a$' and Prandtl number Pr respectively. It shows that increase in couple stress inverse 'a' increases Nusselt number while Prandtl number number decreases it.

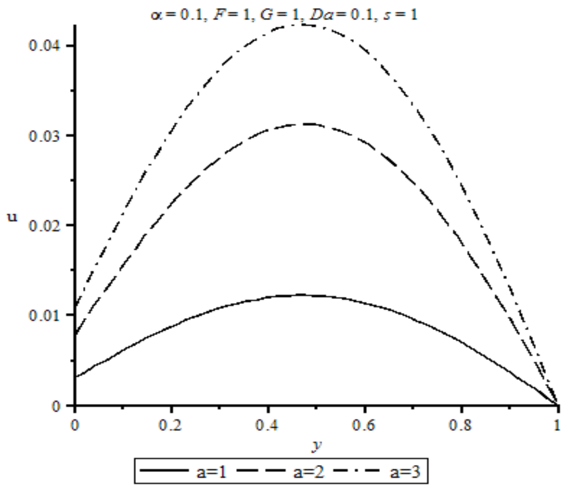

Fig 2: Variation of couple stress inverse on velocity profile

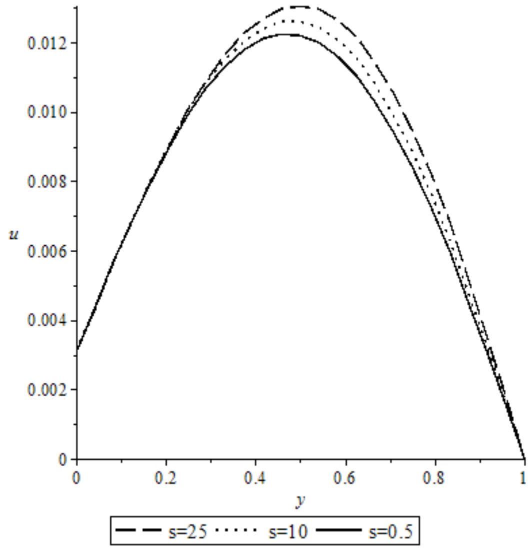

Fig 3: Variation of suction/injection parameter on velocity profile

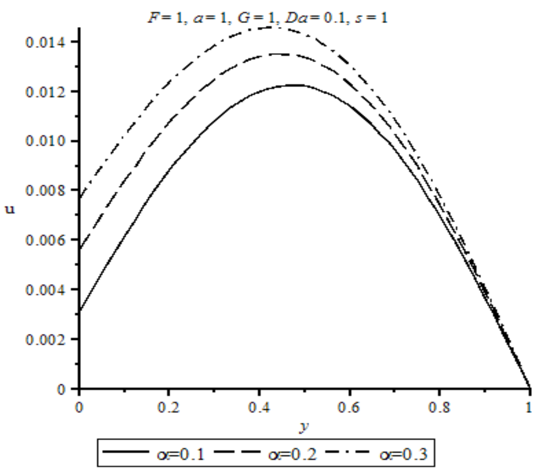

Fig 4: Variation of slip parameter on velocity temperature distribution profile

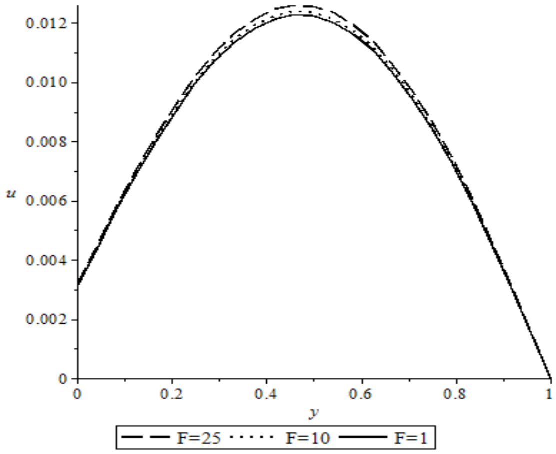

Fig 5: Variation of Forchheimer parameter on velocity profile

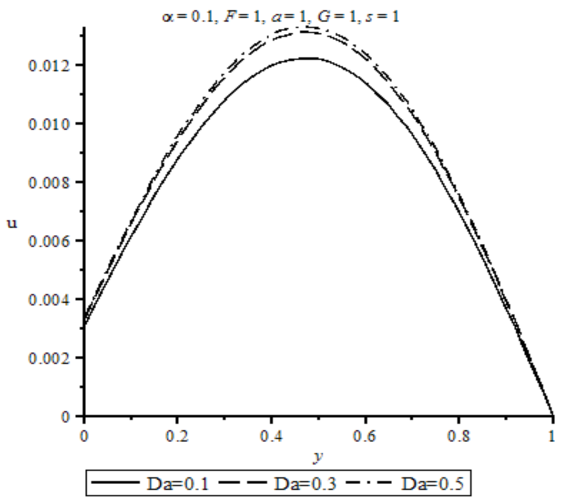

Fig 6: Variation of Da on velocity profile

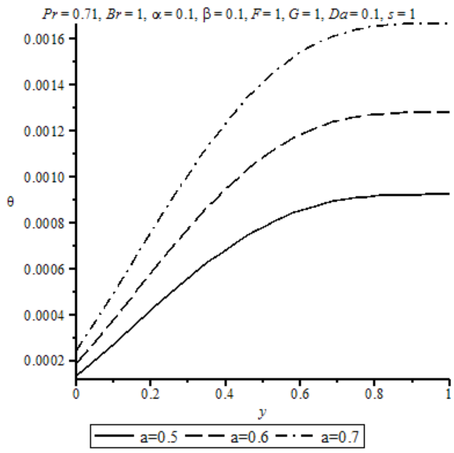

Fig 7: Variation of couple stresses parameter on temperature distribution

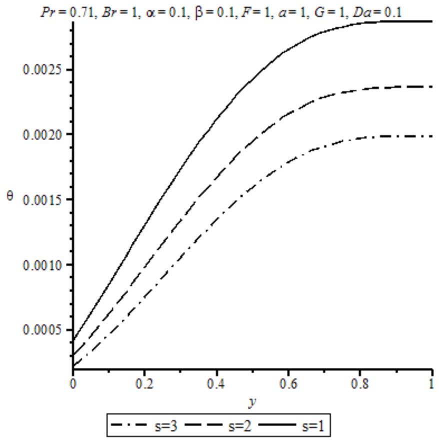

Fig 8: Variation of suction/injection parameter on temperature distribution

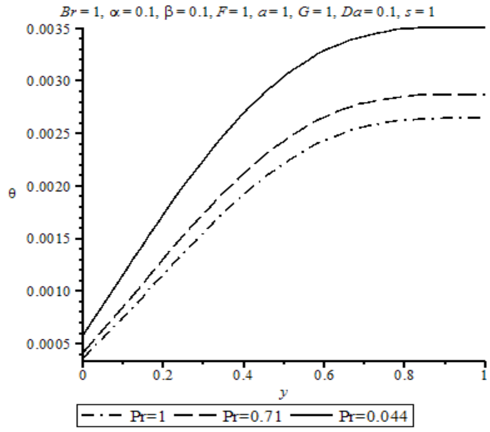

Fig 9: Variation of Prandtl number on temperature

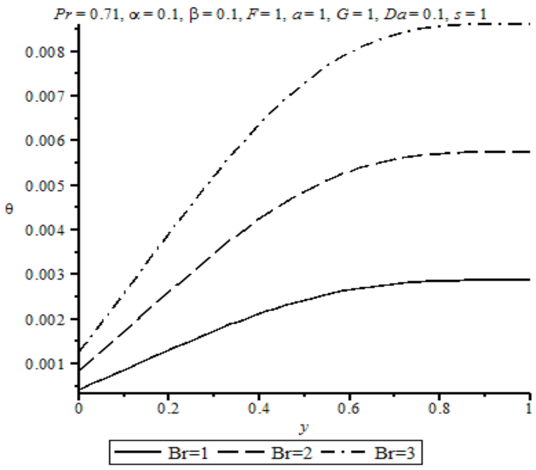

Fig 10: Variation of Brinkman number on temperature distribution

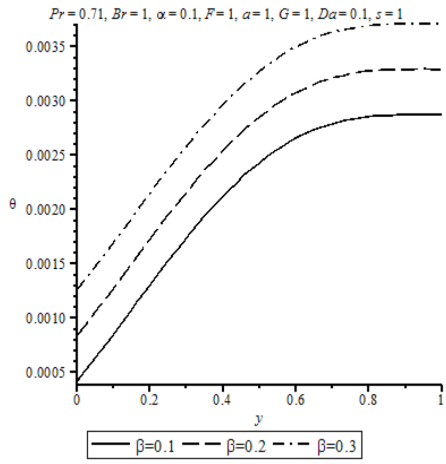

Fig 11: Variation of slip parameter on temperature distribution

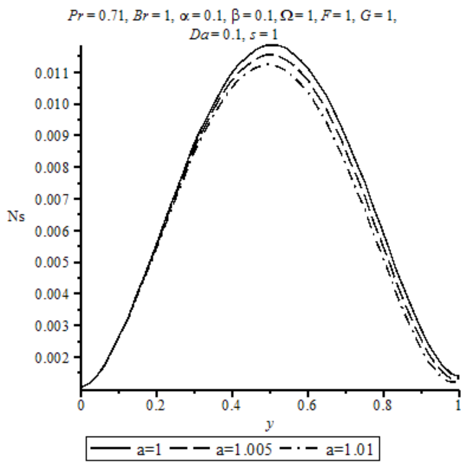

Fig 12: Variation of couple stress inverse on entropy generation rate

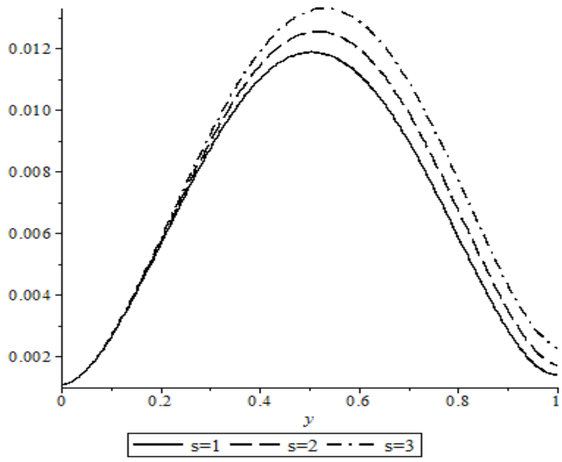

Fig 13: Variation of suction/injection on entropy generation rate

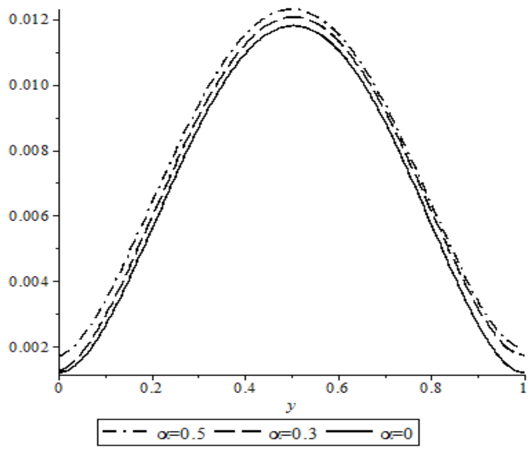

Fig 14: Variation of slip parameter 'α' on entropy generation rate

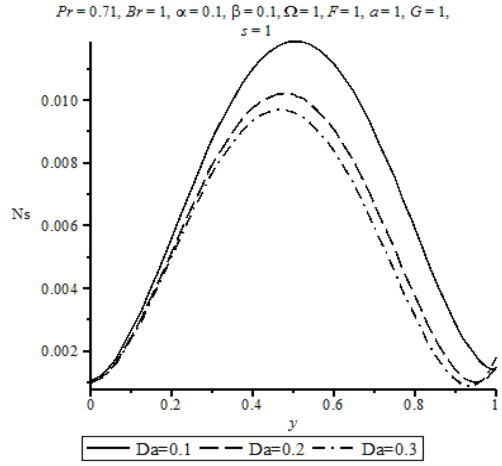

Fig 15: Variation of Darcy number on entropy

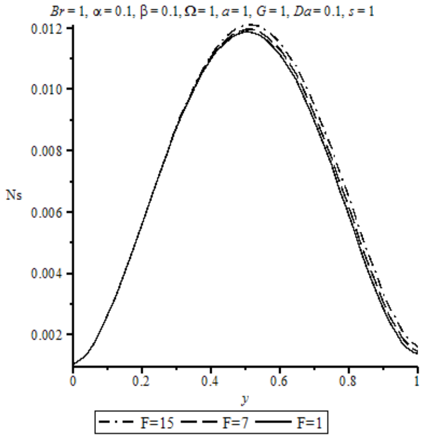

Fig 16: Variation of Forchheimer number on entropy generation rate

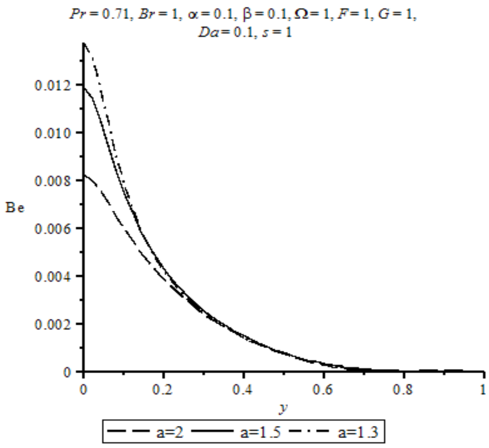

Fig 17: Variation of couple stress parameter on Bejan number

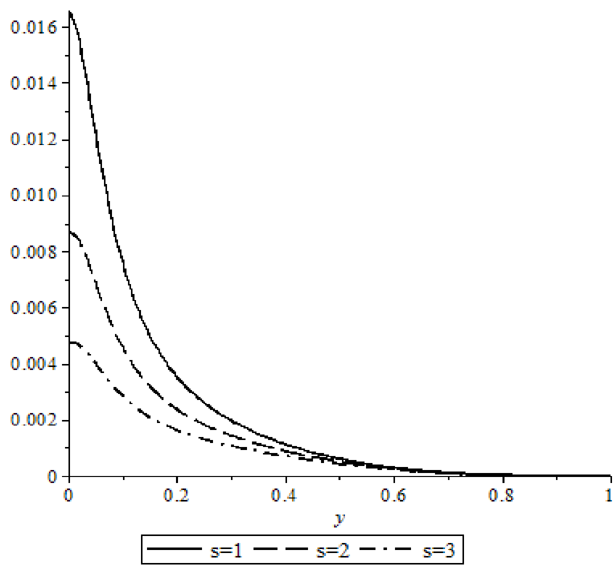

Fig 18: Variation of suction/injection on Bejan number

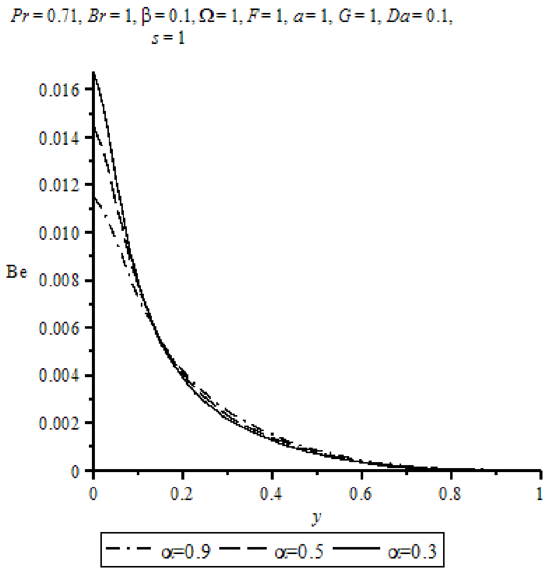

Fig 19: Variation of slip parameter 'α' on Bejan number

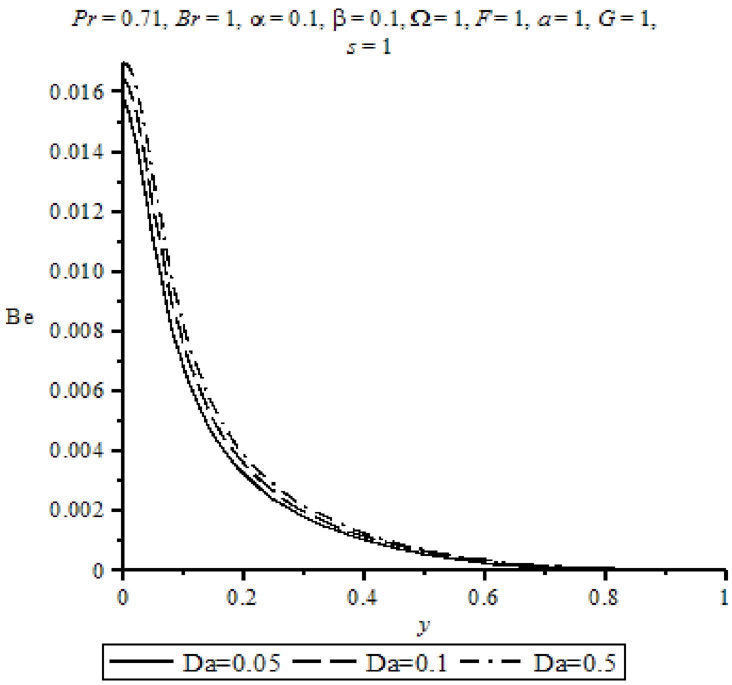

Fig 20: Variation of 'Da' on Bejan number

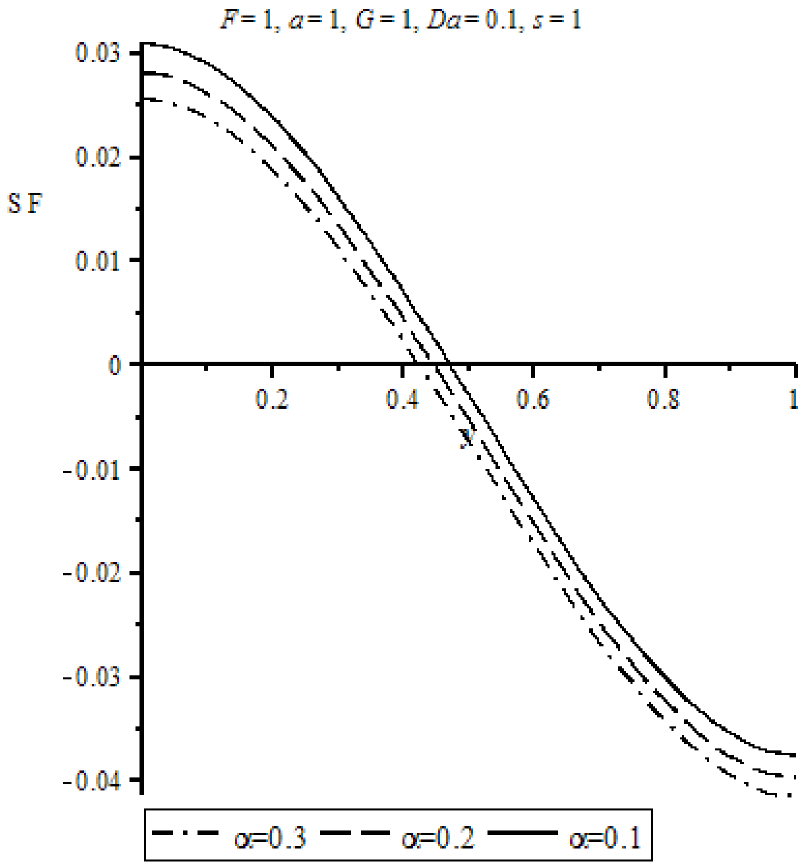

Fig 21: Variation of slip parameter on skin friction

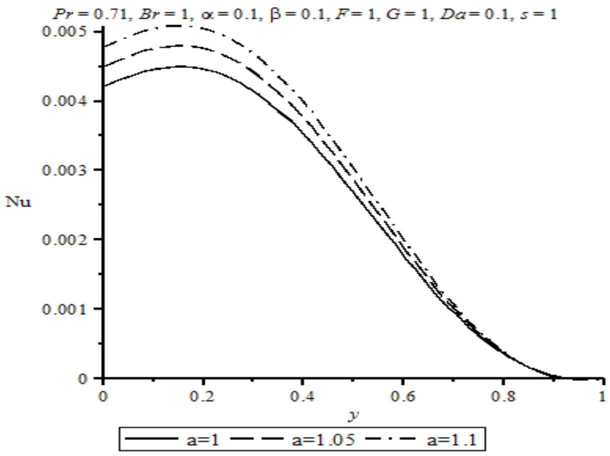

Fig 22: Variation of couple stress parameter on Nusselt number

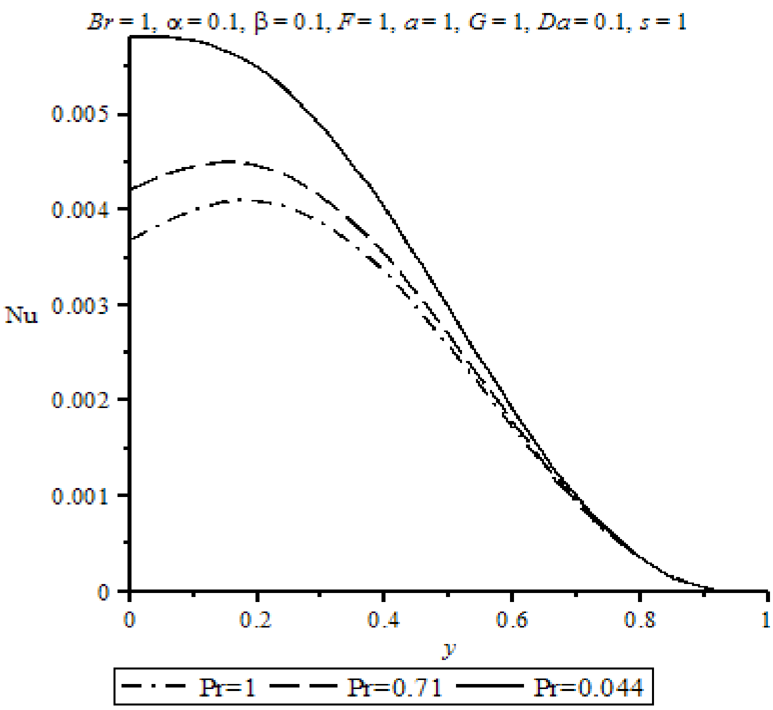

Fig 23: Variation of prandtl number on Nusselt number

5.0 Conclusion

The effects of slip conditions and couple stress fluid on entropy generation rate in a porous channel occupied by a highly porous medium were examined. The following observations and conclusions are drawn

- The effect of slip parameter is to increase the temperature and velocity profile.

- The presence of couple stress inverse $'a'$ also has an increasing effect on the fluid velocity and fluid temperature.
- Increase in slip coefficient increases the entropy generation rate within the channel.
- Increase in couple stress inverse $'a'$ decreases the Bejan number at the injection wall and a slight variation occurs at the suction wall.

References

Adesanya, S.O, Makinde, O.D. (2014): Entropy generation in couple stress fluid flow through porous channel with fluid slippage. *Inter. J. Energy*, **15** (3), 344-362.

Adesanya, S.O, Makinde, O.D. (2015): Effects of couple stresses on entropy generation rate in a porous channel with convective heating. *Comp. Appl. Math.* **34** 293-307.

Adesanya, S.O, Makinde, O.D. (2012): Heat transfer to Magneto hydrodynamic Non-Newtonian couple stress pulsatile flow between two parallel porous plates. *Z Naturforsch*. **67a**, 647-656.

Adesanya, S.O, Gbadeyan, J.A. (2011): Adomian Decomposition Approach to steady visco-elastic fluid Flow with slip through a planer Channel. *Inter. J. Nonlinear Sci.* **11** (1), 86-94.

Ajibade, A.O. Jha, B.K and Omame, A (2011): Entropy Generation Under the effects of suction/injection. *Appl. Math. Mod.* **35**, 4630-4046.

Ayaz, F. (2004): Application of differential transform method to differential-algebraic equations, *Appl. Math. & Comp*. **152**, 649-657.

Bejan, A. (1982): *Entropy generation through heat and fluid flow*. Wiley, New-York.

Bejan, A. (1995): *Entropy generation Minimization.* CRC press, New-York.

Chauhan, D. S. and Kumar, V. (2009): Effect of slip condition on forced convection and entropy generation in a circular channel occupied by highly porous medium: Darcy extended Brinkmann-Forchheimer model. *Turk. J. Eng. Env. Sci*. **33**, 91-104.

Chinyoka, T. Makinde, O.D. (2013): Analysis of entropy generation rate in an unsteady Porous channel flow with Navier slip and convective cooling. *Entropy*. **15**, 2081-2099.

Dada, M. S. and Disu, A.B. (2015): Heat transfer with radiation and temperature dependent heat source in MHD free convection flow in a porous medium between two vertical wavy walls. *J. Math. Soc*. **34**(2), 200-215.

Davakar, M. Sreenivasu, D. and Shankar, B. (2014): Analytical solution of couple stress fluid flows with slip boundary conditions. *Alexandria Eng. J*. (In press).

Eegunjobi, A.S. and Makinde, O. D. (2012): Combined effect of Bouyancy force and Navier slip on entropy generation in a vertical porous channel. *Entropy,* **14**, 1028-1044

Eegunjobi, A. S. and Makinde, O. D. (2012): Effect of Navier slip on entropy generation in a porous channel with suction/injection. *J. Therm. Sci. Tech*. **7**, 522-535

Eldabe, N .T. and Mohammed, M. A. A. (2003): Effect of couple stresses on the M.H.D of Non-Newtonian Unsteady flow between two parallel porous plates. *Z. Naturforsch* **58a**, 204-210.

Ellahi, R. (2009): Effects of the slip boundary condition on non-newtonian flows in a channel. *Comm. Nonlinear Sci. Num. Sim*. **15**, 1377-1384.

Khalid, A. R. and Vafai, A.K. (2004): The effect of the slip condition on stokes and coquette flows due to an oscillatory wall: exact solutions, *Inter. J. Non-linear Mesh*. **39**, 795-809.

Makinde, O. D. and Eegunjobi, A. S. (2013): Effects of convective heating on entropy generation rate in a channel with permeable walls. *Entropy*. **15**, 220-233.

Makinde, O.D. and Eegunjobi, A.S. (2013): Entropy generation in a couple stress fluid flow through a vertical channel filled with saturated porous media. *Entropy*. **15**, 4589-4606.

Makinde, O. D. and Osalusi, E. (2006): Entropy generation in a liquid film falling along an inclined porous heated plate. *Mech. Res. Comm*. **33**, 692-698.

Mohammadyari, R. and Rahimi-Esbo, M, and Khalili, A. A. (2014): Differential Transformation Method to determine Magneto Hydrodynamics flow of compressible fluid in a channel with porous walls. *Bol. Soc. Paran. Mat*. **32**, 249-261.

Srinivasacharya, D. and Srinivasacharyulu, N. D. and Odelu, O. (2010): Flow of couple stress fluid between two parallel porous plates. *Inter. J. Appl. Math*. **41**, 41- 48.