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Unsteady Permeable Magnetohydrodynamic (MHD) Radiating Fluid Flow Bounded by an Oscillating Porous Plate with Slip Flow Condition and Heat Generation.

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Abstract

The problem of unsteady Magnetohydrodynamic (MHD) radiating fluid flow through a porous medium bounded by an oscillating porous plate with slip flow condition with heat generation was investigated. Boundary layer equations were derived and the resulting approximate non-linear ordinary differential equations were solved analytically using perturbation technique. The numerical results reveal that the radiation induces a rise in velocity with a decrease in temperature. Also the heat generation increases at the points – 0.13, 0.5 and 0.13 and decreases at the point -0.1 and 0.1 together with an increase in temperature. Some of the physical parameters like Grashof number, radiation, Prandlt number and chemical reaction has an influence on velocity, temperature and Concentration

Keyword: Unsteady, Porous medium, MHD, Radiation, Heat and mass transfer

1. Introduction

The effect of thermal radiation is significant in some industrial application such as glass production, furnace design and in space technology application such as cosmical flight aerodynamics, rocket, propulsion system, plasma physics which operate at high temperature. Consequently, Chamkha (2003) studied the MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/ absorption and a chemical reaction numerically. Chamkha, *et al*. (2001) studied the radiation effect on the free convection flow past a semi-infinite vertical plate with mass transfer.

Hady *et al.* (2006) researched on the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Gnaneshwara and Bhaskar (2009) investigated the radiation and mass transfer effects on an unsteady MHD free convection flow past a

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heated vertical porous plate with viscous dissipation.

Kim and Fedorov (2004) studied Transient mixed radiative convection flow of a micro-polar fluid past a moving semi-infinite vertical porous plate while Vajravelu and Hadjinicolaou (1993) studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic, endothermic or transfer chemical reactions.

Hossain *et al.* (2004) investigated the problem of natural convection flowing along a vertical wavy surface with uniform surface temperature in the presence of heat generation/ absorption. In this direction, Alam *et al.* (2006) studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Abdus and Mohammed (2006) considered the thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. The importance of radiation in the fluid led Muthucumaraswamy and Chandrakala (2006) to study radiative heat and mass transfer effect on moving isothermal vertical plate in the presence of chemical reaction. Muthucumaraswamy and Senthih (2004) considered a Heat and Mass transfer effect on moving vertical plate in the presence of thermal radiation.

In many chemical engineering processes, the chemical reaction do occur between a mass and fluid in which plate is moving. These processes take place in numerous industrial applications such as polymer production, manufacturing of ceramics or glassware and food processing. In the light of the fact that, the combination of heat and mass transfer problems with chemical reaction are of importance in many processes, and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously.

Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electricity is one in which electrical energy is extracted directly from a moving conducting fluid. Kumar and Gupta (2008) investigated the effect of variable permeability on unsteady two-dimensional free convective flow through a porous bounded by a vertical porous surface. Sharma *et al.* (2011) have studied the Influence of chemical reaction on unsteady MHD free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source. Muthucumaraswamy and Ganesan (2001) studied the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Mohammed (2009) studied double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects.

Soundalgakar (1972) have studied the viscous dissipative effects on unsteady free convective flow past a vertical porous plate with constant suction. Soundalgakar *et al.* (1979) considered the effect of mass transfer and free convection effect on MHD stokes problem for a vertical plate. Jimoh (2012) studied heat and mass transfer of magneto hydrodynamic (MHD) and dissipative fluid flow pass a moving vertical porous plate with variable suction.

Despite all these studies, the unsteady MHD for a heat generating fluid with thermal radiation and chemical reaction has little attention. Hence, the main objective of the present investigation is to study the effect of a second-order homogeneous chemical reaction, thermal radiation, heat source and dissipation on the unsteady MHD fluid flow past a vertical porous plate with variable suction. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field with oscillating free stream.

2. Materials and Methods

Mathematical Analysis

Consider unsteady two-dimensional hydromagnetic laminar, incompressible, viscous, electrically conducting and heat source past a semi-infinite vertical moving heated porous plate embedded in a porous medium and subjected to a uniform transverse magnetic field in the presence of thermal diffusion, chemical reaction and thermal radiation effects. According to the coordinate system, the x-axis is taken along the plate in upward direction and y-axis is normal to the plate. The fluid is assumed to be in a gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction (Chakha, *et al.*, 2001). It is assumed that there is absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium is also involved.

The MHD term is derived from an order-of-magnitude analysis of the full Navier-stokes equation. It is assumed here that the whole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is an approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been considered in the body-force. Since the plate is semi-infinite in length, therefore all physical quantities are functions of y and t only. Hence, by the usual boundary layer approximations, the governing equations for unsteady flow of a viscous incompressible fluid through a porous medium are:

Continuity equation

$$\frac{\partial v^*}{\partial y^*}=0\ . \qquad (2.1)$$

Momentum equation

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*}+v\frac{\partial^2 u^*}{\partial y^{*2}}+g\,\beta\,(T^*-T^*_\infty)+g\,\beta^*\,(C^*-C^*_\infty)+\frac{\sigma\beta_0^2}{\rho}(U^*-u^*)+\frac{v}{K^*(t^*)}(U^*-u^*). \qquad (2.2)$$

Energy equation

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\alpha\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{v}{C_P}\left(\frac{\partial u^*}{\partial y^*}\right)^2-\frac{1}{pC_p}\frac{\partial q_r}{\partial y^*}-\frac{Q_0}{pC_p}(T^*-T^*_\infty). \qquad (2.3)$$

Diffusion equation

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-k_r^{*2}\,(C^*-C^*_\infty). \qquad (2.4)$$

The boundary conditions for the velocity, temperature and concentration fields are:

$$\left.\begin{aligned} &u^*=U_o e^{-\omega i t}+L_1\frac{\partial u^*}{\partial y^*},\ T^*=T^*_\infty,\ C^*=C^*_\infty \ \text{ at } y=o,\\ &u^*\to o,\ T^*\to T^*_\infty,\ C^*\to C^*_\infty \quad \text{as } y^*\to\infty, \end{aligned}\right\} \qquad (2.5)$$

where L_1 =(2 - m_1)(L/ m_1), L = μ (π /2PP)$^{1/2}$ is the mean free path, and m_1 is Maxwell's reflection coefficient.

Where x and y are dimensions coordinates, u^* and v^* are dimensionless velocities, t^* is dimensionless time, T^* is the dimensional temperature, C^* is dimensional concentration, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of thermal expansion with concentration, ρ the density of the fluid, C_p is the specific heat at constant pressure, D is the species diffusion coefficient, k^* is the permeability of the porous medium, q_r is the radiation heat flux, Q_0 is the heat generation/absorption constant, k_r^2 is the chemical reaction parameter, B_O magnetic induction, v the kinematic viscosity, α is the thermal diffusivity, U_o is the scale of free stream velocity, T_w^* and C_w^* are wall dimensional temperature and concentration respectively, T_∞^* the free stream temperature far away from the plate, C_∞^* the free stream concentration in fluid far away from the plate, n^* the constant.

From the continuity equation (2.1), it is clear that the suction velocity normal to the plate is a function of time only and we shall take it in the form:

$$V^* \quad (t)' = - V_o' \, (1 + \epsilon \, \mathrm{A}e^{-\omega it}). \tag{2.6}$$

Let the medium between the plate be filled with a porous material of permeability:

$$K' \quad (t)' = - K_o' \, (1 + \epsilon \, \mathrm{A}e^{-\omega it}), \tag{2.7}$$

where A and B are real positive constant, ϵ and ϵ A are small less than unity, and V_o is a scale of suction velocity which has non- zero positive constant. Outside the boundary layer, equation (2) gives:

$$\rho \frac{dU_\infty^*}{dt^*} = \frac{\partial p^*}{\partial x^*} - \rho_\infty g - \frac{\mu}{K(t)'} U_\infty^* - \frac{\sigma}{\rho} \beta_O^2 U^* . \tag{2.8}$$

Eliminating $\frac{\partial p^*}{\partial x^*}$ between equations (2.2) and (2.8), we have

$$\rho \left(\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*}\right) = (\rho_\infty - \rho)\, g + \rho \frac{dU_\infty^*}{dt^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \sigma \beta_O^2 \, (U_\infty - u) + \frac{\mu}{K(t)'} (U_\infty^* - u^*) \tag{2.9}$$

by making use of the equation of state

$$\rho_\infty - \rho = \rho \, \beta \, (T^* - T_\infty^*) \tag{2.10}$$

and substituting equation (2.10) into equation (2.9), we obtain

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{dU_\infty^*}{dt^*} + v \frac{\partial^2 u^*}{\partial y^{*2}} + g \, \beta \, (T^* - T_\infty^*) + g \, \beta \, (C^* - C_\infty^*) + \sigma \beta_O^2 \, (U_\infty^* - u^*) + \frac{v}{K(t)'} (U_\infty^* - u^*), \tag{2.11}$$

where $v = \frac{\mu}{\rho}$ is the coefficient of the kinematic viscosity. The third term on the RHS of equation (2.11) denotes the body force due to non uniform temperature, the fourth is the bulk matrix linear resistance, i.e. Darcy term, and the fifth is the MHD term.

The radiative heat flux term by using the Roseland approximation is given by

$$q_{\omega}^{*} =-\frac{\partial \sigma^{*}}{3k_{r}^{*}}\left(\frac{\partial T^{*4}}{\partial y^{*}}\right)_{y=o}, \quad (2.12)$$

where σ^{*} is the Stefan-Boltzmann constant and k_{r}^{*} the mean absorption coefficient. It should be noted that by using the Roseland approximation the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficient small, then Equation (2.12) can be linearized by expanding T^{*4} in the Taylor series about T_{∞}^{*} , which after neglecting higher order terms takes the form

$$T^{*4} \cong 4T_{\infty}^{*3} - 3T^{*4} \quad (2.13)$$

$$q_{\omega}^{*} = -\left(\frac{16\sigma T_{\infty}^{*}}{3k_{r}^{*}}\right). \quad (2.14)$$

Substituting equation (2.14) into equation (2.3) gives

$$\frac{\partial T^{*}}{\partial t^{*}}+v^{*}\frac{\partial T^{*}}{\partial y^{*}} =\alpha\frac{\partial^{2} T^{*}}{\partial y^{*2}} + \frac{1}{pC_{p}}\frac{16\sigma_{s}}{3k_{e}}\ T_{\infty}^{*3}\frac{\partial^{2} T^{*}}{\partial y^{*2}}+ \frac{v}{C_{P}}\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2} + Q_{0}\ (T^{*}-T\ ). \quad (2.15)$$

3. METHOD OF SOLUTION

Introducing the following non-dimensional quantities

$$u=\frac{u^{*}}{U_{o}},\ y=\frac{V_{o}y^{*}}{v},\ t=\frac{V_{o}^{2}t^{*}}{v},\ n=\frac{vn^{*}}{V_{o}},\ U_{\infty}=\frac{U_{\infty}^{*}}{U_{o}},\ \theta=\frac{T^{*}-T_{\infty}^{*}}{T_{W}^{*}-T_{\infty}^{*}},\ C=\frac{C^{*}-C_{\infty}}{C_{W}-C_{\infty}},\ P_{r}=\frac{v\rho C_{p}}{k}=\frac{v}{\alpha},$$

$$Sc=\frac{v}{D},\ \mathrm{Gr}=\frac{(T_{W}-T_{\infty})}{U_{o}V_{o}^{2}},\ \mathrm{Gm}=g\beta^{*}v\frac{(C_{W}-C_{\infty})}{U_{o}V_{o}^{2}},\ \mathrm{Ec}=\frac{V_{o}^{2}}{C_{p}(T_{W}-T_{\infty})},\ k_{r}^{2}=\frac{k^{*2}rv}{V_{o}^{2}},\ \mathrm{R}=\frac{4\sigma_{S}T_{\infty}^{\prime 3}}{k_{e}k},$$

$$M=\frac{\sigma B_{o}^{2}uv}{\rho V_{o}^{2}},\ \eta=\frac{vQ_{o}}{V_{o}^{2}\rho C_{p}} \quad (2.16)$$

into the equations (2.4), (2.11) and (2.15) with equation (2.1) identically satisfied, we obtained the following set of differential equations:

$$\frac{\partial u}{\partial t} -(1+ \epsilon Ae^{-\omega it})\frac{\partial u}{\partial y} = \frac{dU_{\infty}}{dt} + \frac{\partial^{2} u}{\partial y^{2}} +\mathrm{Gr}\theta +\mathrm{GmC} - \left(\mathrm{M} + \frac{1}{k_{p}(1+\epsilon Be^{-\omega it})}\right)(U_{\infty} - u), \quad (2.17)$$

$$\frac{\partial \theta}{\partial t} - (1+ \epsilon Ae^{-\omega it})\frac{\partial \theta}{\partial y} = \frac{1}{P_{r}}\left(1+\frac{4R}{3}\right)\frac{\partial^{2}\theta}{\partial y^{2}} + \mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^{2} + \eta\theta, \quad (2.18)$$

$$\frac{\partial C}{\partial t} - (1+ \epsilon Ae^{-\omega it})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^{2} C}{\partial y^{2}} - k_{r}^{2}\mathrm{C}, \quad (2.19)$$

where u and v are dimensionless velocities, t is dimensionless time, ω is dimensionless frequency, ω^{*} is frequency, T_{W}^{*} and C_{W}^{*} are wall dimensional temperature and concentration respectively, T_{∞}^{*} the free stream temperature far away from the plate, C_{∞}^{*} the free stream concentration in fluid far away from the plate, n^{*} the constant, θ is dimensionless temperature function, C is dimensionless concentration function, U_{o} is the scale of free stream velocity, U_∞ the potential flow velocity, β the coefficient of thermal expansion, R_e is the Reynolds number, R is the radiation parameter, Pr is Prandtl number, U is velocity, Sc is Schimdt number, n is the frequency, M is the Hartmann number, K is the permeability parameter, Gr is thermal Grashof number and Gm is species Grashof number, η the heat source parameter, k_r^2 is the chemical reaction parameter and Ec is Eckert number, A is a real positive constant of suction velocity parameter , B is porosity parameter < ϵ, and ϵ A < 1 are small less than unity, i.e ϵ A <<1, V_o is a scale of suction velocity normal to the plate.

The boundary conditions (2.5) are given by the following dimensionless form.

$$y=0: \ u=e^{-\omega it} + R\frac{\partial u}{\partial y}, \theta=1, C=1; \ y \to \infty: u\to 0, \theta \to 0, C \to 0. \quad (2.20)$$

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, the velocities, momentum, temperature, free stream velocity and mass are represented as [12]:

$$u(y,t) = u_o(y) + \epsilon\, e^{-\omega it}\, u_1(y) + 0(\epsilon^2) + \ldots, \quad (2.21)$$

$$\theta(y,t) = \theta_o(y) + \epsilon e^{-\omega it}\ \theta_1(y) + 0(\epsilon^2) + \ldots, \quad (2.22)$$

$$C(y,t) = C_o(y) + \epsilon e^{-\omega it}\ C_1(y) + 0(\epsilon^2) + \ldots \quad (2.23)$$

and the free stream velocity is expressed as

$$U(t) = 1 + \epsilon e^{-\omega it}. \quad (2.24)$$

Substituting equations (2.21) - (2.24) into equations (2.17) - (2.19) and neglecting the coefficient of like powers of ϵ, we get the following set of differential equations.

$$u_o''(y) + u_o'(y) - \left(M + \frac{1}{k_p}\right) u_o(y) = \left(M + \frac{1}{k_p}\right) - Gr\theta_o(y) - GmC_o(y), \quad (2.25)$$

$$u_1''(y) + u_1'(y) - \left(M + \frac{1}{k_p} - \omega i\right) u_1(y) = -\left(M + \frac{1}{k_p} - \omega i - \frac{B}{k_p}\right) - Au_o'(y)$$
$$- Gr\theta_1(y) - Gm\, C_1(y) + \frac{B}{k_p}\ u_o, \quad (2.26)$$

$$(3 + 4R)\, \theta_o''(y) + 3Pr\theta_o'(y) - 3\, Pr\eta\theta_0(y) = -3\, Pr\, Ec\, (u_0')^2(y), \quad (2.27)$$

$$(3 + 4R)\, \theta_1''(y) + 3Pr\theta_1'(y) - 3\, Pr\eta\theta_1(y) - 3\, Prn\theta_1(y) = -3PrA\theta_o'(y) - 6PrEc(u'\_0(y) - u'\_1(y)), \quad (2.28)$$

$$C_o''(y) + Sc\ C_o'(y) - Sck_r^2\ C_0(y) = 0, \quad (2.29)$$

$$C_1''(y) + Sc\, C_1'(y) - Sc(\omega i + k_r^2)\, C_1(y) = -AScC_0'(y) \quad (2.30)$$

and the corresponding boundary conditions reduces to:

$$y=0: \ u_o = e^{-\omega it} + R\frac{\partial u_o}{\partial y}, u_1=0, \theta_o=1, \ \theta_1=1, C_o=1, C_o=1$$

$$\text{as } y \to \infty: u_o \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0. \quad (2.31)$$

In order to obtain the solutions of above coupled differential equations from (2.25) to (2.30), we expand u_o , u_1 , θ_o , θ_1 , C_o, C_1 in powers of Ecket number (Ec); assuming that it is very small.

$u_o(y) = u_{oo}(y)$ +Ec $u_{o1}(y)$ + 0(ϵ^2),
$u_1(y) = u_{1o}(y)$ +Ec $u_{11}(y)$ + 0(ϵ^2),
$\theta_o(y) = \theta_{oo}(y)$ +Ec $\theta_{o1}(y)$ + 0(ϵ^2),
$\theta_1(y) = \theta_{1o}(y)$ +Ec $\theta_{11}(y)$ + 0(ϵ^2),
$C_o(y) = C_{oo}(y)$ +Ec $C_{o1}(y)$ + 0(ϵ^2),
$C_1(y) = C_{1o}(y)$ +Ec $C_{11}(y)$ + 0(ϵ^2). (2.32)

Substituting equation (2.32) into equations (2.25) to (2.30) and equating the coefficients of like powers of Ec and neglecting the higher order terms of Ec, we obtain:

$$u''_{oo}(y) + u'_{oo}(y) - \left(\mathrm{M} + \frac{1}{k_p}\right) u_{oo}(y) = \left(\mathrm{M} + \frac{1}{k_p}\right) - \mathrm{Gr}\theta_{oo}(y) - \mathrm{Gm}\, C_{oo}(y), \tag{2.33}$$

$$u''_{o1}(y) + u'_{o1}(y) - \left(\mathrm{M} + \frac{1}{k_p}\right) u_{o1}(y) = - \mathrm{Gr}\theta_{o1} - \mathrm{Gm}\, C_{o1}, \tag{2.34}$$

$$u''_{10}(y) + u'_{10}(y) - \left(\mathrm{M} + \frac{1}{k_p} - \omega i\right) u_{10}(y) = -\left(\mathrm{M} + \frac{1}{k_p} - \omega i - \frac{B}{k_p}\right) - \mathrm{A}u'_{oo}(y)$$
$$- \frac{B}{k_p}\, u_{oo} - \mathrm{Gr}\theta_{10}(y) - \mathrm{Gm}\, C_{10}(y), \tag{2.35}$$

$$u''_{11}(y) + u'_{11}(y) - \left(\mathrm{M} + \frac{1}{k_p} - \omega i\right) u_{11}(y) = - \mathrm{A}u'_{o1}(y) - \frac{B}{k_p}\, u_{o1} - \mathrm{Gr}\theta_{11}(y) - \mathrm{Gm}\, C_{11}(y), \tag{2.36}$$

$$(3 + 4\mathrm{R})\, \theta''_{oo}(y) + 3\mathrm{Pr}\theta'_{oo}(y) - 3\, \mathrm{Pr}\eta\theta_{00}(y) = 0, \tag{2.37}$$
$$(3 + 4\mathrm{R})\, \theta''_{o1}(y) + 3\mathrm{Pr}\theta'_{o1}(y) - 3\, \mathrm{Pr}\eta\theta_{01}(y) = - 3\, \mathrm{Pr}(u'_{00})^2 , \tag{2.38}$$
$$(3 + 4\mathrm{R})\, \theta''_{10}(y) + 3\mathrm{Pr}\theta'_{10}(y) - 3\, \mathrm{Pr}\eta\theta_{10}(y) -3\, \mathrm{Pr}n\theta_{10}(y) = - 3\mathrm{Pr}A\theta'_{oo}(y) , \tag{2.39}$$
$$(3 + 4\mathrm{R})\, \theta''_{11}(y) +3\mathrm{Pr}\theta'_{11}(y) - 3\, \mathrm{Pr}(\eta+n)\theta_{11}(y) -3\, \mathrm{Pr}n\theta_{11}(y) = - 3\mathrm{Pr}A\theta'_{o1}(y) - 6\mathrm{Pr}(u'_{oo}(y)\, u'_{10}(y)), \tag{2.40}$$
$$\mathrm{C}''_{oo}(y) + \mathrm{Sc}\ \mathrm{C}'_{oo}(y) - \mathrm{Sc}k_r^2\ C_{00}(y) = 0 , \tag{2.41}$$
$$\mathrm{C}''_{o1}(y) + \mathrm{Sc}\ \mathrm{C}'_{o1}(y) - \mathrm{Sc}k_r^2\ C_{01}(y) = 0, \tag{2.42}$$
$$\mathrm{C}''_{10}(y) + \mathrm{Sc}\ \mathrm{C}'_{10}(y) - \mathrm{Sc}(\omega i + k_r^2)\, C_{10}(y) = - \mathrm{ASc}\mathrm{C}'_{00}(y), \tag{2.43}$$
$$\mathrm{C}''_{11}(y) + \mathrm{Sc}\ \mathrm{C}'_{11}(y) - \mathrm{Sc}(\omega i + k_r^2)\, C_{11}(y) = - \mathrm{ASc}\mathrm{C}'_{01}(y) \tag{2.44}$$

and with the corresponding boundary conditions:

y=0: u_{oo} = R $\frac{\partial u_{oo}}{\partial y}$, u_{oo} =1 + R $\frac{\partial u_{oo}}{\partial y}$, u_{10}=0 , u_{11}=0 θ_{oo}=1 , θ_{01}=0 , θ_{10}=1 , θ_{11}=0 ,

C_{oo}=1, C_{o1}=0 , C_{1o}=0 , C_{11}=0, (2.45)

$y \to \infty$: $u_{oo} \to 0$, $u_{01} \to 0$, $u_{10} \to 1$, $u_{11} \to 0$, $\theta_{00} \to 0$, $\theta_{01} \to 0$, $\theta_{10} \to 0$, $\theta_{11} \to 0$

$C_{00} \to 0$, $C_{01} \to 0$, $C_{10} \to 0$, $C_{11} \to 0$. (2.46)

The solutions of equations (2.33) - (2.44) subject to the boundary conditions (2.45) and (2.46) are respectively:

$U_{00}= Q_1\, e^{-b_8 y} + 1+ L_1\, e^{-b_2 y} + L_2\, e^{-b_4 y}$, (2.47)

$U_{01}= Q_2\, e^{-b_{10} y} + L_6\, e^{-b_{12} y} + L_7\, e^{-2b_8 y} +L_8\, e^{-2b_2 y} + L_9\, e^{-2b_4 y}$, (2.48)

$U_{10}= e^{-b_{18} y}$ (- C_2 - L_{12} - L_{13} - L_{14} - L_{15} - L_{16}) + C_2 + $L_{12}\, e^{-b_8 y}$ + $L_{13}\, e^{-b_2 y}$ + L_{14} $e^{-b_4 y}$ + $L_{15}\, e^{-b_{14} y}$ + $L_{16}\, e^{-b_{16} y}$, (2.49)

$u_{11} = e^{-b_{24} y}$ (- L_{33} - L_{34} - L_{35} - L_{36} - L_{37} - L_{38} - L_{39} - L_{40} - L_{41} - L_{42} - L_{43} - L_{44} - L_{45} - L_{46} - L_{47} - L_{48} - L_{49} - L_{50}) +$L_{33}\, e^{-b_{10} y}$ + $L_{34}\, e^{-b_{12} y}$ +$L_{35}\, e^{-2b_8 y}$ + $L_{36}\, e^{-2b_2 y}$ + $L_{37}\, e^{-2b_4 y}$ + $L_{38}\, e^{-b_{20} y}$ + $L_{39}\, e^{-(b_8 y+b_{18} y)}$ + $L_{40}\, e^{-(b_8 y+b_2 y)}$ + $L_{41}\, e^{-(b_8 y+b_4 y)}$ + L_{42} $e^{-(b_8 y+b_{14} y)}$ + $L_{43}\, e^{-(b_8 y+b_{16} y)}$ + $L_{44}\, e^{-(b_2 y+b_{18} y)}$ + $L_{45}\, e^{-(b_2 y+b_4 y)}$ + L_{46} $e^{-(b_2 y+b_{14} y)}$ + $L_{47}\, e^{-(b_2 y+b_{16} y)}$ + $L_{48}\, e^{-(b_4 y+b_{18} y)}$ + $L_{49}\, e^{-(b_4 y+b_{14} y)}$ + L_{50} $e^{-(b_4 y+b_{16} y)}$, (2.50)

$\theta_{00} = e^{-b_8 y}$, (2.51)

$\theta_{01} = e^{-b_{18} y}$ (- L_3 - L_4 - L_5) + $L_3\, e^{-2b_8 y}$ +$L_4\, e^{-2b_2 y}$ + $L_5\, e^{-2b_4 y}$, (2.52)

θ_{10} = - L_{10} ($e^{-b_{14} y}$ - $e^{-b_2 y}$), (2.53)

$\theta_{11} = e^{-b_{20} y}$ (- L_{17} - L_{18} - L_{19} - L_{20} - L_{21} - L_{22} - L_{23} - L_{24} - L_{25} - L_{26} - L_{27} - L_{28} - L_{29} - L_{30} - L_{31}- L_{32}) +$L_{17}\, e^{-(b_{12} y)}$ + $L_{18}\, e^{-(2b_8 y)}$ + $L_{19}\, e^{-(2b_2 y)}$ + $L_{20}\, e^{-(2b_4 y)}$ + $L_{21}\, e^{-(b_8 y+b_{18} y)}$ + $L_{22}\, e^{-(b_8 y+b_2 y)}$ + $L_{23}\, e^{-(b_8 y+b_4 y)}$ + $L_{24}\, e^{-(b_8 y+b_{14} y)}$ + $L_{25}\, e^{-(b_8 y+b_{16} y)}$ + $L_{26}\, e^{-(b_2 y+b_{18} y)}$ + $L_{27}\, e^{-(b_2 y+b_4 y)}$ + $L_{28}\, e^{-(b_2 y+b_{14} y)}$ + $L_{29}\, e^{-(b_2 y+b_{16} y)}$ + $L_{30}\, e^{-(b_4 y+b_{18} y)}$ + $L_{31}\, e^{-(b_4 y+b_{14} y)}$ + $L_{32}\, e^{-(b_4 y+b_{16} y)}$, (2.54)

$C_{00} = e^{-b_4 y}$, (2.55)

C_{01} = 0, (2.56)

C_{10} = - L_{11} ($e^{-b_{16} y}$ - $e^{-b_4 y}$), (2.57)

C_{11} = 0, (2.58)

where

$$b_1 = \frac{3Pr}{2(3+4R)}\left(-1+\sqrt{\frac{3\,\mathrm{Pr}-4\eta(3+4R)}{3Pr}}\right) \quad , \quad b_2 = \frac{3Pr}{2(3+4R)}\left(1+\sqrt{\frac{3\,\mathrm{Pr}-4\eta(3+4R)}{3Pr}}\right),$$

$$b_3 = \frac{1}{2}\left(-Sc-\sqrt{Sc^2 - 4\mathrm{Sc}k_r^2}\right) , b_4 = \frac{1}{2}\left(Sc+\sqrt{Sc^2 - 4\mathrm{Sc}k_r^2}\right) ,$$

$$b_5 = \frac{1}{2}\left(-Sc+\sqrt{Sc^2 - 4\mathrm{Sc}k_r^2}\right) , b_6 = \frac{1}{2}\left(Sc+\sqrt{Sc^2 - 4\mathrm{Sc}k_r^2}\right) ,$$

$$b_7 = \frac{1}{2}\left(-1+\sqrt{1+4\left(M+\frac{1}{k_p}\right)}\right) , b_8 = \frac{1}{2}\left(1+\sqrt{1+4\left(M+\frac{1}{k_p}\right)}\right),$$

$$b_9 = \frac{1}{2}\left(-1+\sqrt{1+4\left(M+\frac{1}{k_p}\right)}\right), \; b_{10} = \frac{1}{2}\left(1+\sqrt{1+4\left(M+\frac{1}{k_p}\right)}\right),$$

$$b_{11} = \frac{3Pr}{2(3+4R)}\left(-1+\sqrt{\frac{3\,\mathrm{Pr}-4\eta(3+4R)}{3Pr}}\right) , b_{12} = \frac{3Pr}{2(3+4R)}\left(1+\sqrt{\frac{3\,\mathrm{Pr}-4\eta(3+4R)}{3Pr}}\right) ,$$

$$b_{13} = \frac{3Pr}{2(3+4R)}\left(-1+\sqrt{\frac{3\,\mathrm{Pr}-4(\eta-\omega i)(3+4R)}{3Pr}}\right) , b_{14} = \frac{3Pr}{2(3+4R)}\left(1+\sqrt{\frac{3\,\mathrm{Pr}-4(\eta-\omega i)(3+4R)}{3Pr}}\right) ,$$

$$b_{15} = \frac{1}{2}\left(-Sc+\sqrt{Sc^2 - 4\mathrm{Sc}\,(\omega i + k_r^2}\right) , b_{16} = \frac{1}{2}\left(Sc+\sqrt{Sc^2 - 4\mathrm{Sc}\,(\omega i + k_r^2}\right) ,$$

$$b_{17} = \frac{1}{2}\left(-1+\sqrt{1+4\left(M+\frac{1}{k_p}-\omega i\right)}\right), b_{18} = \frac{1}{2}\left(1+\sqrt{1+4\left(M+\frac{1}{k_p}-\omega i\right)}\right),$$

$$b_{19} = \frac{3Pr}{2(3+4R)}\left(-1+\sqrt{\frac{3\,\mathrm{Pr}+4(\eta+\omega i)(3+4R)}{3Pr}}\right) , b_{20} = \frac{3Pr}{2(3+4R)}\left(1+\sqrt{\frac{3\,\mathrm{Pr}+4(\eta+\omega i)(3+4R)}{3Pr}}\right) ,$$

$$b_{21} = \frac{1}{2}\left(-Sc+\sqrt{Sc^2 + 4\mathrm{Sc}\,(\omega i + k_r^2}\right) , b_{22} = \frac{1}{2}\left(Sc+\sqrt{Sc^2 + 4\mathrm{Sc}\,(\omega i + k_r^2}\right) ,$$

$$b_{23} = \frac{1}{2}\left(-1+\sqrt{1+4\left(M+\frac{1}{k_p}+\omega i\right)}\right), b_{24} = \frac{1}{2}\left(1+\sqrt{1+4\left(M+\frac{1}{k_p}+\omega i\right)}\right),$$

$$L_1 = \frac{-Gr}{b_2^2-b_2-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; , \; L_2 = \frac{-Gc}{b_4^2-b_4-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; , L_3 = \frac{3PrQ_1b_8^2}{4(3+4R)b_8^2-6Prb_8-3Pr\eta} \; ,$$

$$L_4 = \frac{3Prb_2^2L_1^2}{4(3+4R)b_2^2-6Prb_2-3Pr\eta} \; , L_5 = \frac{3Prb_4^2L_2^2}{4(3+4R)b_4^2-6Prb_4-3Pr\eta} \; , L_6 = \frac{-Gr(-L_3-L_4-L_5)}{b_{12}^2-b_{12}-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; ,$$

$$L_7 = \frac{-GrL_3}{4b_8^2-2b_8-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; , L_8 = \frac{-GrL_4}{4b_2^2-2b_2-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; , L_9 = \frac{-GrL_5}{4b_4^2-2b_4-\left(\mathrm{M}+\frac{1}{k_p}\right)} \; ,$$

$$L_{10} = \frac{3PrAb_2}{(3+4R)b_2^2-3Prb_2-3\mathrm{Pr}\,(\eta+\omega i)} \; , L_{11} = \frac{AScb_4}{b_2^2-Scb_4-\mathrm{Sc}\,(\omega i+k_r^2)} , L_{12} = \frac{Q_1\left(Ab_8+\frac{1}{k_p}\right)}{b_8^2-b_8-\left(M+\frac{1}{k_p}+\omega i\right)} ,$$

$$L_{13} = \frac{-Ab_2L_1-\frac{B}{k_p}L_1-GrL_{10}}{b_2^2-b_2-\left(M+\frac{1}{k_p}+\omega i\right)},\ L_{14} = \frac{-Ab_4L_2-\frac{B}{k_p}L_2-GcL_{11}}{b_4^2-b_4-\left(M+\frac{1}{k_p}+\omega i\right)},\ L_{15} = \frac{-Gr(-1+L_{10})}{b_{14}^2-b_{14}-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{16} = \frac{-GcL_{11}}{b_{16}^2-b_{16}-\left(M+\frac{1}{k_p}+\omega i\right)},\ L_{17} = \frac{3APrb_{12}(-L_3-L_4-L_5)}{(3+4R)b_{12}^2-3Prb_{12}-3\Pr(\eta+\omega i)},\ L_{18} =$$

$$\frac{6Prb_8(AL_3-b_8Q_1L_{12})}{(3+4R)4b_8^2-6Prb_8-3\Pr(\eta+\omega i)},$$

$$L_{19} = \frac{6Prb_2(AL_4-b_2L_1L_{13})}{(3+4R)4b_2^2-6Prb_2-3\Pr(\eta+\omega i)},\ L_{20} = \frac{6Prb_4(AL_5-b_4L_2L_{14})}{(3+4R)4b_4^2-6Prb_4-3\Pr(\eta+\omega i)},$$

$$L_{21} = \frac{-6Prb_8b_{18}Q_1(-C_2-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_8+b_{18})^2-3Pr(b_8+b_{18})-3\Pr(\eta+\omega i)},\ L_{22} = \frac{-6Prb_8b_2Q_1L_{13}+b_2b_8L_1L_{12}}{(3+4R)(b_8+b_2)^2-3Pr(b_8+b_2)-3\Pr(\eta+\omega i)},$$

$$L_{23} = \frac{-6Prb_8b_4(Q_1L_{14}+L_2L_{12})}{(3+4R)(b_4+b_8)^2-3Pr(b_4+b_8)-3\Pr(\eta+\omega i)},\ L_{24} = \frac{-6Prb_8b_{14}Q_1L_{15}}{(3+4R)(b_4+b_{14})^2-3Pr(b_4+b_{14})-3\Pr(\eta+\omega i)},$$

$$L_{25} = \frac{-6Prb_8b_{16}Q_1L_{16}}{(3+4R)(b_8+b_{16})^2-3Pr(b_8+b_{16})-3\Pr(\eta+\omega i)},\ L_{26} = \frac{-6Prb_2b_{18}L_1(-C_2-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_2+b_{18})^2-3Pr(b_2+b_{18})-3\Pr(\eta+\omega i)},$$

$$L_{27} = \frac{-6Prb_2b_4(L_1L_{14}+L_2L_{13})}{(3+4R)(b_4+b_2)^2-3Pr(b_4+b_2)-3\Pr(\eta+\omega i)},\ L_{28} = \frac{-6Prb_2b_{14}L_1L_{15}}{(3+4R)(b_2+b_{14})^2-3Pr(b_2+b_{14})-3\Pr(\eta+\omega i)},$$

$$L_{29} = \frac{-6Prb_2b_{16}L_1L_{16}}{(3+4R)(b_2+b_{16})^2-3Pr(b_2+b_{16})-3\Pr(\eta+\omega i)},\ L_{30} = \frac{-6Prb_4b_{18}L_2(-C_2-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_4+b_{18})^2-3Pr(b_4+b_{18})-3\Pr(\eta+\omega i)},$$

$$L_{31} = \frac{-6Prb_4b_{14}L_2L_{15}}{(3+4R)(b_4+b_{14})^2-3Pr(b_4+b_{14})-3\Pr(\eta+\omega i)},\ L_{32} = \frac{-6Prb_4b_{16}L_2L_{16}}{(3+4R)(b_4+b_{16})^2-3Pr(b_4+b_{16})-3\Pr(\eta+\omega i)},$$

$$L_{33} = \frac{Ab_{10}Q_2-\frac{B}{k_p}}{b_{10}^2-b_{10}-\left(M+\frac{1}{k_p}+\omega i\right)},\ L_{34} = \frac{Ab_{12}L_6-\frac{B}{k_p}L_6-GrL_{17}}{b_{12}^2-b_{12}-\left(M+\frac{1}{k_p}+\omega i\right)},\ L_{35} = \frac{2Ab_8L_7-\frac{B}{k_p}L_7-GrL_{18}}{4b_8^2-2b_8-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{36} = \frac{2Ab_2L_8-\frac{B}{k_p}-GrL_{19}}{4b_2^2-2b_2-\left(M+\frac{1}{k_p}+\omega i\right)},\qquad L_{37} = \frac{2Ab_4L_9-\frac{B}{k_p}-GrL_{20}}{4b_4^2-2b_4-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{38} = \frac{Gr(L_{17}+L_{18}+L_{19}+L_{20}+L_{21}+L_{22}+L_{23}+L_{24}+L_{25}+L_{26}+L_{27}+L_{28}+L_{29}+L_{30}+L_{31}+L_{32})}{b_{20}^2-b_{20}-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{39} = \frac{-GrL_{21}}{(b_8+b_{18})^2-(b_8+b_{18})-\left(M+\frac{1}{k_p}+\omega i\right)},\qquad L_{40} = \frac{-GrL_{22}}{(b_8+b_2)^2-(b_8y+b_2)-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{41} = \frac{-GrL_{23}}{(b_8+b_4)^2-(b_8+b_4)-\left(M+\frac{1}{k_p}+\omega i\right)},\qquad L_{42} = \frac{-GrL_{24}}{(b_8+b_{14})^2-(b_8+b_{14})-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{43} = \frac{-GrL_{25}}{(b_8+b_{16})^2-(b_8+b_{16})-\left(M+\frac{1}{k_p}+\omega i\right)},\qquad L_{44} = \frac{-GrL_{26}}{(b_2+b_{18})^2-(b_2+b_{18})-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$$L_{45} = \frac{-GrL_{27}}{(b_2+b_4)^2-(b_2+b_4)-\left(M+\frac{1}{k_p}+\omega i\right)},\qquad L_{46} = \frac{-GrL_{28}}{(b_2+b_{14})^2-(b_2+b_{14})-\left(M+\frac{1}{k_p}+\omega i\right)},$$

$L_{47} = \frac{-GrL_{29}}{(b_2+b_{16})^2-(b_2+b_{16})-\left(M+\frac{1}{k_p}+\omega i\right)}$, $L_{48} = \frac{-GrL_{30}}{(b_4+b_{18})^2-(b_4+b_{18})-\left(M+\frac{1}{k_p}+\omega i\right)}$,

$L_{49} = \frac{-GrL_{31}}{(b_4+b_{14})^2-(b_4+b_{14})-\left(M+\frac{1}{k_p}+\omega i\right)}$, $L_{50} = \frac{-GrL_{32}}{(b_4+b_{16})^2-(b_4+b_{16})-\left(M+\frac{1}{k_p}+\omega i\right)}$.

In view of the above solutions, the velocity, temperature and concentration distribution in the boundary layer become:

$$U(y,t) = e^{-b_8 y}Q_1 + 1 + L_1 e^{-b_2 y} + L_2 e^{-b_4 y} + Ec(e^{-b_{10} y} Q_2 + L_6 e^{-b_{12} y} + L_7 e^{-2b_8 y} + L_8 e^{-2b_2 y} + L_9 e^{-2b_4 y}) + \epsilon e^{i\omega t}(e^{-b_{18} y}(-C_2 - L_{12} - L_{13} - L_{14} - L_{15} - L_{16}) + C_2 + L_{12} e^{-b_8 y} + L_{13} e^{-b_2 y} + L_{14} e^{-b_4 y} + L_{15} e^{-b_{14} y} + L_{16} e^{-b_{16} y} + Ec(L_{12} e^{-b_{24} y}(- L_{33} - L_{34} - L_{35} - L_{36} - L_{37} - L_{38} - L_{39} - L_{40} - L_{41} - L_{42} - L_{43} - L_{44} - L_{45} - L_{46} - L_{47} - L_{48} - L_{49} - L_{50}) + L_{33} e^{-b_{10} y} + L_{34} e^{-b_{12} y} + L_{35} e^{-2b_8 y} + L_{36} e^{-2b_2 y} + L_{37} e^{-2b_4 y} + L_{38} e^{-b_{20} y} + L_{39} e^{-(b_8 y+b_{18} y)} + L_{40} e^{-(b_2 y+b_8 y)} + L_{41} e^{-(b_4 y+b_8 y)} + L_{42} e^{-(b_8 y+b_{14} y)} + L_{43} e^{-(b_8 y+b_{16} y)} + L_{44} e^{-(b_2 y+b_{18} y)} + L_{45} e^{-(b_2 y+b_4 y)} + L_{46} e^{-(b_2 y+b_{14} y)} + L_{47} e^{-(b_2 y+b_{16} y)} + L_{48} e^{-(b_4 y+b_{18} y)} + L_{49} e^{-(b_4 y+b_{14} y)} + L_{50} e^{-(b_4 y+b_{16} y)})\,, \quad (2.59)$$

$$\theta(y,t) = e^{-b_2 y} + Ec(e^{-b_{12} y}(-L_3 - L_4 - L_5) + L_3 e^{-2b_8 y} + L_4 e^{-2b_2 y} + L_5 e^{-2b_4 y}) + \epsilon e^{i\omega t}\{L_{10}(-e^{-b_{14} y} + e^{-b_2 y}) + e^{-b_{14} y} + Ec(e^{-b_{20} y}(- L_{17} - L_{18} - L_{19} - L_{20} - L_{21} - L_{22} - L_{23} - L_{24} - L_{25} - L_{26} - L_{27} - L_{28} - L_{29} - L_{30} - L_{31} - L_{32}) + L_{17} e^{-b_{12} y} + L_{18} e^{-2b_8 y} + L_{19} e^{-2b_2 y} + L_{20} e^{-2b_4 y}$$
$$L_{21} e^{-(b_8 y+b_{18} y)} + L_{22} e^{-(b_2 y+b_8 y)} + L_{23} e^{-(b_4 y+b_8 y)} + L_{24} e^{-(b_8 y+b_{14} y)} + L_{25} e^{-(b_8 y+b_{16} y)} + L_{26} e^{-(b_2 y+b_{18} y)} + L_{27} e^{-(b_2 y+b_4 y)} + L_{28} e^{-(b_2 y+b_{14} y)} + L_{29} e^{-(b_2 y+b_{16} y)} + L_{30} e^{-(b_4 y+b_{18} y)} + L_{31} e^{-(b_4 y+b_{14} y)} + L_{32} e^{-(b_4 y+b_{16} y)})\}\,. \quad (2.60)$$

$$C(y,t) = e^{-b_4 y} + \epsilon e^{i\omega t}[L_{11}(-e^{-b_{16} y} + e^{-b_4 y})]. \quad (2.61)$$

Skin-friction coefficient is expressed as follows:

$$C_f = \left[\frac{\tau_\omega}{\rho U_0 V_0}\right] = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0(y)}{\partial y} + \epsilon e^{i\omega t}\frac{\partial u_1(y)}{\partial y}\right)_{y=0}$$

$$= - b_8 Q_1 - b_2 L_1 - b_4 L_2 + Ec(b_{10} Q_2 - 2b_{12} L_6 - 2b_8 L_7 - 2b_2 L_8 - 2b_4 L_9) + \epsilon e^{i\omega t}[b_{18}(C_2 + L_{12} + L_{13} + L_{14} + L_{15} + L_{16}) + C_2 - b_8 L_{12} - b_2 L_{13} - b_4 L_{14} - b_{14} L_{15} - b_{16} L_{16} + Ec(b_{24}(L_{33} + L_{34} + L_{35} + L_{36} + L_{37} + L_{38} + L_{39} + L_{40} + L_{41} + L_{42} + L_{43} + L_{44} + L_{45} + L_{46} + L_{47} + L_{48} +$$

L_{49}+ L_{50}) - L_{33} b_{10} -L_{34} b_{12}- $2b_8 L_{35}$ - $2b_2$ L_{36} - $2b_4$ L_{37} - b_{20} L_{38} - (b_8+ b_{18}) L_{39} - (b_2+ b_8) L_{40} - (b_4+ b_8) L_{41} - (b_8+ b_{14}) L_{42} - (b_8+ b_{16}) L_{43} - (b_2+ b_{18})L_{44} - (b_2+ b_4) L_{45} - (b_2+ b_{14})L_{46} -(b_2+ b_{16})L_{47} -(b_4+b_{18})L_{48} -(b_4+b_{14})L_{49} -(b_4+b_{16})L_{50} . (2.62)

The heat transfer coefficient in term of Nusselt number is as follows:

knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer q_ω^*. This is given by:

$$q_\omega^* = - K \left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} - \frac{4\sigma^*}{3k_r'} \left(\frac{\partial T^{*4}}{\partial y^*}\right)_{y=0} . \quad (2.63)$$

Using equation (2.13) we can write equation (2.63) as follow

$$q_\omega^* = -\left(K + \frac{16\sigma^* T_\infty^{*3}}{3k_r^*}\right) \left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} , \quad (2.64)$$

which is written in non-dimensional form as :

$$q_\omega^* = -\left(1 + \frac{4R}{3}\right) \left(\frac{\partial \theta}{\partial y}\right)_{y=0} . \quad (2.65)$$

The non-dimensional Nusselt number is obtained as

NuRe_x^{-1}=-$\left(1 + \frac{4R}{3}\right) \left(\frac{\partial \theta_0(y)}{\partial y} + \epsilon e^{i\omega t} \frac{\partial \theta_1(y)}{\partial y}\right)_{y=0}$ = - b_2 – Ec(b_{12} (L_3 + L_4 + L_5) - $2b_8$ L_3 -2 b_2 L_4 + $2b_4 L_5$) - $\epsilon e^{i\omega t}$[L_{10} (b_{14} - b_2) - b_{14} + Ec(b_{20} (L_{17} + L_{18} + L_{19} + L_{20} + L_{21} + L_{22} + L_{23} + L_{24} + L_{25} + L_{26} + L_{27} + L_{28} + L_{29} + L_{30} + L_{31} + L_{32}) -b_{12} L_{17} - $2b_8$ L_{18} -$2b_2$ L_{19} - $2b_4$ L_{20} - (b_8+ b_{18}) L_{21} – (b_2+ b_8) L_{22} - (b_4+ b_8) L_{23} - (b_8+ b_{14}) L_{24} - (b_8+ b_{16}) L_{25} - (b_2+ b_{18}) L_{26} - (b_2+ b_4) L_{27} - (b_2+ b_{14}) L_{28} - (b_2+ b_{16}) L_{29} - (b_4+ b_{18}) L_{30} - (b_4+ b_{14}) L_{31} - (b_4+ b_{16}) L_{32} , (2.66)

where $\text{Re}_x = \frac{V_0 x}{v}$ is the Reynolds number.

Local Sherwood Number (Sh_w) can be define as

$$\text{Sh} = \frac{Kx}{D} , \quad (2.67)$$

with the help of these equations, one can write

$$\text{ShRe}_x^{-1} = - \left(\frac{\partial C_0(y)}{\partial y} + \epsilon e^{i\omega t} \frac{\partial C_1(y)}{\partial y}\right)_{y=0} =- b_4 + \epsilon e^{i\omega t} L_{11}(b_{16} - b_4) \quad (2.68)$$

3. Results and Discussion

The effects of thermal radiation, heat source and chemical reaction on heat and mass transfer of MHD incompressible, viscous fluid along vertical porous moving plate in a porous medium has been investigated. The numerical calculation for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters are obtained in this study using the following A=0.5, t=1.0, n=0.1 and ϵ =0.10, while R , k_r^2, Sc, Gr, Gc, M, Pr, η, and K are varried in order to account for their effects. The boundary conditions for y is replaced with y_{max} that is when y sufficiently large and the velocity profile u approaches to the relevant free stream velocity.

The velocity profiles for different values of Grashof number Gr are described in the Fig.1 and Fig. 2. It is observed that an increasing in Gr leads to a rise in the values of velocity and the curves show that the peak values of the velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity. Here the Grashof number represents the effect of the free convection currents. Physically, Gr > 0 means heating of fluid of cooling of the boundary surface, Gr<0 means cooling of the fluid of heating of the boundary surface and Gr=0 corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grshof number increases, and then decays to the relevant free stream velocity.

The velocity profiles across the boundary layer for different values of prandtl number Pr are plotted in Fig. 3. The results shows that the effect of increasing values of Pr results in a decreasing the velocity.

Fig. 4 shows the effect of radiation R on velocity. It is observed that as the value of R increases, the velocity increases with an increasing in the flow boundary layer thickness. Thus, thermal radiation enhances the flow.

The influence of chemical reaction parameter k_r^2 on the velocity profiles across the boundary layer are presented in Fig. 5 it is seen that the velocity distribution across the boundary layer decreases with increasing in k_r^2.

The effect of heat generation η on the velocity profiles is shown in Fig.6. From this figure it is observed that the heat is generated the buoyancy force increase which influence the flow rate to increase giving rise to the increase in the velocity profiles.

Fig.7. represents the velocity profile with various Schmidt number Sc. The effect of increasing values of Sc results in a decreasing velocity distribution across the boundary layer.

Fig.8. shows that the effect of increasing values of M parameter results in decreasing velocity distribution across the boundary layer because of the application of transfer magnetic field will result a restrictive type force(Lorenz force) similar to drag force which tends to resist the fluid and this reducing its velocity.

It is observed from Fig.9. That as velocity profiles for different values of the permeability K. Clearly, as K increases the peak value of velocity tends to increase. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering.

Fig.10. show the effect of Schmidt number Sc on temperature, as Schmidt number Sc increases the temperature distribution across the boundary layer increases but however decreases.

The effect of heat generation η on temperature is shown in Fig.11. From this figure it is observed that the heat is generated the buoyancy force increase which influence the flow rate to increase giving rise to the increase in the temperature profiles.

It is observed from Fig.12 that an increase in prandtl number results in a decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence for smaller Pr, the rate of heat transfer is reduced.

For different values of Schmidt number Sc, the concentration profiles are plotted in Fig. 13. It is obvious that the influence of increasing values of Sc, the concentration distribution across the boundary layer decreases.

The influence of chemical reaction parameter k_r^2 on concentration profiles are plotted in Fig. 14. It is obvious that the influence of increasing values of k_r^2 , the concentration distribution across the boundary layer decreases but later increases.

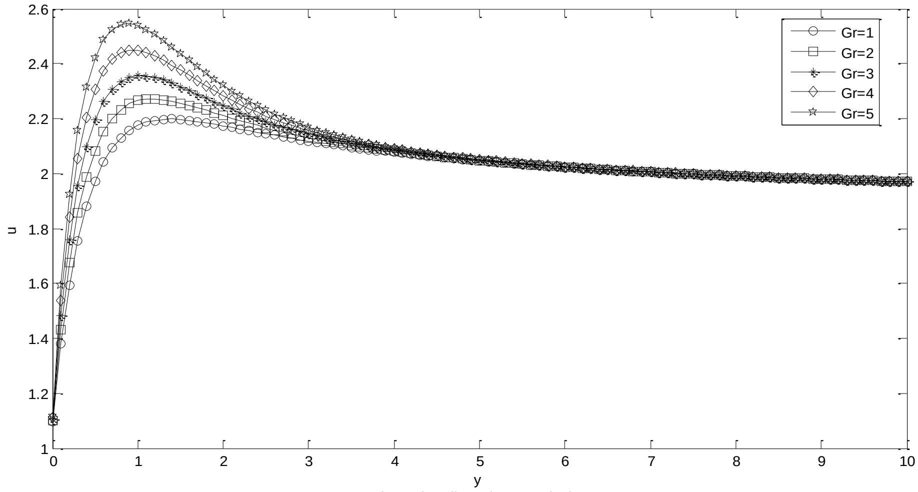

Fig 1: The effect of Gr on velocity

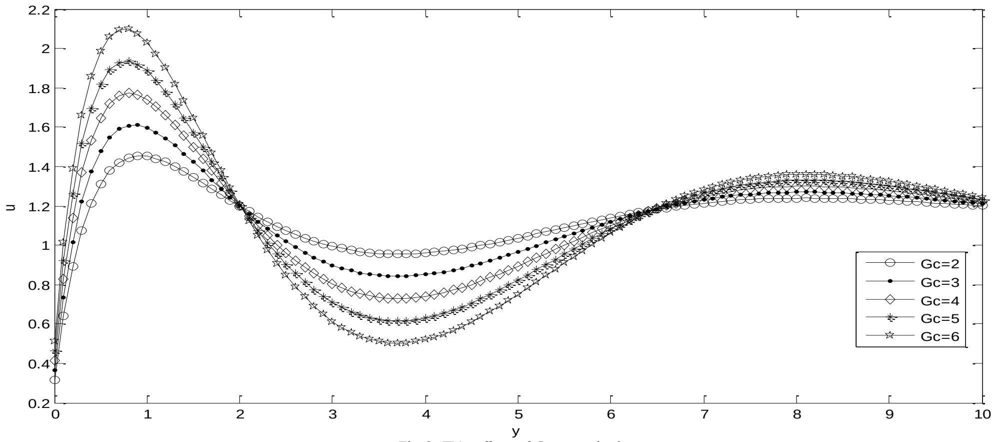

Fig 2: THe effect of Gc on velocity

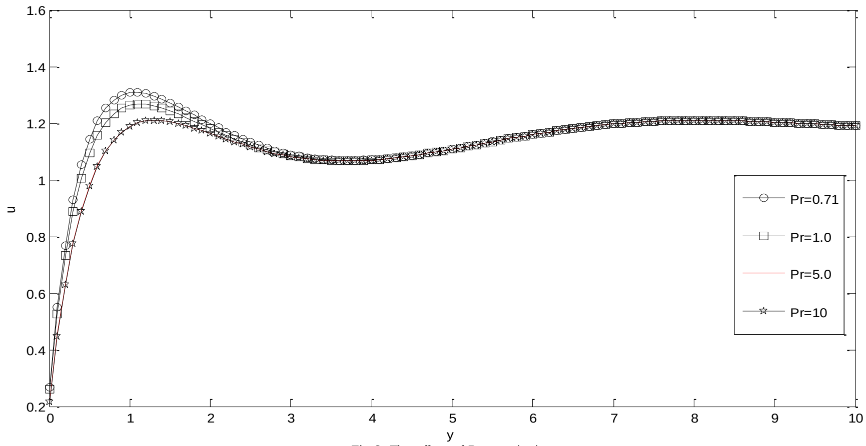

Fig 3: The effect of Pr on velocity

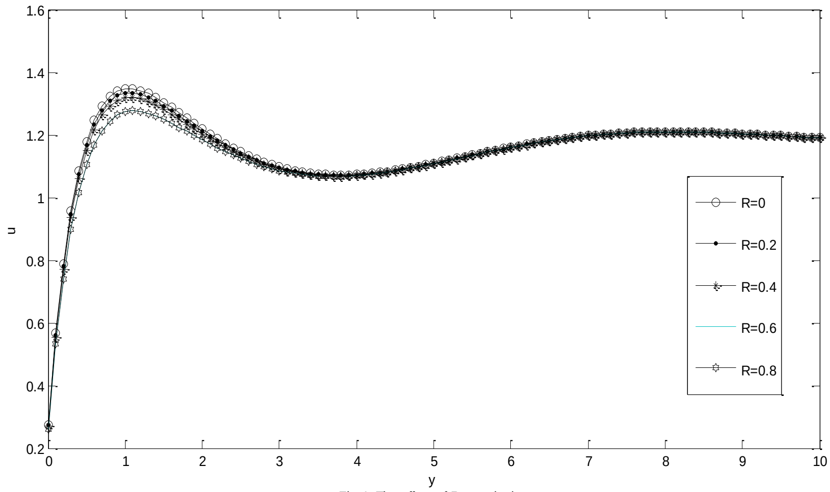

Fig 4: The effect of R on velocity

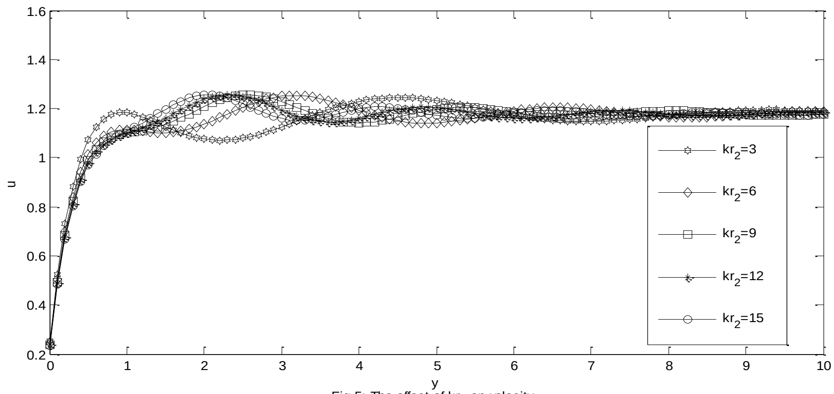

Fig 5: The effect of kr_2 on velocity

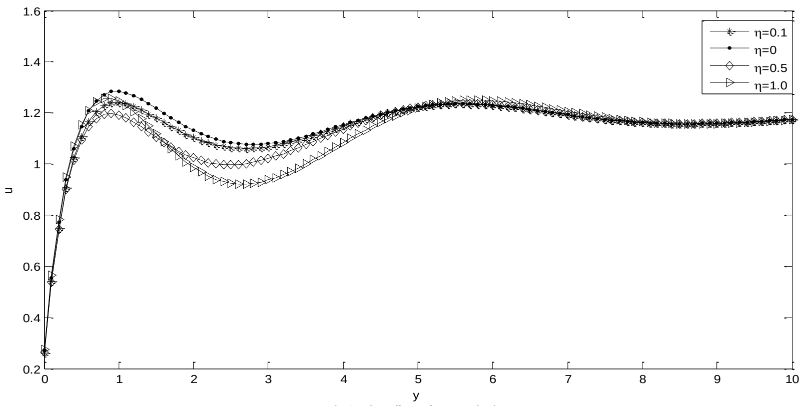

Fig 6: The effect of η on velocity

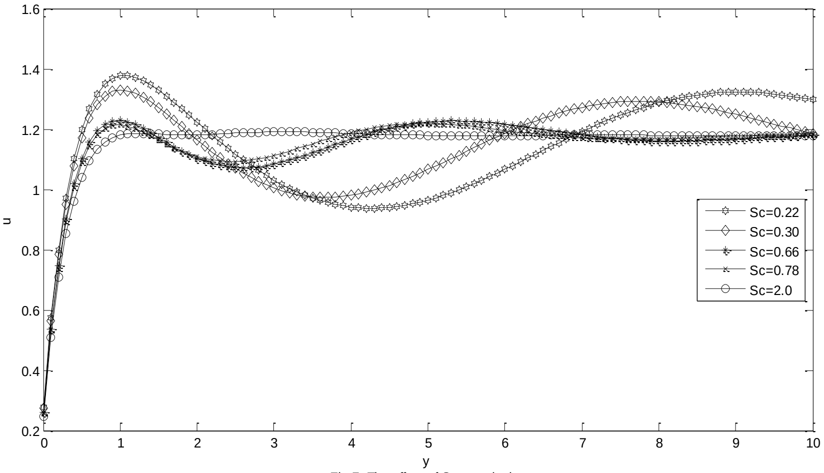

Fig 7: The effect of Sc on velocity

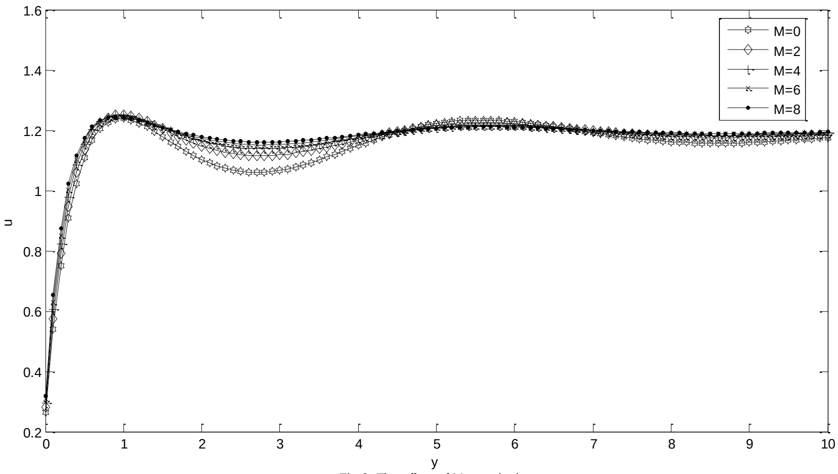

Fig 8: The effect of M on velocity

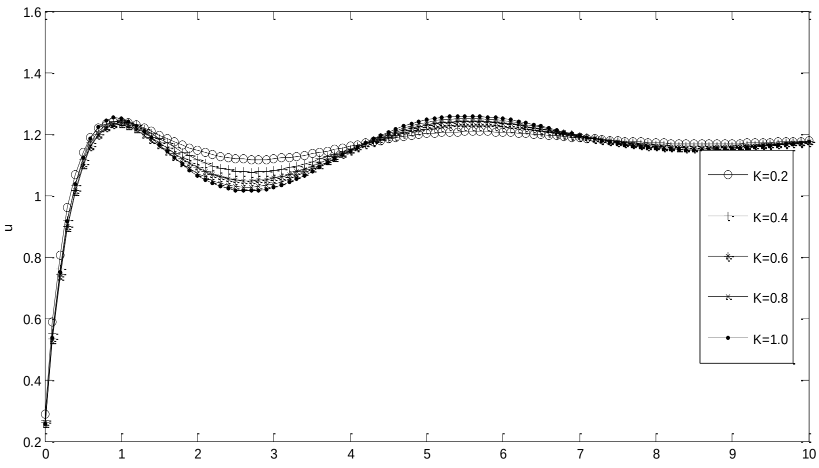

Fig 9: The effect of K on velocity

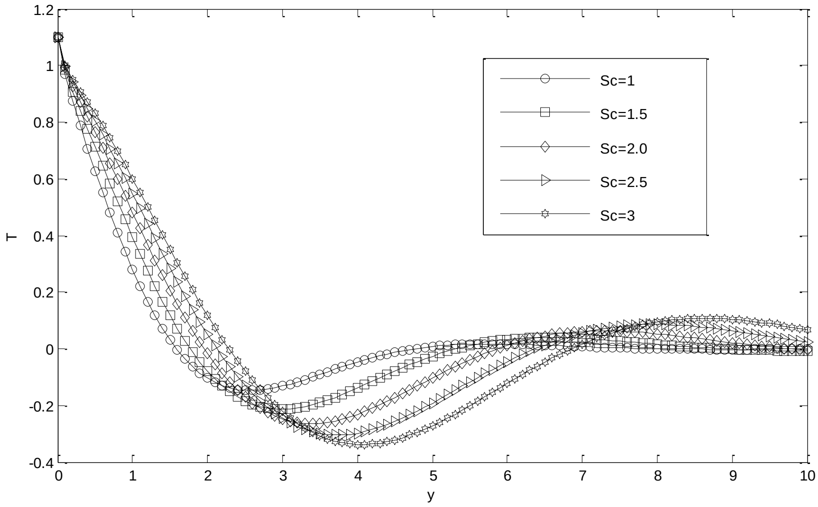

Fig 10: The effect of Sc on temperature

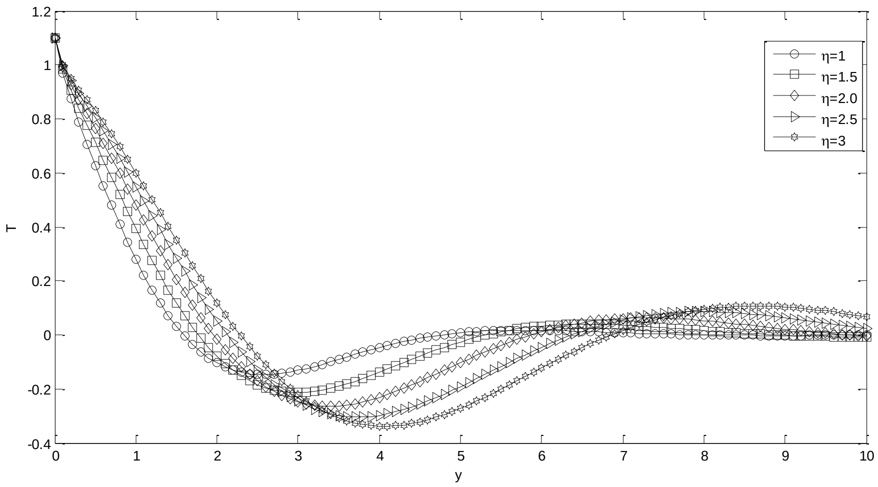

Fig 11: The effect of η on temperature

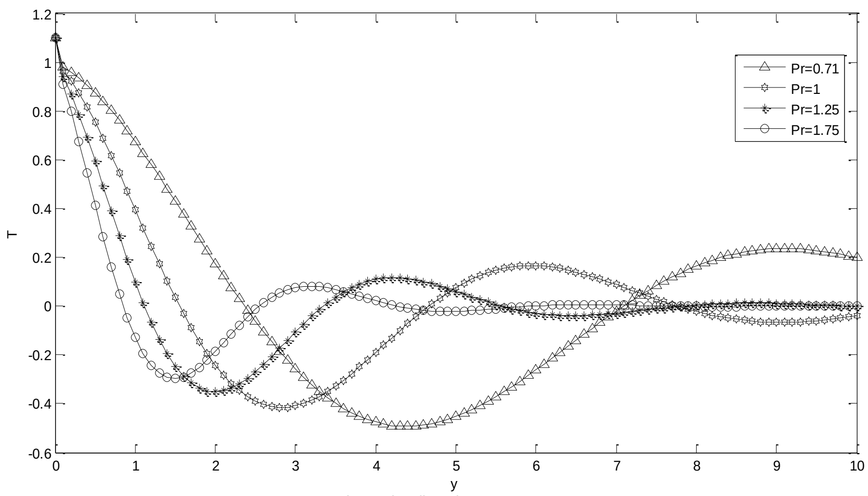

Fig 12: The effect of Pr on temperature

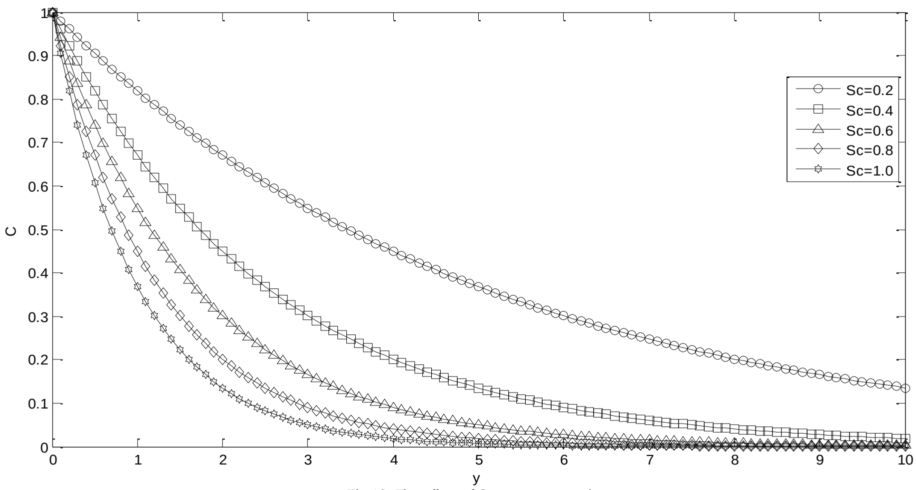

Fig 13: The effect of Sc on concentration

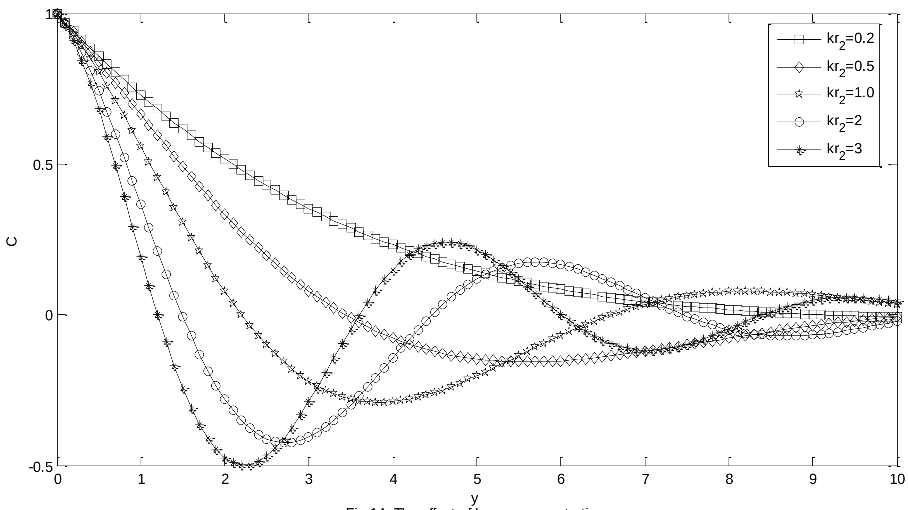

Fig 14: The effect of kr_2 on concentration

Table 1.: Effect of R on velocity, n=ϵ =0.1, t= A=Gc=M=1, Pr=0.71, k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5, Ec=0.01.

R	C_f
0.2	3.2622
0.4	3.2947
0.6	3.3071
0.8	3.3144
1.0	3.3199

Table 2.: Effect of Ec on velocity, n=ϵ =0.1, t= A=Gc=M=1, Pr=0.71,R=k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5

Ec	C_f
0.02	3.0246
0.04	2.8878

0.06	2.7509
0.08	2.6140
0.10	2.4772

Table 3.: Effect of k_r^2 on velocity ,n=ϵ =0.1, t= A=Gc=M=1, Pr=0.71,R=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

k_r^2	C_f
0.2	3.1367
0.5	3.0930
1	3.0394
2	2.9611
3	2.9065

Table 4.: Effect of Gr on velocity, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71, R=k_r^2=0.0, η=-0.13, Sc=0.22, K=0.5,Ec=0.01.

Gr	C_f
4	2.1354
6	-1.7737
8	-10.1304
10	-24.4308
12	-46.1711

Table 5.: Effect of Gc on velocity, n=ϵ =0.1, t= A=M=1, Pr=0.71,R=k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

Gc	C_f
2	3.5930
3	4.0983
4	4.6089
5	5.1249
6	5.6462

Table 6.: Effect of M on velocity, n= ϵ =0.1, t= A=Gc=1, Pr=0.71,R= k_r^2 =0.0, η =-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

M	C_f
2	3.3656
4	3.4926
6	3.6230
8	3.7495
10	3.8682

Table 7.: Effect of Sc on velocity, n= ϵ =0.1, t= A=Gc=M=1, Pr=0.71,R= k_r^2 =0.0, η =-0.13, Gr=2, K=0.5,Ec=0.01.

Sc	C_f
0.30	3.160 0
0.66	3.082 7
0.78	3.064 1
1.00	3.036 1
2.00	2.969 6

Table 8.: Effect of Pr on velocity, n= ϵ =0.1, t= A=Gc=M=1,R= k_r^2 =0.0, η =-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

Pr	C_f

0.71	3.0930
1	3.2071
5	5.1072
7	3.0884
10	2.8915

Table 9.: Effect of η on velocity, n= ϵ =0.1, t= A=Gc=M=1, Pr=0.71,R= k_r^2 =0.0, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

η	C_f
-0.13	3.0930
-0.1	1.8865
0.5	3.1869
0.1	3.2515
0.13	3.2290

Table 10.: Effect of Sc on velocity, n= ϵ =0.1, t= A=Gc=M=1,Pr=0.71, R= k_r^2 =0.0, η =-0.13, Gr=2,K=0.5,Ec=0.01.

Sc	C_f
1	3.3760
1.5	3.4103
2.0	3.4320
2.5	3.4473
3.0	3.4586

Table 11.: Effect of Pr on temperature, n= ϵ =0.1, t= A=Gc=M=1,R= k_r^2 =0.0, η =-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

Pr	Nu/Re
0.71	- 0.6666
1.0	-1.1192
5	- 15.4602
7	- 14.8602
10	-

	21.3426

Table 12.: Effect of R on temperature, n=ϵ =0.1, t= A=Gc=M=1, Pr=0.71,k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

R	Nu/Re
0.2	-0.5284
0.4	-0.3924
0.6	-0.2567
0.8	-0.1210
1.0	-0.0149

Table 13.: Effect of Ec on temperature, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71, R=k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5.

Ec	Nu/Re
0.02	-0.6673
0.04	-0.6687
0.06	-0.6701
0.08	-0.6715
0.10	-0.6729

Table 14.: Effect of η on temperature, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71, R=k_r^2 =0.0, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

η	Nu/Re
-0.13	-0.6666
-0.1	-0.6732
0.5	-0.4128
0.1	-0.8798
0.13	-0.9729

Table 15.: Effect of K on temperature, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71,R=k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, Ec=0.01.

K	Nu/Re
1	-0.6690
1.5	-0.6720
2.0	-0.6747
2.5	-0.6770

3.0	-0.6789

Table 16.: Effect of M on temperature, n=ϵ =0.1, t= A=Gc=1, Pr=0.71, R=k_r^2=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

M	Nu/Re
2	-0.6667
4	-0.6678
6	-0.6690
8	-0.6700
10	-0.6708

Table 17.: Effect of Sc on concentration, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71,R=k_r^2=0.0, η=-0.13, Gr=2, K=0.5,Ec=0.01.

Sc	Sh/Re
0.30	-0.3000
0.66	-0.6600
0.78	-0.7800
1	-1.0
2	-2.0

Table 18.: Effect of k_r^2 on concentration, n=ϵ =0.1, t= A=Gc=M=1,Pr=0.71,R=0.0, η=-0.13, Gr=2, Sc=0.22, K=0.5,Ec=0.01.

k_r^2	Sh/Re
0.2	-0.3000
0.5	-0.3000
1.0	-o.3000
2.0	-0.3000
3.0	-0.3000

Tables 1- 10 show the effects of the radiation parameter, chemical reaction, Eckert number, Grashof numbers, Schmidt number , and Prandtl number on the skin- friction coefficient. It is observed from this table that as radiation parameter, Grashof number for mass transfer, and Permeability K increases, the skin-friction coefficients increases, while as others increases skin-friction coefficient decreases. Also decreases in the heat generation Parameter effect, the skin-friction coefficient to increase.

Table 11 - 16 presents the radiation parameter increases the Nusselt number also increases, increase in chemical reaction, magnetic field, heat generation, Prandtl number, Eckert number, the Nusselt number decreases.

Table 17 - 18 reflects that the Sherwood number at the plate decreases with the increase of Schmidt number. As chemical reaction increases, Sherwood remains unchanged.

4. Conclusion

Heat and mass transfer of MHD and dissipative fluid flow past a moving vertical porous plate with variable suction in the presence of chemical reaction, heat source, transfer magnetic field and oscillating free stream are carried out and the following conclusions were made:

i. a rise in the Grashof number causes an increase in the heavy flow of fluid velocity owing to the increase in quality of buoyancy force. The highest point of the velocity goes up quickly near the porous plate as buoyancy force for heat movement rises and rots the free stream velocity;

ii. the size of fluid velocity reaches a very high value with the rise of buoyancy force for a large amount of movement in the drops appropriately to reach a free stream velocity;

iii. the size of fluid velocity drops with the rise of molecular diffusivity of the magnetic field, while it rises with the increase of heat source;

iv. a rise in the chemical reaction parameter leads to reduction of the velocity as well as the species concentration. The hydrodynamic and the concentration boundary surface get thick as the reaction parameter goes up.

v. the size of fluid concentration reduces with the rise in chemical reaction parameter;

vi. a rise in the radiation heat transfer leads to a fall in the size of fluid velocity and the height of fluid temperature inside the boundary surface and also a fall in the thickness of the velocity as well as thermal boundary layer;

vii. a rise in Prandtl number generates a fall in the thermal boundary layer and in totality less average temperature inside the boundary area being the lesser values of PR are same to the rise in the thermal conductivity of the fluid. Hence, heat is able to pass away from the heated surface quickly for higher values of PR. Due to this, for smaller Pr, the rate of heat movement are reduced; and

viii. the level of fluid temperature falls with the rise of chemical reaction parameter, viscous dissipation effect and molecular diffusivity; while it rises alongside an increase of level of magnetic field and heat source.

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