Ilorin Journal of Science
Volume 1, Number 1, 2014, pp. 87 – 99 (Printed in Nigeria)
ISSN: 2408 – 4840 © 2014 Faculty of Physical Sciences, University of Ilorin
https://doi.org/10.54908/iljs.2014.01.01.007

ILORIN
JOURNAL OF SCEINCE

ILJS-14-032

Analysis of a Three-Queue Polling System with Probabilistic Routing

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Abstract

In this paper, we consider a three-queue polling system with probabilistic routing. The choice of the queue to be visited next may depend on the current state of the system through the polling probability. Using the embedded Markov chain technique, we derived expressions for the steady-state joint and marginal queue length distribution at the switch points, as well as the waiting time distribution at each queue. The relation between the queue length and waiting time distributions becomes inherent in the model, providing a platform for easily computing waiting time moments.

Keywords: Polling model, probability generating function, Laplace-Stieltjes transform, super cycle

1. Introduction

Probabilistic routing is a very important router discipline that allows different types of traffic to receive the appropriate quality-of-service requirements they need individually. The probabilistic routing scheme dynamically allocates the available server resource to each traffic class based on the class's polling probability. This discipline plays a vital role in several applications in telecommunications, traffic management and logistics. Consider a server who polls the three queues of a queueing network in a cyclic order, but with a tendency to skip queue 2 in a non-empty situation, with probability $p<1$. This might be the case for an inherent priority system. Examples are systems with a greedy-type routing mechanism, where the server chooses a queue with a certain proportion of units (or customers) waiting at the start. With each of the queues, there is associated an infinite buffer (or waiting room) fed by a homogeneous Poisson arrival stream with intensity, λ_i, i = 1,2,3. The overall intensity, or total arrival rate is

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$$\lambda = \lambda_1 + \lambda_2 + \lambda_3$$

A typical polling system is shown in figure 1.

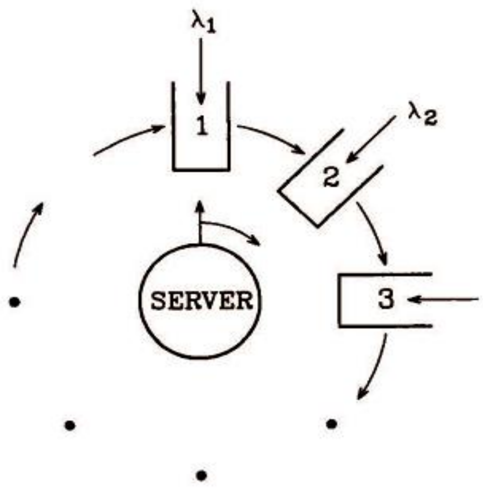

Figure 1: A cyclic polling system (Levy and Sidi, 1990)

The server employs the exhaustive service policy at each queue. The services are independent of the arrival stream and each served unit departs from the system. The service times at queue i are independent and identically distributed random variables B_i with finite k^{th} moments $b_i^{(k)}$, k=1,2,.... When the server has finished the batch of services at queue i or if he found the queue empty, then he switches to the next queue without incurring a switchover time. The choice may depend on the current state of the system through the polling probability, p.

The probability p can be independent of the state of the system or p depends on the state. The case p=0 can be viewed as a *polite* system, in which case the server movement is cyclic. On the other hand, the case p=1 implies an *aggressive* system where the server must switch to queue 3, after exhausting queue 1.

A necessary and sufficient condition for stability is that the overall system load ρ, is less than 1. We also assume that the ergodicity conditions are fulfilled and we restrict ourselves to results for the system in equilibrium. All references to queue indices greater than 3 or less than 1 are implicitly assumed to be modulo 3.

A unique property of the present model is that the server's routing rule depends on the actual configuration of units in the system.

The model is a generalization of a three-queue polling model with exhaustive service at each queue. It reduces to the conventional three-queue model when p=0, and corresponds to a two-queue model when p=1.

A few examples where the model is appropriate will suffice:

i. A 3-station robotic system where heavier traffic is experienced in a particular stream of arrivals thus tending to clog the waiting space available to other arrivals.
ii. A traffic controller at a three-way junction where traffic build-up is very high on a particular lane. We may desire to investigate the optimum value of p that minimizes the average waiting time of an arbitrary customer.
iii. A central processing unit which has to attend to three sets of jobs but one of the sets has more arrivals.
iv. A brewery producing three different brands of drinks with greater demand for one of them than the others.
v. A dynamically controlled traffic light at an intersection.

We mention a few relevant researches on polling systems. Eisenberg (1972) analyzed a polling system where the polling order was periodic, with exhaustive discipline at each queue. The analysis involved the study of the embedded process at four points namely: service beginning, service completion, visit beginning and visit completion. A similar work by Boxma and Down (1997) obtained closed and exact expressions for some key performance measures of the system in a two-queue model. Schassberger (1993) worked on a polling system with probabilistic order of service, called Bernoulli scheduling. The Bernoulli scheduling system has been solved only approximately. Another probabilistic case mentioned by Schassberger is one in which, after the completion of service at any queue, the next polled queue is queue j with probability p_j, where $\sum_{j=1}^{N} p_j = 1$, called random polling system.

Obilade (1983) derived a processor-sharing approximation to the strictly alternating switch (SAS) polling model. The SAS is basically an endogenous priority queueing model in which the next unit for service is selected not only based on what priority class it belongs but also on what priority class was last served. It represents a particular extreme case of switching from one queue to the other after a specified k≥1 number of units have been served in a two-queue system. The two queues are M/M/1-type and a feedback model was integrated into the system, in which either unit may require further service with some probability.

Coffman and Gilbert (1987) provided an analysis of a polling system with a greedy server on a circle and a line as well as insights about the stability condition. Schassberger (1993) solved the stability problem for polling systems with state-dependent routing, where the ergodicity of a symmetric ring-like network with a limited service policy has been proved. These results were generalized in Foss and Last (1995) which dealt with polling systems with a special greedy routing mechanism on a graph but with rather general service policies for each station. There are many other papers establishing comparison and stability results for polling systems with state-independent routing.

Brill and Hlynka (2000) studied an M/M/c queueing system in which there is a single special customer. This special customer is viewed as competing for service with the regular customers in the system. They obtained the waiting time distribution of this special customer under various modes of probability p, of the customer starting service at some regular service completion epoch.

In Wierman *et. al* (2007), a mean value analysis framework for analyzing the effect of scheduling within queues in classical asymmetric polling systems with gated or exhaustive service was presented. Their framework illustrated that a large class of scheduling policies behave similarly in the exhaustive polling model and the standard M/G/1 model, whereas scheduling policies in the gated polling model behave differently than in an M/G/1. They showed that the impact on mean response time from scheduling within a queue of a polling system can be dramatic.

Two symmetric M/G/1 type polling systems were investigated by Cooper *et. al* (1999) to pinpoint the effect on the efficiency of the system when the server configuration is such that it incurs switchover times even when the station is empty. In these models, the server spends time not only in inter-queue switchovers but also on warming up, that is, getting ready for queue service. The mean waiting times were determined and compared for different parameters of the times of switchover between the queues and warming-up times in order to verify whether they are constants or random variables.

In section two, we develop the steady-state system equations at the switch points, while the moments of the queue length distribution and the waiting times are presented in sections 3 and 4, respectively. Finally, our conclusions are expressed in section 5.

2 Steady-state system equations at the switch points

We shall use the embedded Markov chain technique to obtain the steady state system equations. This technique is applicable to systems with Poisson inputs. We are concerned at any instant t, with a group of random variables **N**(t), the number of customers in the system at time t, and X(t) the service time already received by the customer in service, if any. {N(t), t≥0} is non-Markovian, but {N(t),X(t), t≥0} is a Markov process.

Now, by observing the number in the system at switch points - the instant the server exhausts a particular queue and is about to switch to the next, rather than at all points in time t, it is possible to simplify matters to a great extent.

Using the principles developed by Takagi and Kleinrock (1984), we proceed to obtain the system equations at steady state.

Let $\tau_1, \tau_2, \dots, \tau_N$ be the time instants at the switch points. At each of these time instants τ_i, X(τ_i)=0, since the last customer just completed service, thus effectively reducing the dimension of the embedded Markov Chain to {N(τ_i),0, $\tau_i \geq 0$}.

Define $P_i(q_{i-1}, 0, q_{i+1}) = \Pr\{N_{i-1}(\tau) = q_{i-1}, N_i(\tau) = q_{i+1}\}$ as the joint probability that at an arbitrary switch point, the server has just completed a visit to queue i and q_j units are waiting in queue j, (j=i-1,i+1), i=1,2,3. $P_i(q_{i-1}, 0, q_{i+1})$ is the probability that at the instant when the server switches from queue i, the number of units waiting at queue i-1 and queue i+1 are q_{i-1} and q_{i+1}, respectively.

This state (i: $q_{i-1}, 0, q_{i+1}$) can occur through the following exhaustive and mutually exclusive events:

1. The server leaves queue i-1 and finds $k_i \geq 1$ units in queue i, where it spends k_i busy periods.
2. The server leaves queue i-1 and finds k_i=0 units waiting for service in queue i but at least one unit waiting for service elsewhere in the system, so that the server then passes through queue i in zero time.
3. The server leaves some queue and finds the system empty. With probability λ_i/λ, the next arrival (which reinitiates the process) occurs at queue i, where the server spends a single busy period.

The possible server cycles according to the model are:

1) 1-2-3 (Cycle 1, C1), Pr(C1)=1-p; or
2) 1-3 (Cycle 2, C2), Pr(C2)=p.

The probability state equations for i=1,2 are obtained using the information from the possible system states as specified above.

$$P_i(q_{i-1},0,q_{i+1}) = \sum_{k_i=1}^{\infty}\sum_{k_{i+1}=0}^{q_{i+1}} P_i(0,k_i,k_{i+1}) \int_0^{\infty} \frac{(\lambda_{i+1}t)^{q_{i+1}-k_{i+1}}}{(q_{i+1}-k_{i+1})!} e^{-\lambda_{i+1}t} \frac{(\lambda_{i-1}t)^{q_{i-1}}}{(q_{i-1})!} e^{-\lambda_{i-1}t} d\theta_i^{(k_i)}(t) + P_i(0,0,q_{i+1})\big(1-\delta(q_{i+1})\big)\delta(q_{i-1}) + \frac{\lambda_i}{\lambda} P(0) \int_0^{\infty} \frac{(\lambda_{i+1}t)^{q_{i+1}}}{(q_{i+1})!} e^{-\lambda_{i+1}t} \frac{(\lambda_{i-1}t)^{q_{i-1}}}{(q_{i-1})!} e^{-\lambda_{i-1}t} d\theta_i(t) \quad (2.1)$$

where $P(0) = \sum_{i=1}^{3} P_i(0,0,0)$

$$\delta(x) = \begin{cases} 1, & if\ x = 0 \\ 0, & if\ x \neq 0 \end{cases}$$

and $\theta_i(t)$ is the busy period distribution function while $\theta_i^{(k_i)}$(t) is its k_i-fold convolution.

The normalization condition is given by

$$\sum_{i=1}^{3}\sum_{q_{i-1}=0}^{\infty}\sum_{q_{i+1}=0}^{\infty} P_i(q_{i-1},0,q_{i+1}) = 1$$

In order to compute $P_3(q_1,q_2,0)$, we have to condition it on the server cycle so as to correctly specify the system at a 3-switch point. The conditional probabilities $P_3(q_1,q_2,0|C1)$ and $P_3(q_1,q_2,0|C2)$ are first derived:

$$P_3(q_1,q_2,0|C1) = \sum_{k_3=1}^{\infty}\sum_{k_1=0}^{q_1} P_2(k_1,0,k_3) \int_0^{\infty} \frac{(\lambda_1 t)^{q_1-k_1}}{(q_1-k_1)!} e^{-\lambda_1 t} \frac{(\lambda_2 t)^{q_2}}{(q_2)!} e^{-\lambda_2 t} d\theta_3^{(k_3)}(t) + P_3(0,0,q_1)\big(1-\delta(q_1)\big)\delta(q_2) + \frac{\lambda_3}{\lambda} P(0) \int_0^{\infty} \frac{(\lambda_1 t)^{q_1}}{(q_1)!} e^{-\lambda_1 t} \frac{(\lambda_2 t)^{q_2}}{(q_2)!} e^{-\lambda_2 t} d\theta_3(t) \quad (2.2)$$

and

$$P_3(q_1,q_2,0|C2) = \sum_{k_3=1}^{\infty}\sum_{k_2=0}^{\infty} P_1(0,k_2,k_3)\int_0^{\infty}\frac{(\lambda_1 t)^{q_1}}{(q_1)!}e^{-\lambda_1 t}\frac{(\lambda_2 t)^{q_2-k_2}}{(q_2-k_2)!}e^{-\lambda_2 t}d\theta_3^{(k_3)}(t)$$
$$+ P_1(0,q_2,0)\big(1-\delta(q_2)\big)\delta(q_1)$$
$$+\frac{\lambda_3}{\lambda}P(0)\int_0^{\infty}\frac{(\lambda_1 t)^{q_1}}{(q_1)!}e^{-\lambda_1 t}\frac{(\lambda_2 t)^{q_2}}{(q_2)!}e^{-\lambda_2 t}d\theta_3(t) \qquad (2.3)$$

The probability, $P_3(q_1,q_2,0)$ is the weighted average of both $P_3(q_1,q_2,0|C1)$ and $P_3(q_1,q_2,0|C2)$. That is

$$P_3(q_1,q_2,0) = P_3(q_1,q_2,0|C1)\Pr(C1) + P_3(q_1,q_2,0|C2)\Pr(C2) \qquad (2.4)$$

Equation (2.4), which is a combination of equations (2.2) and (2.3), reflects the impact of the polling probability p on the routing of the server when switching from queue 3. The equation reduces to equations (2.2) and (2.3) respectively, when p=0 and p=1, respectively.

The embedded Markov chain probability state equations derived above are very important in describing the system. The usefulness of these state equations is enhanced when they are transformed through their probability generating functions so that the moments could be easily obtained, which are of major interest in queueing theory.

We now define the joint probability generating function

$$G_i(z_{i-1},0,z_{i+1}) = \sum_{q_{i-1}=0}^{\infty}\sum_{q_{i+1}=0}^{\infty} P_i(q_{i-1},0,q_{i+1})\prod_{\substack{j=1\\ j\neq i}}^{3}(z_j)^{q_j}$$

The joint probability generating function for i=1,2 is

$$G_i(z_{i-1},0,z_{i+1}) = G_{i-1}\big(0,\theta_i^*\big[\textstyle\sum_{j\neq i}\lambda_j(1-z_j)\big],z_{i+1}\big) - P_{i-1}(0,0,0)\frac{\lambda_i}{\lambda}P(0)\theta_i^*\big[\textstyle\sum_{j\neq i}\lambda_j(1-z_j)\big] \qquad (2.5)$$

where $\theta_i^*(s)$ is the Laplace-Stieltjes transform (LST) of the type-i busy period distribution function, and is given by

$$\theta_i^*(s) = B_i^*[s + \lambda_i - \lambda_i \theta_i^*(s)] \tag{2.6}$$

We note that $G_i(0,1,1)$ is the probability that an arbitrary switch point is associated with queue i.

The joint probability generating function for i=3 is given by

$$G_3(z_1, z_2, 0) = (1-p)G_2\left(z_1, 0, \theta_3^*\left[\sum_{j\neq 3} \lambda_j(1-z_j)\right]\right) + pG_1\left(0, z_2, \theta_3^*\left[\sum_{j\neq 3} \lambda_j(1-z_j)\right]\right)$$
$$- pP_1(0,0,0) - (1-p)P_2(0,0,0)$$
$$+ \frac{\lambda_3}{\lambda} P(0)\theta_3^*\left[\sum_{j\neq 3} \lambda_j(1-z_j)\right] \tag{2.7}$$

Equation (2.7) gives the joint probability generating function of the state of the system at the 3-switch point. This result reflects the contribution of $G_1(.)$ and $G_2(.)$ to the joint probability generating function of the distribution of the system size at a 3-switch point. It also contains the probabilities of an idle system at both the 1- and 2-switch points, as well as the LST of the busy period distribution at queue 3.

3 Moments of number of units at the switch points

The essence of the joint probability generating functions of the system size at the switch points, which were derived in section 2, is to enable us obtain the moments of the underlying distribution. They also serve as useful tools in obtaining the distribution of waiting times.

We now define for $i \neq j$ and $j = i-1, i+1$, the moments of the steady state system size at the switch points.

$$g_i(j) = \frac{\partial}{\partial z_j} G_i(z_{i-1}, 0, z_{i+1})|_{z_{i-1}=z_{i+1}=1}$$

and

$$g_i(j,k) = \frac{\lambda(1-\rho)}{P(0)} \frac{\partial^2}{\partial z_j \partial z_k} G_i(z_j, 0, z_k)|_{z_j=z_k=1}$$

The marginal queue length, obtained as the solution to the first derivative of the state equations joint generating functions, is given as

$$g_{i-1}(i) = \frac{\lambda_i}{\lambda} P(0) \frac{\rho_i - \rho}{1 - \rho} \quad (3.1)$$

P(0), the overall probability of an idle system at an arbitrary switch point, can be estimated using a novel technique introduced in Mapp *et. al* (2010), called the Zero-server Markov chain for exhaustive service polling systems. It should be noted however, that in general,

$$P(0) \neq 1 - \rho$$

That is, for the polling model with probabilistic routing, the probability of an idle system at an arbitrary switch point is not the same with the general probability of an empty system, which is equal to $1 - \rho$.

By differentiating $\{G_i(z_j, 0, z_k);\ \mathrm{i, j, k} = 1{,}2{,}3, \mathrm{i} \neq \mathrm{j, k}\}$ with respect to z_j and z_k and then setting z_j=$z_k = 1$ we have a set of $3^2 = 9$ recursive equations for $\{g_i(j,k)$; i,j,k=1,2,3, i≠j,k$\}$. The expression $g_i(j,k)$ is the cross correlation of the mean queue lengths at queues j and k at an i-switch point. The values of these 9 cross correlations:

$(g_1(2{,}3), g_1(2{,}2), g_1(3{,}3), g_2(1{,}1), g_2(1{,}3), g_2(3{,}3), g_3(1{,}1), g_3(1{,}2), g_3(2{,}2))$ are obtained by solving the set of 9 equations with 9 unknowns, and this is achieved numerically. Such equations are amenable to numerical solutions, as results would converge numerically in a reasonable number of steps.

$$\begin{aligned} g_i(j,k) = {} & g_{i-1}(j,k) + g_{i-1}(i,j)\lambda_k\theta_i \\ & + g_{i-1}(i,k)\lambda_j\theta_i + g_{i-1}(i,i)\lambda_j\lambda_k\theta_i^2 \\ & + \lambda_j\lambda_k\lambda_i(1-\rho_i)\theta_i^{(2)}, \quad j \neq i, i-1, \quad k \neq i, i-1 \end{aligned} \quad (3.2)$$

$$\begin{aligned} g_i(i-1,k) = {} & g_{i-1}(i,k)\lambda_{i-1}\theta_i + g_{i-1}(i,i)\lambda_{i-1}\lambda_k\theta_i^2 \\ & + \lambda_{i-1}\lambda_k\lambda_i(1-\rho_i)\theta_i^{(2)}, \qquad k = i, i-1 \end{aligned} \quad (3.3)$$

$$g_i(i-1, i-1) = g_{i-1}(i,i)(\lambda_{i-1}\theta_i)^2 + \lambda_{i-1}^2\lambda_i(1-\rho_i)\theta_i^{(2)} \quad (3.4)$$

4 Waiting times

For the system, we define a *super cycle* as the elapsed time between the arrival instant of a unit at any queue when the system is empty, and the first instant at which the system becomes empty again.

The units that arrive into queue i can be classified into two exclusive and exhaustive types:

1) arrivals that either initiate a super cycle or occur during the first busy period in a super cycle; or
2) all other arrivals that occur after (and including) the second busy period at queue i in a super cycle.

The Laplace-Stieltjes transform (LST) of the distribution of the waiting time of type 1 and type 2 units are given as

$$W_i^*(s|type\ 1) = \frac{s(1-\rho_i)}{s-\lambda_i+\lambda_i B_i^*(s)} \tag{4.1}$$

and

$$W_i^*(s|type\ 2) = \frac{\lambda_i(1-\rho_i)[G_{i-1}(0,1,1) - G_{i-1}\left(0,1-{}^{s}/_{\lambda_i},1\right)]}{g_{i-1}(i)[s-\lambda_i+\lambda_i B_i^*(s)]} \tag{4.2}$$

The mean number of arrivals into queue i during an interval of length t is $\lambda_i t$. The probability that an arbitrary arrival at queue i finds the system empty is $(1-\rho)$, so that $\lambda_i t(1-\rho)$ is the mean number of arrivals at queue i that initiate a super cycle during any length of time t. The mean number of units served in a busy period generated by each such arrival is$1/(1-\rho_i)$, hence the mean number of arrivals at queue i which initiate a super cycle during any elapsed time t is

$$\frac{\lambda_i t(1-\rho)}{(1-\rho_i)}$$

The probability that an arbitrary arrival into queue i is of type 1, $P_i(type\ 1)$ is given as

$$P_i(type\ 1) = \frac{\lambda_i t(1-\rho)/(1-\rho_i)}{\lambda_i t} = \frac{(1-\rho)}{(1-\rho_i)} \tag{4.3}$$

and

$$P_i(type\ 2) = 1 - P_i(type\ 1) = \frac{(\rho-\rho_i)}{(1-\rho_i)} \tag{4.4}$$

Thus the Laplace-Stieltjes transform $W_i^*(s)$, of the waiting time distribution is the weighted average of both types:

$$W_i^*(s) = P_i(type\ 1)W_i^*(s|type\ 1) + P_i(type\ 2)W_i^*(s|type\ 2)$$

Hence

$$W_i^*(s) = \frac{s(1-\rho)}{s-\lambda_i+\lambda_i B_i^*(s)} + \frac{\lambda_i(\rho-\rho_i)[G_{i-1}(0,1,1) - G_{i-1}\left(0,1-{}^{s}/_{\lambda_i},1\right)]}{g_{i-1}(i)[s-\lambda_i+\lambda_i B_i^*(s)]} \quad (4.5)$$

We shall now consider the LST $W_3^*(s)$.

$$W_3^*(s) = \frac{s(1-\rho)}{s-\lambda_3+\lambda_3 B_3^*(s)} \quad (4.6)$$
$$+ \frac{\left[(\rho-\rho_3)\left\{G_1(0,1,1)\left[G_2(1,0,1)-(1-p)G_2\left(1,0,1-{}^{s}/_{\lambda_3}\right)\right]-G_2(1,0,1)pG_1\left(0,1,1-{}^{s}/_{\lambda_3}\right)\right\}\right]}{[(1-p)G_1(0,1,1)g_2(3)+pG_2(1,0,1)g_1(3)](s-\lambda_3+\lambda_3 B_3^*(s))}$$

The expected waiting time at queue i, $E(W_i)$ for i=1,2, is given as

$$E(W_i) = -\frac{\partial}{\partial s}W_i^*(s)|_{s=0} = \frac{\lambda_i b_i^{(2)}}{2(1-\rho_i)} + \frac{g_{i-1}(i,i)}{2\lambda_i^2(1-\rho_i)} \quad (4.7)$$

and the expected waiting time at queue 3, $E(W_3)$ is given as

$$E(W_3) = \frac{\lambda_3 b_3^{(2)}}{2(1-\rho_3)} + \frac{(1-p)\lambda_1 g_2(3,3) + p\lambda_2 g_1(3,3)}{2\lambda_3^2(1-\rho_3)[(1-p)\lambda_1 + p\lambda_2]} \quad (4.8)$$

Equation (4.8) thus establishes the mean waiting time at queue 3 and its relationship with the second factorial moments of the marginal queue length distribution in queue 3 when the server switches either from queue 1 ($g_1(3,3)$) or queue 2 ($g_2(3,3)$). Hence $E(W_3)$ is a function of the amount of dispersion in the marginal queue length at queue 3, and it is inversely proportional to the polling or selection probability, p.

It is interesting to note that for the case p=0, $E(W_3)$ reduces to

$$E(W_3) = \frac{\lambda_3 b_3^{(2)}}{2(1-\rho_3)} + \frac{g_2(3,3)}{2\lambda_3^2(1-\rho_3)} \quad (4.9)$$

The above expression coincides with previously obtained results in Eisenberg (1972) and Takagi and Kleinrock (1984).

5 Conclusion

The steady-state system equations for the three-queue polling model with probabilistic routing, which were described in section 3 are a fundamental aspect in deriving various performance measures of the system. They are involved in the formulation of the queue length distribution, the intervisit-time distribution and the waiting time distribution.

A unified embedding scheme as presented in this paper greatly simplifies the process of obtaining the distribution of the performance measures of the polling model. Moreover, operations of dynamically controlled and automated traffic control systems can be enhanced by the insights provided in this work.

Acknowledgement

The authors are grateful to the anonymous reviewers who made useful and invaluable comments and suggestions. We also acknowledge the Faculty of Physical Sciences, University of Ilorin for sponsoring this maiden edition.

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