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On The Free Vibration Analysis of Simply Supported Beam with Shear Deformation

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Abstract

This paper considered the free vibration analysis of a simply supported beam with shear deformation. The governing equation of fourth order partial differential equation was reduced to an ordinary differential equation using series solution which is the solution of the governing equation. Numerical result was presented and plotted against x, it was found that the response amplitude of the beam increases with increase in the length of the beam and radius of gyration but reduces with an increase in foundation modulus and shear modulus. To keep the deflection of the beam in check, there's need to keep the length of the beam and the radius of gyration in check to avoid over deflection of the beam which can cause damage in building structures and so on.

Keywords: Shear Modulus, Foundation Modulus, Shear Deformation, Radius of Gyration, Amplitude.

1. Introduction

A beam is a structural element that primarily resists loads applied laterally to the beam's axis. Beams are widely used in various engineering fields, such as bridges, tall buildings, and helicopter rotor blades. A large number of studies can be found in literature about the free vibrations of beams. Free vibrational problems of beams are often described by, partial differential equations and in most cases it is extremely difficult to find their exact solutions. Consequently, several methods of approximate analytical and numerical solutions such as Rayleigh-Ritz, separation of variable, Fourier Transform, Galerkin, finite difference, finite element, and spectral finite element methods have been used to obtain the free vibration of beams. An exact invention of the beam problem was first studied by Chree (1889).

They deduced the equations that described a vibration of a solid cylinder. However, it is impractical to solve the full problem because it results in more information than actually

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needed in applications. Therefore, approximate solutions for transverse displacement are adequate. Free vibration equation of the beam on partially elastic foundation including only bending moment effect was analytically solved by Doyle and Pavlovic (1982) while the eigenvalues for free vibration of beam-column systems on elastic foundation were obtained using a numerical approach in West and Mafi (1984). Chen and Ho (1996, 1999) used differential transform method (DTM) to solve eigenvalue problems for free and transverse vibration problems of a rotating twisted Timoshenko beam under axial loading.

The dynamic behaviour of this beam vibrating at its first mode was canvassed using this fatigue model. Duan and Wang (2013) demonstrated the free vibration of beams with multiple step changes using the modified discrete singular convolution (DSC). The jump conditions at the steps were used to overcome the difficulty in using ordinary DSC for dealing with ill-posed problems. A transfer matrix method and the Frobenius method were adopted by Lee and Lee (2016) to solve the free vibration characteristics of a tapered Bernoulli-Euler beam and obtain the power series solution for bending vibrations. Guojin *et al.* (2016) examined the free flexural free vibration of multi-step non-uniform beams. Using appropriate transformations, the differential equation for flexural free vibration of one-step beam with variable cross section was reduced to a fourth-order differential equation with constant coefficients. According to different types of roots for the characteristic equation of fourth-order differential equation with constant coefficients, two kinds of modal shape functions are obtained, and the general solutions for flexural free vibration of one-step beam with variable cross section are presented.

An exact approach to solve the natural frequencies and modal shapes of multistep beam with variable cross section is presented by using transfer matrix method, the exact general solutions of one-step beam, and iterative method. Numerical examples reveal that the calculated frequencies and modal shapes are in good agreement with the finite element method (FEM). Usman *et al.* (2018) presented an analysis of free vibrations of a cantilever beam and simply supported beam using series solution.

2. Materials and Methods

2.1 Mathematical Formulation

The free vibration of Euler-Bernoulli beam with shear deformation has the governing equation of the form:

$$EI\frac{\partial^4 V(x,t)}{\partial x^4} - \rho A\frac{\partial^2 V(x,t)}{\partial t^2} + \rho A r_G^2\left[1+\frac{E}{KG}\right]\frac{\partial^4 V(x,t)}{\partial t^2 \partial x^2} + \left[\frac{\rho^2 A r_G}{KG}\right]\frac{\partial^4 V(x,t)}{\partial t^4} = 0, \tag{1}$$

where x is the spatial coordinate, t is the time, $V(x,t)$ is the deflection of the beam, E is the Young's modulus, I is the moment of inertia of the beam's cross section A, the neutral axis ρ is the Mass per unit length of the beam, K is the foundation Modulus, G is the Shear Modulus, r_G is the radius of gyration. With the boundary conditions:

$$V(0,t) = 0 = V(l,t), \tag{2}$$

$$\frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(l,t)}{\partial x^2}. \tag{3}$$

Without loss of generality, one can consider the initial conditions of the form

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \tag{4}$$

2.2 Method of Solution

Assume a solution such that the transverse vibration of the beam may be expressed in the following series form:

$$V(x,t) = U(x)\cos(\omega_n t). \tag{5}$$

Substituting the above equation and its derivatives into equation (1), we obtain the following:

$$EIU^{iv}(x)\cos(\omega_n t) - \omega_n^2\rho A U(x)\cos(\omega_n t) + \omega_n^2 \rho A r_G^2\left[1+\frac{E}{KG}\right]U''(x)\cos(\omega_n t) + \left[\frac{\rho^2 A r_G}{KG}\right]\omega_n^4 U(x)\cos(\omega_n t) = 0 \tag{6}$$

Equation (6) can also be rewritten as

$$EIU^{iv}(x) - \omega_n^2\rho A U(x) + \omega_n^2 \rho A r_G^2\left[1+\frac{E}{KG}\right]U''(x) + \left[\frac{\rho^2 A r_G}{KG}\right]\omega_n^4 U(x) = 0 \tag{7}$$

with the boundary conditions:

$$\frac{\partial^2 U(0)}{\partial x^2} = \frac{\partial^2 U(L)}{\partial x^2} = 0; U(0) = U(L) = 0. \tag{8}$$

Equation (7) can be rewritten as:

$$U^{iv}(x) - \frac{\omega_n^2 \rho A}{EI} U(x) + \frac{\omega_n^2 \rho A r_G^2}{EI}\left[1 + \frac{E}{KG}\right] U''(x) + \left[\frac{\omega_n^4 \rho^2 A r_G}{EIKG}\right] U(x) = 0, \tag{9}$$

and representing $B = \frac{\omega^2 n A r_G{}^2}{EI}\left[1 + \frac{E}{KG}\right]; C = \sqrt{\frac{\omega_n{}^2 \rho A}{EI}\left[1 - \frac{\rho \omega_n{}^2 r G}{KG}\right]}$.

Equation (9) becomes

$U^{iv}(x) + BU'''(x) - C^2U(x) = 0,$ (10)

Using the notation $C^2 = \lambda^2\gamma^2$ and $B = \lambda^2 - \gamma^2$ to have $U(x) = D\cos\gamma x + E\sin\gamma x + F\cosh\lambda x + G\sinh\lambda x$. Using the boundary condition in equation (8), we have that $Sin\ \gamma x = Sin \frac{n\pi}{L} x$ and hence

$$V(x,t) = \sin\frac{n\pi}{L}x\cos\omega_n t \tag{11}$$

Next, we find ω_n, substituting back into the governing equation, that is, equation (1), we have

$$\begin{aligned} &EI\frac{n^4\pi^4}{L^4}\sin\frac{n\pi}{L}x\cos\omega_n t - \rho A r^2\frac{n^2\pi^2}{L^2}\omega_n^2\sin\frac{n\pi}{L}x\cos\omega_n t \\ &+ \rho A r_G^2\left[1 + \frac{E}{KG}\right]\frac{n^2\pi^2}{L^2}\omega_n^2\sin\frac{n\pi}{L}x\cos\omega_n t + \omega_n^4\left[\frac{\rho^2 A r_G^2}{KG}\right]\sin\frac{n\pi}{L}x\cos\omega_n t = 0 \end{aligned} \tag{12}$$

and following from equation (12), we have that

$$EI\frac{n^4\pi^4}{L^4} - \rho A r_G^2\left[1 + \frac{E}{KG}\right]\frac{n^2\pi^2}{L^2}\omega_n^2 + \left[\frac{\rho^2 A r_G^2}{KG}\right]\omega_n^4 = 0.$$

The above equation can be rearranged as:

$$\left[\frac{\rho^2 A r_G^2}{KG}\right]\omega_n^4 - \rho A r_G^2\left[1 + \frac{E}{KG}\right]\frac{n^2\pi^2}{L^2}\omega_n^2 + EI\frac{n^4\pi^4}{L^4} = 0$$

$$Hw_n^4 - Iw_n^2 + J = 0,$$

where $H = \left[\frac{\rho^2 A r_G^2}{KG}\right]$; $I = \rho A r_G^2\left[1 + \frac{E}{KG}\right]\frac{n^2\pi^2}{L^2}$; $J = EI\frac{n^4\pi^4}{L^4}$ and $\omega_{n_{1,2}}^2 = \frac{I \pm \sqrt{I^2 - 4HJ}}{2H}$,

hence

$$\omega_{n_{1,2}} = \pm\sqrt{\frac{I \pm \sqrt{I^2 - 4HJ}}{2H}}. \tag{13}$$

Equation (12) together with (13) is the deflection of the beam.

3. Result and Discussion

Beam dimension and specification:

Table 1: Beam Specification and Dimension. Source: Oni and Jimoh (2014).

PARAMETERS	VALUES
The beam was made of steel E	$2.10 \times 10^{1}1$N
Length(L)	10, 12, 14
Breadth	0.05 m
Height	0.15 m
Shear Modulus G	150000, 210000, 270000
Density of the mass (ρ)	3.6777×10^{5} Kg/m^3
Radius of gyration r_G	1, 11, 21
Foundation Modulus K	40000,80000, 120000 N/m^3
Surface area of the beam cross section A	7.5×10^{-3} m^2
Moment of Inertia	I8.33×10^{-17}m^4

Figure (1) shows the deflection profile of the beam for different values of Shear modulus G (150000) for fixed value of foundation modulus, 40000 N/m^3. It is observed that there is periodic deflection in the response amplitude such that there is a decrease in the response amplitude as G increases. Figure (2) displays the deflection of beam for various values of K for fixed values of Shear modulus G (210000). It is observed that the deflection is also periodic such that there is a decrease in the response amplitude as K (80000 N/m^3) increases. Figure (3) shows the deflection of beam for different values of L for fixed values of Shear modulus G (270000) and foundation modulus K, 120000 N/m^3. The response amplitude of the deflection increases with an increase in the length of the beam. Figure (4) depicts the deflection of the beam for G = 150000 at various values of radius of gyration with fixed value of foundation modulus K, 40000 N/m^3. It is seen that the response amplitude of the beam increases as the radius of gyration increases.

Figures (5) displays the deflection of the beam for G = 210000 at various values of radius of gyration with fixed value of foundation modulus K, 80000 N/m^3. It is seen that the response amplitude of the beam increases as the radius of gyration increases. Figures (6) shows the deflection of the beam for G = 270000 at various values of radius of gyration with fixed value of foundation modulus K, 120000 N/m^3. It is seen that the response amplitude of the beam increases as the radius of gyration increases. The above results presented in this chapter confirm the results in Oni and Jimoh (2014).

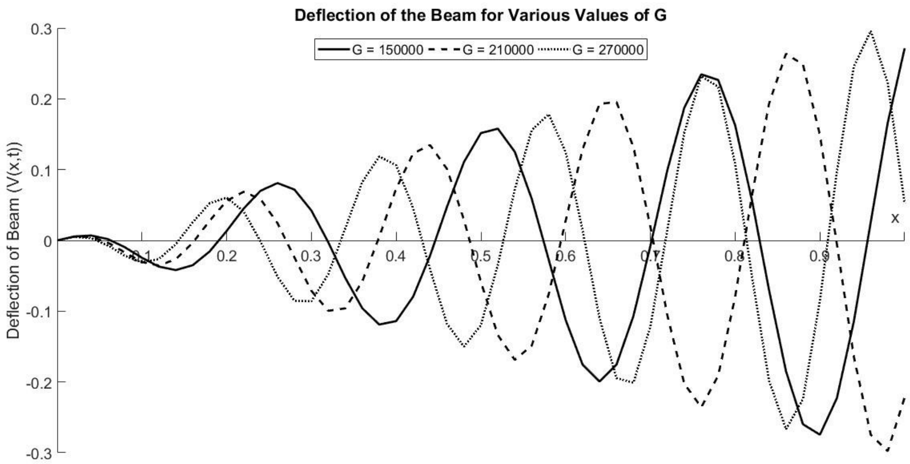

Figure 1: Deflection of Beam for various values of G.

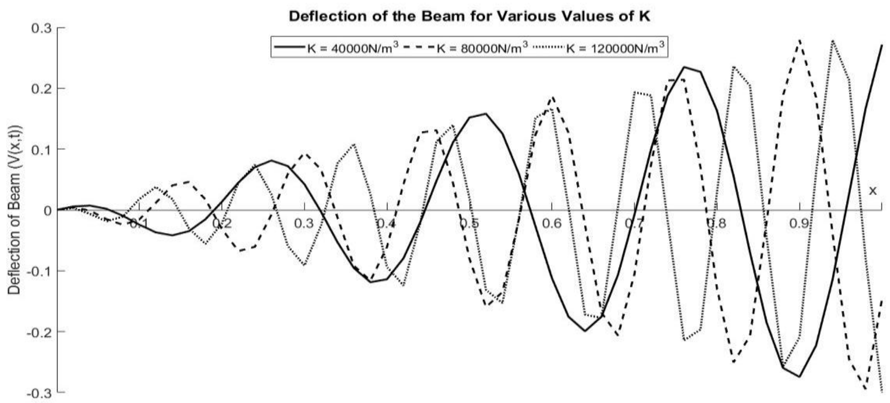

Figure 2: Deflection of Beam for various values of K.

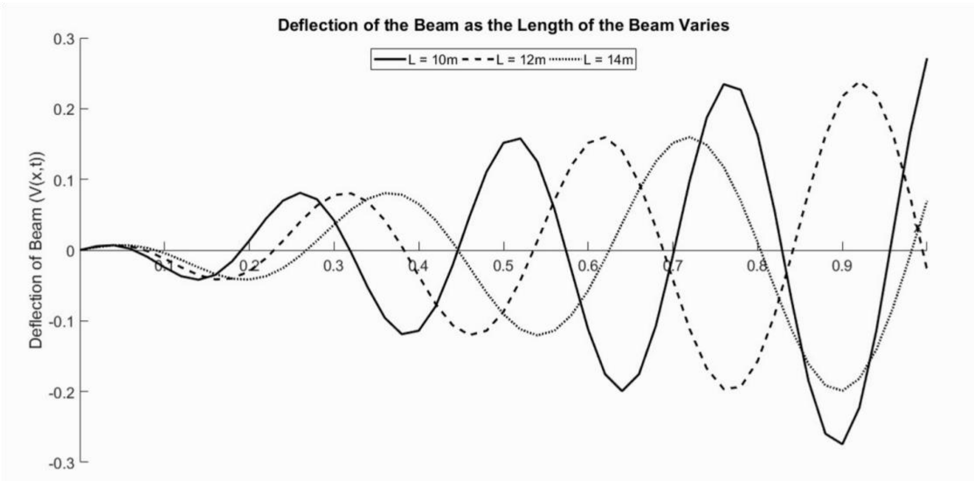

Figure 3: Deflection of Beam for various values of L.

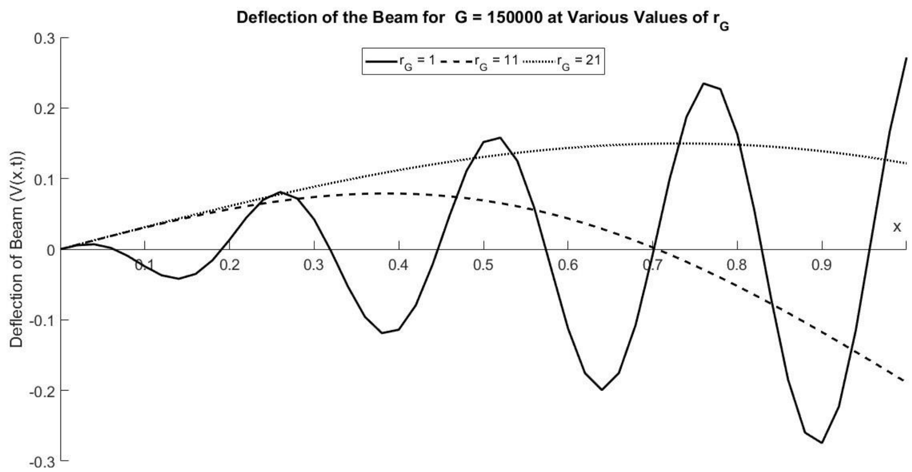

Figure 4: Deflection of Beam at G = 150000 for various values of r_G.

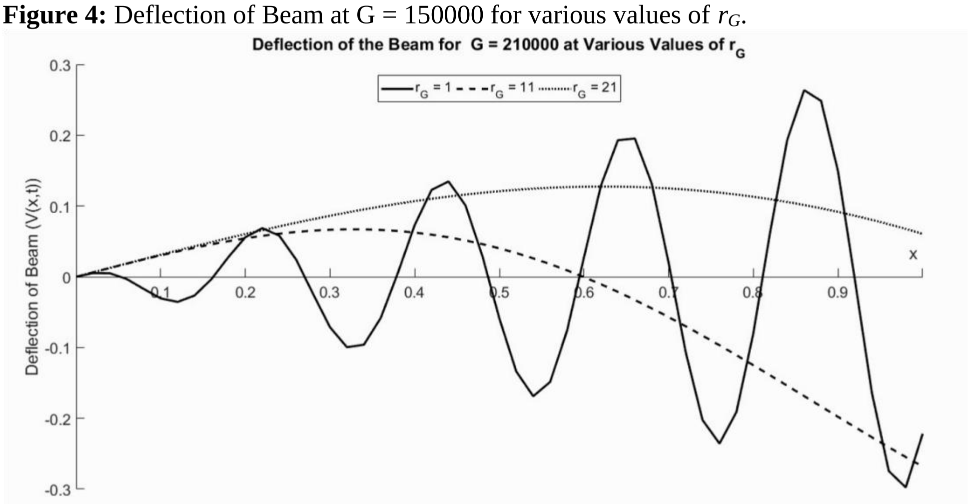

Figure 5: Deflection of Beam at G = 210000 for various values of r_G.

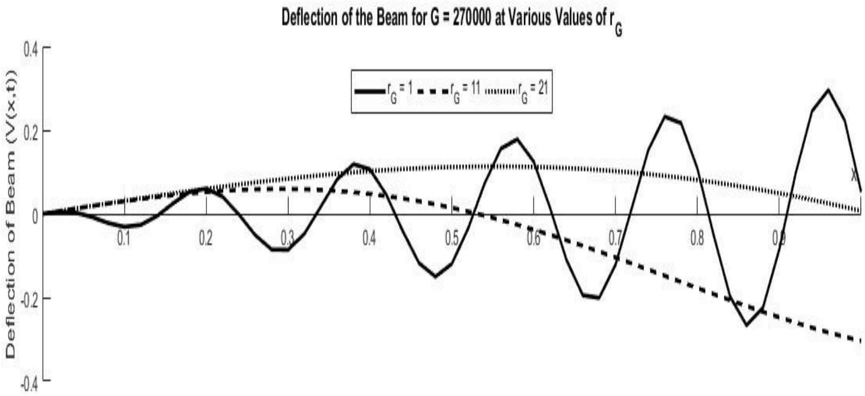

Figure 6: Deflection of Beam at G = 270000 for various values of $r_{G.}$

4. Conclusion

Free vibration analysis of a simply supported beam with shear deformation was considered in this paper work. The governing equation of fourth order partial differential equation was reduced to an ordinary differential equation using an assumed solution. The frequency of the oscillation was found and substituted back into the assumed series solution which is the solution of the governing equation. Numerical result was presented and plotted against x for various parameters like length of the beam, shear modulus, foundation modulus, and radius of gyration using a computer program (MATLAB). From the numerical results, we conclude that the response amplitude of the beam increases with increase in the length of the beam and radius of gyration but reduces with an increase in foundation modulus and shear modulus. To keep the deflection of the beam in check, there's need to keep the length of the beam and the radius of gyration in check to avoid over deflection of the beam which can cause damage in building structures and so on.

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