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Optimization of Apiary Products using Crisp Deterministic Method

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Abstract
Apiculture is the scientific method of beekeeping that enhance the availability of honey bee products. Designing a model that considers the foraging activities of the worker bees is the main focus in this paper. Crisp deterministic method was adopted in solving the problem while foragers were classified into pollen foragers, nectar foragers and water foragers using real data obtained from a bee farmer in Obior, Aniocha North Local Government Area, Delta State. The result obtained shows that distribution of beehives in the apiary in connection to source of food and water will maximize the products of honey bee, reduce competition and death among foragers and thereby improve the output.

Keyword: Apiculture, beehives, forager, colony survival, crisp deterministic model

1. Introduction
Apiculture is the modern scientific method of beekeeping. It is the maintenance of bee colonies, usually in man-made hives, by humans (Das *et al*, 2022). According to Food and Agriculture Organization of the United Nations (FAO) (2021), beekeeping deals with the practical management of social bee species, mostly within farming systems, and significantly contributes to food and nutrition security, poverty reduction and economic growth of the society. Honey bee production (honey beekeeping) is the rearing of honey bee for honey and other by-products (Khan and Khan, 2018). The products of honey bee include: honey, royal jelly, wax and propolis etc (Asem and Machathoibi, 2021).

Honey bees are social insects which are considered to be one of the most highly developed social insects in the family of invertebrate animals due to their social life which involves exhibiting very well adapted collective behaviors. They live in colonies comprise of thousands of workers, some few drones and only one queen (Ndu, 2017). Honey bees have the capacity of laying up to 1500 eggs or more per day depending on some factors like season and colony strength (Abou-Shaara *et al*, 2021). According to Bagheri and Mirzaie (2019), honey bees have four main development stages in their life cycle: egg, larva, pupa, and adult.

In setting up an apiary, the foraging capabilities of the honey bee is an essential factor to consider. According to Bagheri and Mirzaie (2019), the major duties in a bee hive are been carried out by worker bees and could be divided into two groups, young and older worker bees. Honey bees mostly forage for nectar, pollen and water within a kilometre of their hive and up to about five kilometres for exceptionally rewarding sources.

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Optimizaton of honey bee products is at moment paramount in the nation's economy as it will contribute to food security, poverty reduction, employment creation and income generation. A mathematical optimization problem is one in which some real-valued function is either maximized or minimized relative to a given set of feasible alternatives. The optimal solution to an optimization problem is given by the values of the decision variables that attain the maximum (or minimum) value of the objective function over the feasible region.

In a large apiary site, to determine optimal placement of bee colonies can be regarded as a combinatorial problem solvable using mathematical programming tools. Formulated mixed - integer linear programs can be used in deciding the best location of beehives taking into account the preferred location of the beekeeper, number and strength of available colonies, carrying capacity of the plant clusters, maximum flight distance that the bees can travel, and spatial orientation of the apiary. Komasilova *et al* (2021) designed a model for finding the number of honey bee colonies needed for the optimal foraging process in the specific location, taking into account several assumptions. Atanas and Ivan (2021), applied multi-criteria mathematical model to determine the optimal location of honey bee colonies in regions without overpopulation. Aderinto *et al* (2021), designed a crisp deterministic mixed integer honey bee production model that basically considered nectar and pollen to optimize the distribution of beehives in the apiary in order to maximize production of honey and minimize unhealthy competition among foraging bees.

In this paper, the intention is to extend the model of Aderinto *et al* (2021) to include compartment of honey bee foraging for water which is one of the requirements for colony survival and sustainability as well as justifying maximize production.

2. Formulation of Model

Crisp deterministic method applied in solving mixed-integer problems possesses a certain degree of effectiveness, firmness and freshness if given enough time and provided the problem satisfies certain conditions such as convexity, will terminate with a guaranteed solution or an indication that the problem has no integer solution.

In this paper, an optimization model for efficient apicultural management that will enhance honey production through optimal placement of bee colonies in the apiary for successful foraging using deterministic approach is constructed. The constructed model was solved using modified nature inspired optimization method. Lingo 17.0 software capable of dealing with mixed-integer linear programming problems which will help in determining the best location of beehives for pollen, nectar and water foragers was applied to run the inputted data. The following decision variables and parameters in Table 1 shall be used in the formulation of the model.

2.1 Decision Variable

FP_{ij} : Total number of pollen foragers of apiary site i that can be accommodated by pollen plant j, $FP_{ij} \in$ R

FN_{ib} : Total number of nectar foragers of apiary site i that can be accommodated by nectar plant b, $F\ Nib$ $\in$ R

Li : Total number of foragers in apiary site i not accommodated by plant clusters, $Li \in$ R

$$V_{ki} = \begin{cases} 1 & if \text{ hive k is located in apiary site i} \\ 0 & if \text{ hive k is not located in apiary site i} \end{cases}$$

DFi : Total number of foragers that died during foraging activities in apiary site i.

Table 1: Specified Parameters and Its Definition

PARAMETER	DESCRIPTION
H	total number of beehives
P	total number of pollen plant clusters
N	total number of nectar plant clusters
M	total number of apiry sites
R	total number of water source
T_k	strength of hive k
y_j	carrying capacity of pollen plant cluster j, $y_j \in$ R
y_b	carrying capacity of nectar plant clusters b, $y_b \in$ R
y_s	carrying capacity of water source s, $y_s \in$ R
μ_j	mean of y_j
μ_b	mean of y_b
σ_j	standard deviation of y_j
σ_b	standard deviation of y_b
pw_i	priority weight given to apiary site i, $pw_i \in$ R
N	large positive number applied as penalty weight to minimize overcrowding
$j \in Q_i$	indicates that pollen plant cluster j is connected to apiary site i
$b \in Q_i$	indicates that nectar plant cluster b is connected to apiary site i
$s \in Qi$	indicates that water source s is connected to apiary site i
$i \in R_j$	indicates that apiary site i is connected to pollen plant cluster j
$i \in R_b$	indicates that apiary site i is connected to nectar plant cluster b
$i \in R_s$	indicates that apiary site i is connected to water source s

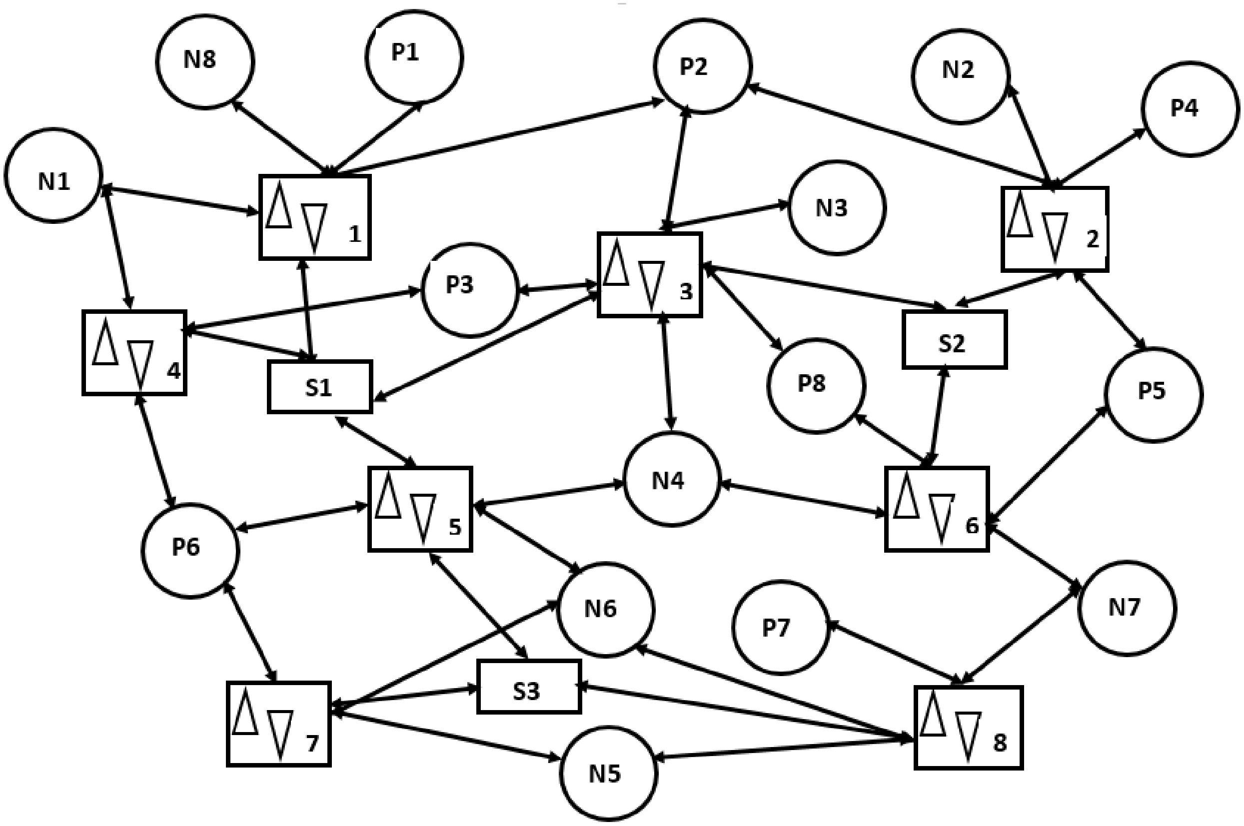

Figure 1: Graphical Representation of Apiaries in Connection to Foraging Activities.

Figure 1 is a graphical representation showing the number of sites, beehives, plant clusters, water sources and their connections that linked them up. The apiary sites are indicated by the square shapes and in it are triangular shapes representing beehives of pollen and nectar foragers. The circular shapes ($N1, N2, N3, N4, N5, N6, N7$ *and* $N8$) represents the nectar plant clusters while the circular shapes ($P1, P2, P3, P4, P5, P6, P7$ *and* $P8$) represents the pollen plant clusters. The rectangular shapes $S1, S2, S3$ indicate the sources of water for the foraging bees. The arrows signify the connections between the apiary sites to the source of feed (nectar/pollen) and water.

2.2 Crisp Deterministic Mixed-integer Honey Bee Productivity Model

The development of a colony is strongly associated with the weather, pollen and the availability of nectar. High yields of honey can be obtained when colony strength get to its peak at the time of high nectar flow. With plenty of pollen supply and nectar flow, the queen bee is fed more and as a consequence she lays more eggs. On the contrary, when there is scarcity of food, hive bees will feed the queen less, and as a result she lays fewer eggs and the bee population declines. Generally, when the peak colony strength reaches its maximum, most brood cells are capped and honey is gathered in the honeycombs.

There are problems often encounter by beekeepers such as death of foraging bees, competition due to insufficient plant clusters in a given area, inadequate water sources and swarming. To address these challenges, a mathematical program, crisp deterministic mixed-integer productivity model to maximize the foraged feed is formulated as:

Maximize $\sum_{i=1}^{m}\left[\left(pw_i \sum_{j\in Q_i} FP_{ij} + \sum_{b\in Q_i} FN_{ib} + \sum_{s\in Q_i} F_{is}\right) - (NL_i + DF_i)\right]$ (1)

Subject to

$$\sum_{i\in R_j} FP_{ij} \leq y_j \quad j = 1,2,\dots,p \tag{2}$$

$$\sum_{i\in R_b} FN_{ib} \leq y_b \quad b = 1,2,\dots,n \tag{3}$$

$$\sum_{i\in R_s} F_{is} \leq y_s \quad s = 1,2,\dots,r \tag{4}$$

$$\sum_{i=1}^{m} V_{ki} \quad k = 1,2,\dots,h \tag{5}$$

$$\sum_{i=1}^{h} T_k V_{ki} - \sum_{j\in Q_i} FP_{ij} - \sum_{i\in R_b} FN_{ib} - L_i \quad \forall i = 1,2,\dots,m \tag{6}$$

The beekeeper's preference weight is influenced by some factors relating to the site such as nearness to residence, source of water, availability of feeds (plant clusters), disease free environment and other relevant factors. It is imperative to first maximize the apiary sites with adequate carrying capacities and higher preference weight before those of lower preference weights. The rate of death of bees is attributed to strenuous foraging activities, insufficient feeds, pest/disease infections which will be minimized in the objective function.

The objective function is meant to maximize the strength of the bee hives through the accumulation of sufficient feed (nectar and pollen) and water by the foraging bees. The penalty weight *N* may exceed the beekeeper's expected weight. Hence minimization of the overcrowd and the rate of death of foraging bees which occurs as a result of flight stress, disease, age as well as other environment hazards. In equation (2), it is expected that the number of pollen bee foragers must not exceed the carrying capacities of the pollen plants Equation (3), states that the number of nectar bee foragers must not exceed the carrying capacities of the nectar plants. Equation (4), shows that the bees foraging for water must not exceed the carrying capacities of the water source. In equation 5, it is expected that each hive is located to exactly one apiary site. In equation 6, It is expected that if V_{ki} = 1 for some *k*, then $\sum_{i=1}^{h} T_k V_{ki} > 0$, which makes $\sum_{j\in Q_i} FP_{ij} + \sum_{i\in R_b} FN_{ib} + L_i > 0$. The implication of this, is that the plant clusters connected to apiary site *i* is compelled to accommodate all the strength of the hives allocated to apiary site *i*. Nevertheless, the values of $\sum_{j\in Q_i} FP_{ij}$ and $\sum_{i\in R_b} FN_{ib}$ are restrained by constraints (1) and (2) respectively. On the other hand, overpopulation arises whenever the plant clusters connected the apiary site *i* can not accommodate all the strength of the hives thus *Li* is compelled to be greater than zero.

2.3 Data Collection

The data obtained from a bee farmer in Obior, Aniocha North Local Government Area, Delta State, was used to implement the model.

Table 2: Strength of each hives

Hives	Strength (x 15,000)
h_1	[1.2, 1.6]
h_2	[1.7, 2.0]
h_3	[0.5, 1.7]
h_4	[0.2, 1.2]
h_5	[1.6, 2.0]
h_6	[0.4, 1.3]
h_7	[0.9, 1.9]
h_8	[1.5, 2.2]

Table 3: Carrying capacity of pollen plant cluster (×10, 000)

	$w_{1,j}$	$w_{2,j}$	$w_{3,j}$	$w_{4,j}$	$w_{5,j}$	$w_{6,j}$	$w_{7,j}$	$w_{8,j}$	$w_{9,j}$	$w_{10,j}$	$w_{11,j}$	$w_{12,j}$	μ_j	σ_j	$\Phi_j^{-1}(0.1)$
$PPC1$	0.97	1.30	0.81	1.42	0.69	1.00	1.51	1.33	0.88	1.03	0.53	2.69	1.18	0.54	0.49
$PPC2$	0.60	0.51	0.29	0.72	0.63	0.84	0.48	0.92	1.20	0.16	0.64	0.53	0.63	0.27	0.28
$PPC3$	1.51	0.70	1.08	0.40	1.80	1.65	0.79	1.80	0.74	1.00	0.92	1.20	1.22	0.39	0.72
$PPC4$	0.81	2.11	1.22	3.06	2.60	0.92	2.61	1.86	0.67	2.01	1.79	0.18	1.65	0.88	0.52
$PPC5$	1.73	1.08	2.11	0.47	2.19	0.28	1.60	1.50	2.00	0.74	2.40	1.00	1.43	0.67	0.57
$PPC6$	2.10	0.15	1.64	1.72	0.44	0.99	2.71	1.52	1.06	0.54	1.24	0.53	1.22	0.72	0.30
$PPC7$	3.11	0.42	1.12	2.61	0.74	0.31	1.50	2.02	0.09	1.11	2.04	1.83	1.41	0.91	0.24
$PPC8$	1.41	2.31	2.01	1.91	0.05	1.96	1.60	1.60	0.33	0.68	2.66	1.39	1.49	0.76	0.52

Table 4: Carrying capacity of nectar plant cluster (×10, 000)

	$w_{1,b}$	$w_{2,b}$	$w_{3,b}$	$w_{4,b}$	$w_{5,b}$	$w_{6,b}$	$w_{7,b}$	$w_{8,b}$	$w_{9,b}$	$w_{10,b}$	$w_{11,b}$	$w_{12,b}$	μ_j	σ_j	$\Phi_j^{-1}(0.1)$
$NPC1$	0.76	0.99	1.36	0.71	0.74	1.57	3.16	0.36	2.91	0.39	2.01	2.11	1.42	0.91	0.25
$NPC2$	1.06	1.44	4.01	0.61	0.89	2.27	0.18	0.14	2.22	1.67	3.17	0.16	1.49	1.18	0.02
$NPC3$	1.37	0.52	0.16	3.12	1.27	0.33	1.24	0.67	0.81	0.82	3.01	0.91	1.19	0.91	0.02
$NPC4$	2.00	2.21	2.91	0.74	1.40	0.81	0.10	1.37	0.72	0.12	0.91	0.07	1.11	0.86	0.01
$NPC5$	1.41	0.16	0.68	1.33	2.15	0.32	1.40	2.00	3.22	1.24	0.36	1.11	1.28	0.87	0.17
$NPC6$	0.41	1.51	1.80	1.40	0.57	1.77	2.19	2.16	0.46	0.97	1.24	1,63	1.26	0.70	0.36
$NPC7$	3.01	0.09	2.32	2.62	1.12	0.36	0.81	3.12	1.61	2.17	0.56	2.71	1.71	1.04	0.38
$NPC8$	0.89	1.32	0.41	1.07	2.00	1.66	2.13	0.99	3.12	0.06	0.84	3.17	1.47	0.94	0.26

Table 5: Carrying capacity of water source (×10, 000)

	$w_{1,b}$	$w_{2,s}$	$w_{3,s}$	$w_{4,s}$	$w_{5,s}$	$w_{6,s}$	$w_{7,s}$	$w_{8,s}$	$w_{9,s}$	$w_{10,s}$	$w_{11,s}$	$w_{12,s}$	μ_j	σ_j	$\Phi_j^{-1}(0.1)$
$S1$	0.04	1.00	0.41	0.56	0.93	1.03	0.13	0.72	1.24	0.39	1.00	1.21	0.72	0.39	0.22
$S2$	0.39	0.81	0.52	1.60	0.18	0.77	2.00	1.30	0.82	0.42	0.81	0.21	0.82	0.53	0.14
$S3$	1.14	1.06	1.72	1.60	2.33	0.92	0.88	1.66	0.29	0.71	1.21	0.17	1.14	0.64	0.32

The beekeeper preference for the various apiary sites assessed using a ten-point system, are as follows: w_1 = 5, w_2 = 4, w_3 = 7, w_4= 1, w_5 = 6, w_6 = 3, w_7 = 2 and w_8 = 3

3. Result and Discussion

Using LINGO optimizer to solve the model, the results obtained are presented in Tables 7 - 10 for the data obtained from the beekeeper, the table below shows the result.

Table 6: Optima solution/result of the mathematical programming model

Decision variable	Optima value
$F\ P_{11}$	0.49
$F\ P_{24}$	0.52
$F\ P_{25}$	0.57
$F\ P_{32}$	0.28
$F\ P_{33}$	0.72
$F\ P_{38}$	0.52
$F\ P_{56}$	0.30
$F\ P_{87}$	0.24
$F\ N_{11}$	0.25
$F\ N_{18}$	0.26
$F\ N_{22}$	0.02
$F\ N_{33}$	0.02
$F\ N_{34}$	0.01
$F\ N_{56}$	0.36
$F\ N_{85}$	0-17
$F\ N_{87}$	0.38
$F\ _{31}$	0.22
$F\ _{32}$	0.14
$F\ _{53}$	0.32
L_1	3.10
L_2	2.19
L_3	1.35
L_4	1.34
L_8	0.81
L_5, L_6, L_7	0
$V_{18}, V_{24}, V_{33}, V_{43}, V_{52}, V_{62}, V_{71}, V_{81}$	1

Table 7: The number of pollen foragers in apiary site *i* that are accommodated by pollen plant cluster *j* (×20, 000)

	1	2	3	4	5	6	7	8
1	0.49	0	0	0	0	0	0	0
2	0	0	0	0.52	0.57	0	0	0
3	0	0.28	0.72	0	0	0	0	0.52
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0.30	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0.24	0

Table 8: The number of nectar foragers in apiary site *i* that are accommodated by nectar plant cluster *b* (×20, 000)

	1	2	3	4	5	6	7	8
1	0.25	0	0	0	0	0	0	0.26
2	0	0.02	0	0	0	0	0	0
3	0	0	0.02	0.01	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0.36	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0.17	0	0.38	0

Table 9: The number of foragers in apiary site *i* that are accommodated by water source *s* (×20, 000)

	1	2	3	4	5	6	7	8
1	0	0	0.22	0	0	0	0	0
2	0	0	0.14	0	0	0	0	0
3	0	0	0	0	0	0.32	0	0

Table 10: The number of beehives/colonies *k* to be relocated to apiary site *i*.

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	1
2	0	0	0	1	0	0	0	0
3	0	0	1	0	0	0	0	0
4	0	0	1	0	0	0	0	0
5	0	1	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0

3.1 Discussion of Result

The results obtained from the numerical experiment are presented in Tables 6 - 10. Table 6 shows an optimal solution to the mathematical programming model which enables the beekeeper to take a robust decision that can enhance or maximize honey bee products in the apiary sites.

Table 7 shows that 0.49×(20, 000) pollen foragers in site 1 can successfully forage in pollen plant cluster 1; from site 2, pollen plant cluster 4 and 5 accommodate 0.52×(20, 000) and 0.57×(20, 000) respectively. The foragers in site 3 can access pollen plant cluster 2, 3 and 8 with capacity 0.28×(20, 000), 0.72 × (20, 000) and 0.52 × (20, 000) respectively. 0.30 × (20, 000) pollen foragers from site 5 can obtain pollens successfully from pollen plant cluster 6 and those in site 8 can forage in pollen plant cluster 7 which accommodates only 0.24 × (20, 000) bees. The foragers in sites 4, 6 and 7 need to search for other pollen plant clusters or the colonies relocated to other sites.

Table 8 represents the nectar foragers in different sites and the nectar plant clusters that can accommodate them with adequate feeds obtained from each of them. From site 1, 0.25 × (20, 000) nectar foragers can forage at nectar plant cluster 1 while 0.26 × (20, 000) can forage at nectar plant cluster 8 conveniently.

Nectar plant cluster 2 can successfully accommodate 0.02×(20, 000) nectar foragers from site 2. In site 3 0.02×(20, 000) bees forage at nectar plant cluster 3 while 0.01 × (20, 000) foragers from the same site forage at nectar plant cluster 4. From site 5, a total number of 0.36×(20, 000) nectar foragers can successfully obtain feeds from nectar plant cluster 6. The nectar foragers in site 8 could access two different clusters, nectar plant cluster 5 hosting 0.17 × (20, 000) while nectar plant cluster 7 accommodates 0.38×(20, 000) respectively. The colonies in site 4, 6 and 7 should be relocated to other sites if the nectar foragers can not access the available nectar plant clusters.

Table 9 shows the number of foragers that can be accommodated by different water sources. Water source 1 accommodates 0.22 × (20, 000) foragers from site 1 while 0.14 × (20, 000) foragers from the same site access water from water source 2. The foragers in site 6 get water from water source 3 which accommodates only 0.32× (20, 000) foragers. From the above, there is need for the bee farmer to provide additional source of water especially for the sites that could access the available ones.

Table 10 shows the possible relocation of beehives/colonies to enhance the products of the honey bees for the beekeeper. Beehive 1 should be relocated to apiary site 8, beehive 2 to apiary site 4, beehives 3 and 4 to apiary site 3; beehives 5 and 6 to site 2 and beehives 7 and 8 to apiary site 1.

The decision variable table clearly shows that overpopulation occurred at L_1, L_2, L_3, L_4 and L_8 while L_5, L_6 and L_7 have no trace of overpopulation. Finally there was record of death among the bees.

4. Conclusion

In this paper, crisp deterministic mixed-integer honey productivity model was designed. The model was solved using LINGO optimizer to obtain the desired optimality of the model which reflected in Table 6. Beekeepers adopting the results in Tables 7 - 10 will enhance the foraging activities in the colonies as well as increase the functionality of the hive bees that are responsible for beehive maintainace. The result also shows that solving the problem of distribution of beehives in the apiary and its connection to source of food and water will maximize the products of honey bee, reduce competition and death among foragers.

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