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Mathematical Modeling of the Spread of False Information within Social Media

[1, 2*]Ibrahim, J. O., [1]Ibrahim, M. O. and [1, 3]Abdurrahman, N. O.

[1]Department of Mathematics, University of Ilorin, Nigeria.
[2]Department of Mathematical and Computer Sciences, Fountain University, Osogbo, Nigeria.
[3]Department of Mathematics, Federal University of Technology, Minna, Nigeria.

Abstract

One of the societal pollutions in our environment that requires overhauling intervention is the spread of false information. In this paper, we modelled the spread of rumor in a continuous and dynamic population of five compartments. We considered an incubation period which allows rumormongers to verify the authenticity of information received before spreading. Stability analysis of the rumor-free equilibrium (RFE) and the rumor-present equilibrium (RPE) was carried out. The RPE is a function of the reproduction number R_0. In order to annihilate the rumor, the results suggest that we should reduce R_0 continuously below 1. The results are numerically validated and discussed.

Keyword: modeling, information, social media, reproduction number, stability

1. Introduction

The spread of false information, or rumors, is a societal menace exacerbated by Information and Communication Technology (ICT). This phenomenon affects all aspects of human interaction, with social media enabling easy generation and rapid dissemination of misinformation (Del Vicario et al., 2016; Wu et al., 2019). Rumor spreading is a social contagion process, similar to epidemiological models, but with intentional transmission.

Rumors lack effective verification and can significantly impact public opinion, financial markets, and society during wars and epidemics (Galam, 2003). Researchers have studied rumor propagation modeling, exploring various network topologies (Yu et al., 2021; Linhe et al., 2019). However, minimizing the spread of misinformation in social networks is an NP-hard problem, requiring approximate solutions (Budak et al., 2011). To address this issue, studies have employed Twitter content modeling, sentiment analysis, and linguistic methods to identify rumors (Castillo et al., 2011; Qazvinian et al., 2011). Others have proposed deterministic mathematical models to explain rumor propagation using epidemiological approaches (Musa and Fori, 2019).

Social media's vast reach, with 4.7 billion users (60% of the world's population), amplifies rumor spreading. In 2023, 94.8% of users accessed chat and messaging apps, and 94.6% used social platforms (investopedia.com). To combat rumor spread, it's essential to discern between true and false information, verify information before transmission, and implement effective mitigation strategies. Understanding rumor dynamics and developing targeted interventions can help mitigate their harmful effects.

Corresponding Author: Ibrahim, J. O.
Email: ibrahimjamiu342@gmail.com

2. Model Formulation

The deterministic mathematical model is used to study the dynamics of false information in this model. Individuals in the population is divided into compartments depending on the stage each belongs to. The compartments are classified into five, namely: Ignorant A(t), Incubator C(t), Spreaders targeting communities via social media, P(t), Spreaders targeting communities via mainstream media, Q(t), and Stiflers R(t).

2.1 The Flow Diagram of the Model

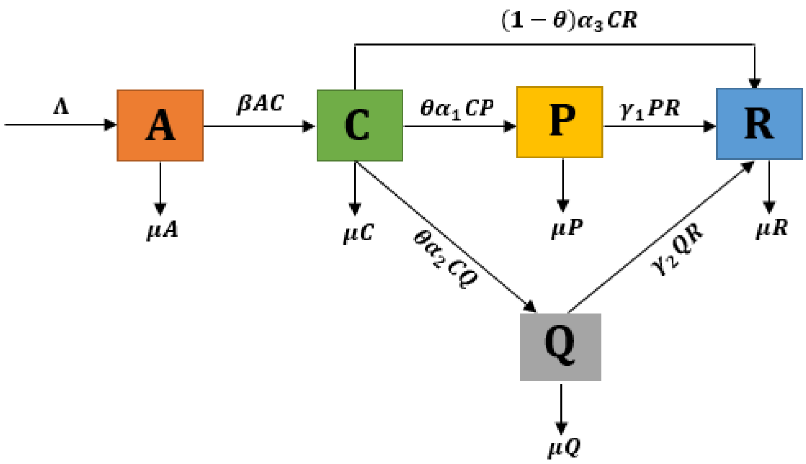

Figure 1: Flow diagram of the Model

2.2 Model Equation

The model is described by the following system of nonlinear differential equations:

$$\frac{dA}{dt} = \Lambda - \beta AC - \mu A \qquad (1.1)$$

$$\frac{dC}{dt} = \beta AC - \theta\alpha_1 CP - \theta\alpha_2 CQ - (1-\theta)\alpha_3 CR - \mu C \qquad (1.2)$$

$$\frac{dP}{dt} = \theta\alpha_1 CP - \gamma_1 PR - \mu P \qquad (1.3)$$

$$\frac{dQ}{dt} = \theta\alpha_2 CQ - \gamma_2 QR - \mu Q \qquad (1.4)$$

$$\frac{dR}{dt} = \gamma_1 PR + \gamma_2 QR + (1-\theta)\alpha_3 CR - \mu R \qquad (1.5)$$

The total population size, $N(t)$ satisfies the equation $\frac{dN}{dt} = \Lambda - \mu N$, $N(0) = N_0$ (1.6)

2.3 Description of Parameters of the Model

Table 1: Description of Parameters of the Model

Parameters	Description
Λ	Recruitment rate (birth or immigration)
β	Rate by which an Ignorant becomes Incubator
θ	Probability of spreading new false information
α_1	Rate at which an Incubator becomes Spreader through Social Media
α_2	Rate at which an Incubator becomes Spreader through Mainstream Media
α_3	Rate at which an Incubator becomes Stifler
γ_1	Rate at which a Spreader through Social Media becomes Stifler

γ_2	Rate at which a Spreader through Mainstream Media becomes Stifler
μ	Dismissal rate (death or emigration)

2.4 Model Assumption

- We assume that Λ, μ are positive constants, and that emigration is independent of rumor class.
- When a spreader or incubator contacts an ignorant, the spreader or incubator transmits the rumor at a constant frequency, and the ignorant gets to know about it and requires time to discern between true and false and becomes rumor latent.
- The incubator does not always become a spreader, but may doubt its credibility and consequently become a stifler.

2.5 Basic Properties

Since the equations (1.1) to (1.5) considers human populations, all the variables and the associated parameters are non-negative at all time. It is necessary to show that the variables of the model remain non negative for all non-negative initial conditions.

Theorem 1: The region $\Omega = \left\{(A, C, P, Q, R) \in R_+^5 : N \leq \frac{\Lambda}{\mu}\right\}$ is positively invariant and attract all solutions in R_+^5.

Proof: From (1.6):

$$\frac{dN}{dt} = \Lambda - \mu N \Rightarrow \int \frac{dN}{\Lambda - \mu N} = \int dt$$

$$-\frac{1}{\mu}\ln(\Lambda - \mu N) = t + C$$

$$-\frac{1}{\mu}\ln(\Lambda - \mu N) \leq t$$

$$\Lambda - \mu N \leq e^{-\mu t}$$

$$N(0) = N_0 \Rightarrow \mu N \leq \Lambda$$

$$N \leq \frac{\Lambda}{\mu}$$

Hence, the model is epidemiologically and mathematically well posed.

3. Analysis of the Model

3.1 Stability Analysis of Rumor Free Equilibrium

In this section, we discuss the existence of Rumor Free Equilibrium (RFE) of the model and its analysis. The model Equations (1.1) to (1.5) has an RFE given by

$$(A, C, P, Q, R) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$$

The local stability of the RFE will be discussed using the Next Generation Matrix method. We calculate the next generation matrix for the system of equation (1.1) to (1.5) by listing the:

(i) number of ways that new spreaders appear

(ii) number of ways that individuals can move but only one way to create a spreader.

So, let

F = rate of appearance of new spreaders into the compartment

V = rate of transfer into (out) of compartment

$$F = \begin{pmatrix} \beta A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \theta\alpha_1 P + \theta\alpha_2 Q + (1-\theta)\alpha_3 R + \mu & \theta\alpha_1 C & \theta\alpha_2 C \\ -\theta\alpha_1 P & \gamma_1 R - \theta\alpha_1 C + \mu & 0 \\ \theta\alpha_2 Q & 0 & \gamma_2 R - \theta\alpha_2 C + \mu \end{pmatrix}$$

At Rumor Free Equilibrium, we have

$$F = \begin{pmatrix} \frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, V = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}, \; FV^{-1} = \begin{pmatrix} \frac{\beta\Lambda}{\mu^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

To compute the eigenvalues of the above matrix, we proceed as follows:

$$|FV^{-1} - \lambda I| = \begin{vmatrix} \frac{\beta\Lambda}{\mu^2} - \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

It follows that the eigenvalues are: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = \frac{\beta\Lambda}{\mu^2}$

The Reproduction Number is the largest eigenvalue which is $R_0 = \frac{\beta\Lambda}{\mu^2}$

The Jacobian matrix of (1.1) to (1.5) at the equilibrium point $E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$ is

$$J = \begin{bmatrix} -\mu & -\frac{\beta\Lambda}{\mu} & 0 & 0 & 0 \\ 0 & \frac{\beta\Lambda}{\mu} - \mu & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & -\mu \end{bmatrix}$$

Now, we calculate the eigenvalues of the Jacobian matrix by finding the characteristic equation using the formula $|J - \lambda I| = 0$

$$|J - \lambda I| = \begin{vmatrix} -\mu-\lambda & -\frac{\beta\Lambda}{\mu} & 0 & 0 & 0 \\ 0 & \frac{\beta\Lambda}{\mu} - \mu - \lambda & 0 & 0 & 0 \\ 0 & 0 & -\mu-\lambda & 0 & 0 \\ 0 & 0 & 0 & -\mu-\lambda & 0 \\ 0 & 0 & 0 & 0 & -\mu-\lambda \end{vmatrix} = 0$$

The characteristic polynomial of the above matrix is given as:

$$\lambda^5 - \frac{(\beta\Lambda - 5\mu^5)}{\mu}\lambda^4 - (4\beta\Lambda - 10\mu^2)\lambda^3 - (6\beta\Lambda\mu - 10\mu^3)\lambda^2 - (4\beta\Lambda\mu^2 - 5\mu^4)\lambda - \mu^3(\beta\Lambda - \mu^2) = 0$$

Solving the above equation, we get:

$$\lambda_1 = \frac{\Lambda\beta - \mu^2}{\mu}, \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = -\mu$$

Theorem 2: The Rumor Free Equilibrium (RFE) of the system (1.1 to 1.5) is locally asymptotically stable if $R_0 < 1$ and unstable otherwise

Proof:

For an equilibrium point to be asymptotically stable, all the eigenvalues must be negative (i.e. $\lambda_i < 0, i = 1, 2, 3, 4, 5$). Since λ_2, λ_3, λ_4 and λ_5 are all negative, we need to show that $\lambda_1 = \frac{\Lambda\beta - \mu^2}{\mu} < 0$.

Then we have:

$$\Lambda\beta - \mu^2 < 0$$
$$\frac{\Lambda\beta}{\mu^2} - 1 < 0$$
$$\frac{\Lambda\beta}{\mu^2} < 1$$
$$R_0 < 1$$

This shows that the Rumor Free Equilibrium point $(A, C, P, Q, R) = \left(\frac{\beta\Lambda}{\mu^2}, 0, 0, 0, 0\right)$ is locally asymptotically stable if $R_0 < 1$ and unstable otherwise.

3.2 Stability Analysis of Rumor Present Equilibrium

In order to establish the existence of Rumor Present Equilibrium of the model (that is equilibria where at least one of the Rumor components of the model is non-zero). Let $E_1 = (A^*, C^*, P^*, Q^*, R^*)$ be an arbitrary Rumor Present Equilibrium point of the model (1.1) to (1.5). Solving the model equation at steady state gives:

$$A^* = -\frac{\theta\Lambda(\alpha_1\gamma_2 - \alpha_2\gamma_1)}{(-\theta\alpha_1\gamma_2 + \theta\alpha_2\gamma_1 + \beta\gamma_1 - \beta\gamma_2)}, \quad C^* = \frac{\mu(\gamma_1 - \gamma_2)}{\theta(\alpha_1\gamma_2 - \alpha_2\gamma_1)}$$

Bellman and Cooke's theorem to test the stability of an equilibrium point was employed.

4. Numerical Simulation and Discussion of Results

We performed some numerical simulations using SciPy from PYTHON programming language and MAPLE to study the behavior of the systems on the Ignorant, Incubator, Spreaders through social media (spreader 1), Spreader through mainstream media (spreader 2) and Stiflers population.

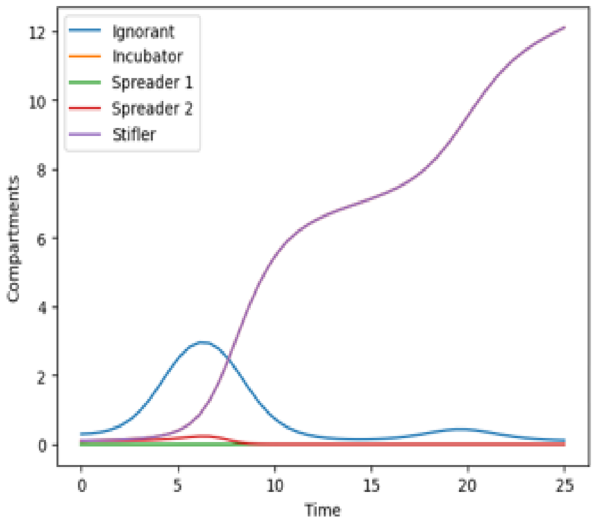

Figure 1: Graph of the compartments against time

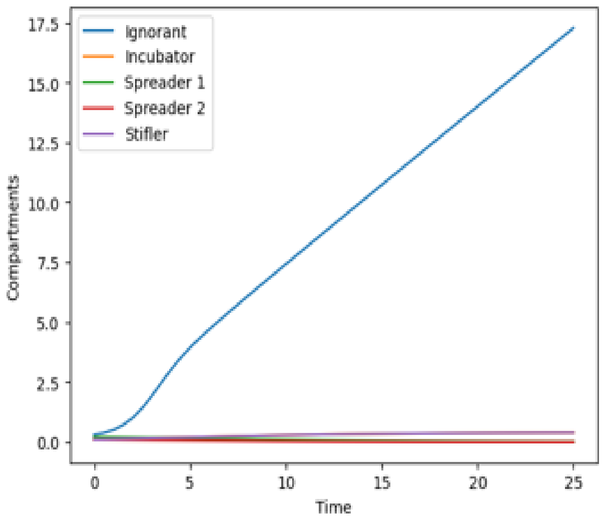

Figure 2: Graph of the compartments against time ($\alpha_1 = \alpha_2 = \alpha_3 = 0$)

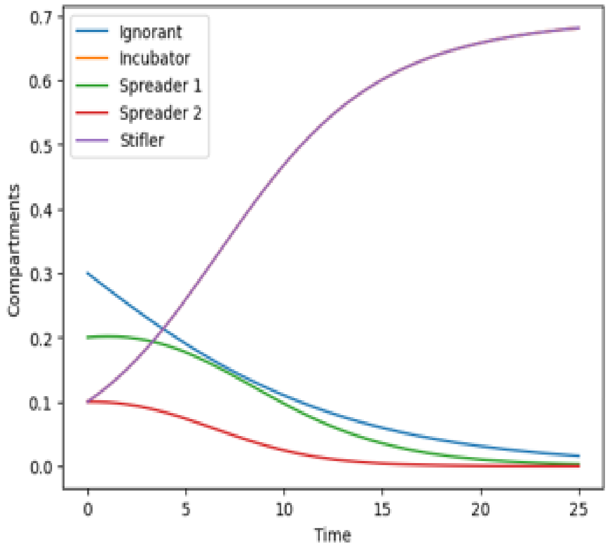

Figure 3: Graph of the compartments against time with $\beta = 0$

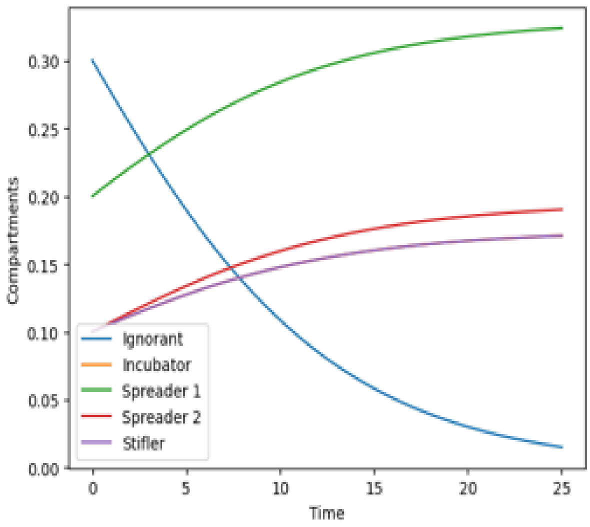

Figure 4: Graph of the compartments against time with $\gamma_1 = \gamma_2 = 0$

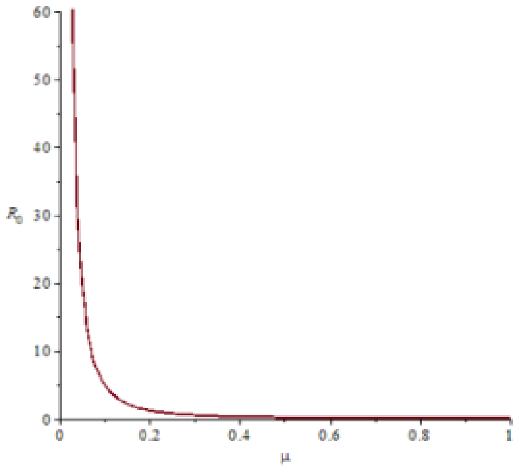

Figure 5: The effect of death rate (μ) on R_0

In figure 1, the relationship between the compartments and time is presented using the parameters: $\Lambda = 0.65, \mu = 0.000005, \beta = 0.5, \gamma_1 = 0.4, \gamma_2 = 0.7, \alpha_1 = 0.3, \alpha_2 = 0.4, \alpha_3 = 0.5, \theta = 0.6$. There is an increase in the population of Stifler with no significant change in the population of the spreaders. Also, the Ignorant population takes a bell shape, signifying that it is symmetrical about the x-axis.

In figure 2, the relationship between the compartments and time is presented using the parameters: $\Lambda = 0.65, \mu = 0.000005, \beta = 0.5, \gamma_1 = 0.4, \gamma_2 = 0.7, \alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \theta = 0.6$. The population of Ignorant increases with time as there is no change in the population of the actors who spread rumor.

In figure 3, the relationship between the compartments and time is presented using the parameters: $\Lambda = 0.65, \mu = 0.000005, \beta = 0, \gamma_1 = 0.4, \gamma_2 = 0.7, \alpha_1 = 0.3, \alpha_2 = 0.4, \alpha_3 = 0.5, \theta = 0.6$. When the Ignorant and Spreader populations decrease with time, there is a corresponding significant increase in the number of those who represses from spreading rumor.

In figure 4, the relationship between the compartments and time is presented using the parameters: $\Lambda = 0.65, \mu = 0.000005, \beta = 0.5, \gamma_1 = 0, \gamma_2 = 0, \alpha_1 = 0.3, \alpha_2 = 0.4, \alpha_3 = 0.5, \theta = 0.6$. There is a significant decrease in the number of Ignorant as a result of rising numbers of spreaders.

5. Conclusion

The spread of false information within social media is modeled using the deterministic approach. The incorporation of incubation period to allow an Ignorant process the rumor he heard before spreading it has no significant impact. The spread of rumor through social media is higher that through the mainstream media (as indicated in figure 4). The results show that when $R_0 < 1$, the rumor will disappear and this will be achieved by increasing the dismissal rate.

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