Ilorin Journal of Science
Volume 9, Number 1, 2022, pp. 20 – 32 (Printed in Nigeria)
ISSN: 2408 – 4840 © 2022 Faculty of Physical Sciences, University of Ilorin
https://doi.org/10.54908/iljs.2022.09.01.002

ILORIN
JOURNAL OF SCEINCE

ILJS-22-008

Performance of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Models in Modeling Volatility of Brent Crude Oil Price

Gbolagade[1*], S.D., Oyeyemi[2], G.M., Abidoye[2], A.O., Adejumo[1], T.J. and Okegbade[1], A.I.

[1] Department of Statistics, Ladoke Akintola University of Technology, Ogbomoso, Oyo State, Nigeria.

[2] Department of Statistics, University of Ilorin, Ilorin, Kwara State, Nigeria.

Abstract

Fluctuations in the price of crude oil determines the economic state of many nations especially the oil producing ones, it becomes important to examine these fluctuations, hence the GARCH models were applied to model the volatility in the price of brent crude oil to determine the best model suitable for predicting future volatility. Descriptive statistics of the stock price and its returns were obtained, some inferential tests were employed to examine the stationarity and the goodness of fit of the daily price and the return under different distributions – the Gaussian, the Student t and the Generalized Error Distributions. Results showed that the price of crude oil dropped between 2014 and 2016 and drastically dropped in 2020. Among the competing models, Exponential GARCH(1,1) with Student t distribution was the best model with the least values of AIC, SBIC and HQIC respectively. The result showed negative significant value in the coefficient of its asymmetric parameter, suggesting that bad news or vital event such as COVID19 has larger effect on the volatility in the price of the Brent crude oil. This study therefore recommended the use of Exponential GARCH model in modelling or predicting volatility in the stock price of crude oils.

Keywords: Akaike Information Criteria (AIC), Autoregressive Conditional Heteroskedasticity (ARCH), Autoregressive Conditional Heteroskedasticity Langrange Multiplier (ARCH LM), Hannan-Quinn Information Criteria (HQIC), Schwarz Information Criteria (SIC), Volatility

1. Introduction

One of the nonrenewable resources is Crude oil, a naturally occurring petroleum product composed of hydrocarbon deposits and other organic materials. It is a global commodity that trades in markets around the world due to several products such as gasoline, jet fuel, diesel and other petroleum products it contains

Corresponding Author: Gbolagade, S.D.

Email: sundaydanielgbolagade@gmail.com

after it is refined. Brent crude oil is extracted from oilfields in the North Sea between the United Kingdom and Norway. Its price is one of the factors that greatly affect the trend of international financial markets because it is used as a benchmark for pricing crude oil in the world market because its lightness and sweetness with relatively low sulfur content (Abdollahi and Ebrahimi, 2020). The movements in the Brent oil price is one of the factors that dampening the impact of sharp fluctuations on the dollar price of oil in many oil producing countries. Nigeria, for example, 80 percent of the total revenue are generated from crude oil (Oyeyemi, 2013). Hence, a small change in the price usually has a large impact on the economy. This kind of impact was observed in Nigeria during the COVID-19 lockdown period when there was economic loss, excessive withdraw of funds by investors from the market due to fall in price of oil (Adenomon, 2022). This effect is also similar with what Zhou *et al*. (2021) reported that whenever there is a sharp fluctuation in the price of oil, there is inverse relationship between the US dollar index and the high-frequency oil prices become more obvious in the trend, but after the prices returned to stable fluctuation, the reverse relationship between the stocks and high-frequency becomes gradually weakened. It is therefore important to examine modeling the price of Brent Crude Oil.

In recent time, Modeling time series are obtained through the use of artificial intelligence (AI) models such as Artificial Neural Networks (ANNs), Adaptive Neuro-Fuzzy Inference System (ANFIS) and intelligent optimization algorithms (Azadeh *et al*., 2012; Hamdi and Aloui, 2015; Yu *et al*., 2016), linear regression, autoregressive integrated moving average (ARIMA), random walk or exponential smoothing model. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) models were however employed for modeling the price of Brent crude oil from January 1st, 2010 through April 27th, 2022. This is because direct statistical analysis of financial prices is difficult, since the consecutive prices are highly correlated and the variances of the prices often increase with time.

In a bid to model and forecast the price of crude oil, several authors have extensively written numerous articles within the past few years. Moshiri and Foroutan (2006) proposed a model using ARIMA and GARCH for forecasting daily oil future prices. Abdollahi and Ebrahimi (2020) developed a new hybrid model for forecasting Brent crude oil price, making use of the Adaptive Neuro Fuzzy Inference System (ANFIS), Autoregressive Fractionally Integrated Moving Average (ARFIMA) and markov-switching models. It was revealed that the hybrid model weighted by genetic algorithm generally outperformed the constituent models, hybrid model with equal weights and hybrid model weighted based on the error values. The forecasted values seem beneficial to the producer and importer nations. In order to develop a forecasting model to predict the price of oil that aid management to reduce operational costs and to increase profit, Bollapragada *et al*. (2021) implemented a target capacity utilization rule recursive simulation model and tested it on historical data. The forecast value of this model was not significantly different from the actual value, suggesting the recursive model was a good model.

Alvarez-Diaz (2020) comparatively examined the performances of linear parametric models (the ARIMA, the ARFIMA and the autoregressive model), a nonlinear parametric model (the GARCH-in-Mean model) and a nonlinear autoregressive artificial neutral network, genetic programming. It was observed that all methods are capable of predicting accurately both the value and the change in the Brent oil prices since there were no significant forecast differences among the methods. Karasu *et al*. (2020) developed a new model of forecasting the price of crude oil, capable of handling chaotic behavior and inherent factuality problems. This model was based on the Support Vector Regression (SVR) with a wrapper-based feature selection using multi-objective optimization technique. The model used Simple Moving Average (SMA), Exponential Moving Average (EMA) and Kaufman's Adaptive Moving Average (KAMA) as indicators. The results showed that the model can capture nonlinear properties of crude oil price. A markov analysis was used to predict the future behavior of the crude oil by means of long run probability but suggested ARIMA (2,1,3) for a short time forecast. Vinothini and Varathan (2015)

(Wang *et al*, 2016; Zhang and Wang, 2013) considered Brent spot and futures price vitality persistence from the period 1990 to 2011, results showed that vitality was very persistent in both spot and future prices. The results also indicated that the spot and futures prices can change in an unpredictable manner. Zavadska *et al*. (2020) also examined the Brent crude oil spot and futures prices during four major crises that significantly affected the oil markets: the First Gulf war in 1990-1998; the Asian Financial crisis between 1997 and 1998; the US terrorist attack in 2001; and the Global Financial crisis in 2008. The result revealed higher levels of volatility during crisis in terms of supply and demand disruptions and higher volatility persistence during financial or economic crises. Meanwhile, Cheong (2009) investigated the time varying volatility of the West Texas Intermediate (WTI) and Europe Brent, where Autoregressive Conditional Heteroscedasticity (ARCH) model was used. There was evidence of long-persistence volatility in the WTI than in the Brent. Similar impact was also observed in the appreciation and depreciation shocks levels WTI and Brent. When random walk hypothesis for the crude oil markets was examined on WTI and Brent, it was found that the Brent crude oil market is weak-form efficiency while WTI seems to be inefficiency on the 1994-2008 sub-period, suggesting that the deregulation have not improved the efficiency on the WTI crude oil market in the sense of making returns less predictable, Charles and Darne (2009). This study therefore aimed to examine the performance of the Standard and -the Exponential GARCH Models at different distributions in modelling the volatility in the price of Brent Crude oil.

2. Materials and Methodology

A secondary data obtained from investing.com between January 1st 2010 and April 27th, 2022 was used in this study. The daily prices of Brent crude oil was an historical data containing 3184 observations. This period was used to observe changes that occurred during the period of COVID-19 and possible effect it could have on the proposed model. Since statistical analysis of financial prices is difficult because the consecutive prices are highly correlated and the variances of the prices often increase with time, daily geometric returns was employed to measure the fluctuations that occurred in the price of the crude oil. The daily geometric returns are defined by:

$$d_r = \ln(p_t) - \ln(p_{t-1}) \tag{1}$$

where d_r is the return at current time, $\ln(p_t)$ is the natural logarithm of the price of the brent crude oil at current time t, while $\ln(p_{t-1})$ is the natural logarithm of the price of daily brent crude oil at the previous time t -1.

The descriptive analysis of the data was performed which includes; time plot which was employed to study the trends of the actual price of daily Brent crude oil and the returns of the price. Mean, median, maximum and minimum price, standard deviation, skewness, kurtosis and Jaque Bera test were used to investigate if there is loss in the price returns meanwhile the skewness of the data was also examined.

2.1 ARCH Effect

It is important before estimating a GARCH-type model to compute the test for ARCH effects to ensure that this class of models is appropriate for the data. The test is based on the null hypothesis that all q lags of the residuals have coefficients values that are not significantly different from zero. The null hypothesis is rejected, if the value of the test statistic is greater than the critical value from the χ^2 distribution.

2.2 Unit Root Test

Augmented Dickey Fuller (ADF) Test: This test examines if the unit root is present in a series sample (null hypothesis) or if it is stationary (alternative hypothesis). The series are denoted by xt, with the null hypothesis H0: $\beta = 0$ against the alternative H1: $\beta < 0$ using the regression

$$x_t = c_t + \beta x_{t-1} + \sum_{i=1}^{p-1} \varphi_i \Delta x_{t-1} + e_t \tag{2}$$

where c_t is a definitive function of the time t, the t ratio of $\hat{\beta} - 1$ is the ADF test defined as

$$ADF_{test} = \frac{\hat{\beta}-1}{std(\hat{\beta})} \tag{3}$$

$\hat{\beta}$ is denoted as the least square estimate of β

2.3 Phillips-Perron (PP) Test: This is a nonparametric test proposed for serial correlation in gauging unit root. The test statistic is defined as

$$\bar{t}_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0}\right)^{1/2} - \frac{T(f_0-\gamma_0)se(\hat{\alpha})}{2f_0^2 s} \tag{4}$$

where $\bar{t}_\alpha$ is the calculation of PP test, $se(\hat{\alpha})$ is the coefficient of standard error. f_0 is denoted as an estimator of the residual spectrum at frequency zero while γ_0 is the error variance.

Dickey Fuller – Generalized Least Square (DF-GLS): The DF-GLS test for a unit root was developed by Elliot *et al*. (1996), which has higher power than the ADF test when the autoregressive root is large but less than one. It has a higher probability of rejecting the false null of a stochastic trend when the sample data stems from a time series that is close to be integrated.

2.4 The Models

2.4.1 Generalized ARCH (GARCH) Model

Generalized ARCH (GARCH) Model was proposed by Bollerslev (1986) and Taylor (1986). This model allows the conditional variance to be dependent upon previous own lags, so that the conditional variance equation in the simplest case is

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{5}$$

eqn. (4) is a GARCH (1,1) model σ_t^2 is known as the conditional variance since it is a one-period ahead estimate for the variance calculated based on any past information, $\alpha_1 u_{t-1}^2$ is the information about volatility during the previous period while $\beta\sigma_{t-1}^2$ is the fitted variance from the model during the previous period.

The general GARCH (p,q) models is written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \cdots + \beta_p\sigma_{t-p}^2 \quad (6)$$

2.4.2 The Exponential GARCH (EGARCH) Model

Exponential GARCH (EGARCH) Model was proposed by Nelson (1991) to overcome weaknesses observed in standard GARCH models. The model can be written as follows:

$$a_t = \sigma_t \varepsilon_t \quad (7)$$

$$\ln(\sigma_t^2) = w + \sum_{i=1}^{s} \alpha_i \frac{|a_{t-1}| + \theta_i a_{t-1}}{\sigma_{t-1}} + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-1}^2) \quad \text{(Emenogu } et\ al.\text{, 2020)} \quad (8)$$

2.5 Model Selection Criteria

The Akaike Information Criteria - AIC (Akaike, 1973), Schwarz Information Criteria – SBIC (Schwarz, 1978) and Hannan-Quinn Information Criteria – HQIC (Hannan and Quinn, 1979) were used as measure of goodness of fit for the models. The model with the lowest values of AIC, SBIC and HQIC is considered as the best model among the competing ones.

$$SBIC = -2\log(\hat{\sigma}^2) + klog(n) - \log(2\pi) - 1 \quad (9)$$

$$AIC = -2\log(\hat{\sigma}^2) + 2k - \log(2\pi) - 1 \quad (10)$$

$$HQIC = -\log(\hat{\sigma}^2) + 2klog(log(n)) - \log(2\pi) - 1 \quad (11)$$

where n is the number of observation, k is the number of parameters in the model and $\hat{\sigma}^2$ is the estimated model error variance

3. Results and Discussion

The summary of the price and the returns of the price of Brent crude oil is presented in Tables 1, 2 and 3 and Figures 1 and 2 respectively. Table 1 presents the descriptive statistics for the price and the returns of the crude oil, it can be observe that there is a positive mean in the return of the crude oil, indicating that there is no loss in the stock price, the return is also positively skewed and is characterized with high peak and fat tail – leptokurtic. The price of the crude goes its peak between 2011 and 2014, but a sudden drop is observed in 2016 and 2020 respectively as shown in Figure 1, the drop in 2020 may be due to the effect of COVID-19. This is also established in Figure 2 which shows that there is volatility between 2014 and 2016, the same is observed in 2020. Table 2 shows that the unit root test is not significant for the actual stock price of the crude oil at lag 28 but significant for the stock returns. This shows that the actual stock price is not stationary but the return is stationary. ARCH effect was checked, the result in Table 3 reveals the presence of ARCH effects at lag 5 using ARCH LM test, and this justifies the use of GARCH models

Table 1: Summary of Price and Returns of Brent Crude Oil

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**	**Skewness**	**Kurtosis**	**JB**
Price	76.8254	72.46	127.98	19.33	25.87554	0.170612	1.792696	208.7538 (0.000)
Return	8.50E-05	0.00082	0.190774	-0.27976	0.022743	0.975384	22.42609	50537.91 (0.000)

Table 2: Unit Root Test of Price and Returns of Brent Crude Oil

		ADF	**DF-GLS**	**PP**
Price	t-Statistic	-1.29517	-1.46357	-1.36505
	p-value	0.8887	-1.9409	0.6009
Return	t-Statistic	-56.08734	-20.69600	-56.08842
	p-value	0.0001	-1.9409	0.0001

Table 3: Result of ARCH Effect Test

	F-statistic	Df	p-value
Return	44.54053	5	0.000

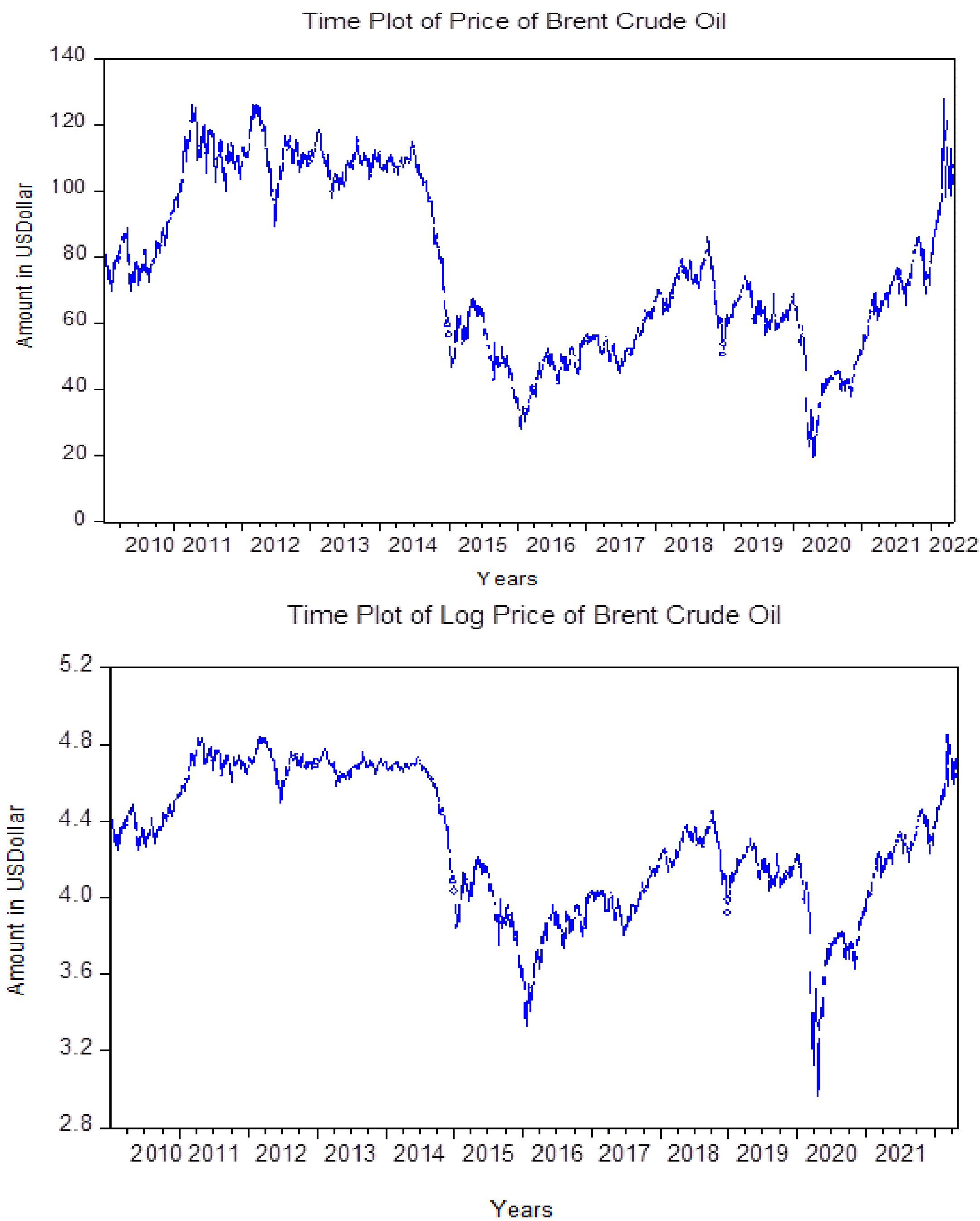

Figure 1: Time Plot of Price of Brent Crude Oil (upper panel) and Time Plot of Log Price of Brent Crude Oil (lower panel)

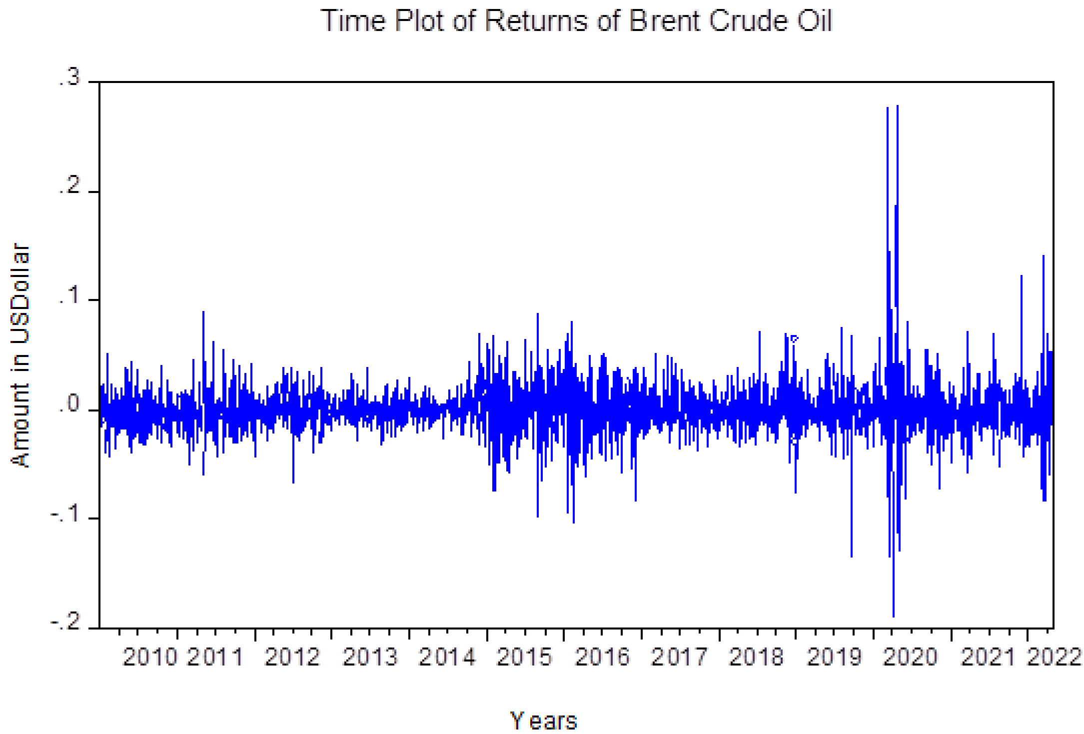

Figure 2: Time Plot of Returns of Brent Crude Oil

3.1 Results of Models selection

Table 4 presents the performances of the Standard GARCH and EGARCH models with Gaussian distribution (Normal), Student t distribution (STD) and Generalized Error Distribution (GED) using Akaike Information Criteria, Schwarz Information Criteria and Hannan-Quinn Information Criteria to choose the best model. The result revealed that GARCH(1,1) with STD model emerged the best model with the least AIC, SC and HQIC values, though EGARCH(1,1) with STD has close values. Hence Exponential GARCH(1,1) with STD was used to model the data. The EGARCH(1,1) model with STD is presented in Table 5. The model reveals significant values in the coefficients of its parameters, suggesting significant effect on the volatility in the price of the crude oil.

Table 4: Results of GARCH Models

Distributions	**Models**	**AIC**	**SBIC**	**HQIC**
Normal	GARCH(1,1)	-5.119841	-5.114123	-5.117790
	GARCH(2,1)	-5.120727	-5.113102	-5.117992
	GARCH(1,2)	-5.120508	-5.112884	-5.117774
STD	GARCH(1,1)	-5.205474	-5.197850	-5.202740
	GARCH(2,1)	-5.205384	-5.195854	-5.201967
	GARCH(1,2)	-5.205109	-5.195578	-5.201691
GED	GARCH(1,1)	-5.195651	-5.188026	-5.192916
	GARCH(2,1)	-5.195791	-5.186261	-5.192373
	GARCH(1,2)	-5.195493	-5.185963	-5.192075
Normal	EGARCH(1,1)	-5.137002	-5.125562	-5.132899
	EGARCH(2,1)	-5.137250	-5.123904	-5.132464
	EGARCH(1,2)	-5.137292	-5.123945	-5.132505
STD	**EGARCH(1,1)**	**-5.216056**	**-5.202709**	**-5.211269**
	EGARCH(2,1)	-5.215886	-5.200633	-5.210416
	EGARCH(1,2)	-5.215562	-5.200309	-5.210092
GED	EGARCH(1,1)	-5.206959	-5.193613	-5.202173
	EGARCH(2,1)	-5.206846	-5.191593	-5.201376
	EGARCH(1,2)	-5.206616	-5.191363	-5.201146

Table 5: Exponential GARCH(1,1) with STD Model

		Variance Equation		
Variable	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
φ	-0.226940	0.035379	-6.41440	0.000
η	0.156508	0.017489	8.94885	0.000
λ	-0.062270	0.010303	-6.04360	0.000
θ	0.986212	0.003573	276.0482	0.000
T-DIST. DOF	5.364436	0.480688	11.15991	0.000

φ = constant, η = ARCH Effect, λ = Asymmetric Effect, θ = GARCH Effect

4. Conclusion

This study employed Standard GARCH(1,1) and Exponential GARCH(1,1) models to model the volatility in the stock price of Brent Crude oil. It is discovered that Exponential GARCH(1,1) under Student t Distribution emerged as the best model among the competing ones. The result showed negative significant values in the coefficients of its asymmetric parameter, suggesting that bad news or vital event such as COVID19 has larger effect on the volatility in the price of the Brent crude oil. This study therefore recommended the use of Exponential GARCH model in modelling or predicting volatility in the stock price of crude oils.

Acknowledgement

All peer reviewers are appreciated for their meaningful contributions.

References

Abdollahi, H., and Ebrahimi, S. B. (2020): A new hybrid model for forecasting Brent crude oil price. Energy, 117520. doi:10.1016/j.energy.2020.11752

Adenomon M.O. (2022): The Effects of COVID-19 outbreak on the Nigeria Stock Exchange Performance: Evidence from GARCH models. *Journal of Statistical Modeling and Analytics*. **4(2)**, 25-38.

Akaike, H. (1973): Information Theory and an Extension of the Maximum Likelihood Principle. Proc. *2nd Intl Symposium of Information Theory*, 267-281.

Alvarez-Diaz, M. (2020): Is it possible to accurately forecast the evolution of Brent Crude Oil Prices? An Answer Based on Parametric and Nonparametric Forecasting Methods. *Empirical Economics*. **59(3**), 1285-1305.

Azadeh, A., Moghaddam, M., Khakzad, M.and Ebrahimpour, V. (2012): A flexible neutral network-fuzzy mathematical programming algorithm for improvement of oil price estimation and forecast. *Comput Ind Eng*. **62(2)**, 421-430.

Bollaprageda, R., Mankude, A. and Udayabhanu, V. (2021): Forecasting the price of crude oil. *Decision* **48(2)**, 1-25.

Bollerslev, T. (1986): Generalised Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics* **31**, 307-327.

Charles, A. and Darne, O. (2009): The efficiency of the crude oil markets: Evidence from variance ratio tests. *Energy Policy*. **37(11)**, 4267-4272.

Cheong, C.W. (2009): Modeling and Forecasting Crude Oil Markets Using ARCH-type Models. *Energy Policy*. **37(6)**, 2346-2355.

Elliot, Graham, Rothenberg T.J. and Stock J.H. (1996): Efficient Tests for an Autoregressive Unit Root. *Econometrica*, **64(4)**, 813-836.

Emenogu, G.N., Adenomon, M.O. and Nweze, N.O. (2020); On the Votality of Daily Stock Returns of Total Petroleum Company of Nigeria: Evidence from GARCH Models, Value-at-Risk and Backtesting, *Financial Innovation*, **6**, 18, https://doi.org/10.1186/s40854-020-00178-1

Hamdi, M. and Aloui, C. (2015): Forecasting crude oil pricing using artificial neutral networks: a literature survey. *Econ Bull*., **3(2)**, 1339-1359.

Hannan, E.J. and Quinn, B.G. (1979): The Determination of the Order of an Autoregression. *J. of the Royal Statistical Society*, Series B, **4**1, 190-195.

Karasu S., Altan A., Bekiros S. and Ahmad W. (2020): A new forecasting model with wrapper-based feature selection using multi-objective optimization technique for chaotic crude oil time series. *Energy* **212**, 118750. https://doi.org/10.1016/j.energy.2020.118750

Moshiri, S. and Foroutan, F. (2006): Forecasting Nonlinear Crude Oil Futures Prices. *Energy J*. 81-95

Nelson, D.B. (1991): Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica: Journal of the Econometric Society, 59, 347-370. https://doi.org/10.2307/2938260

Oyeyemi, A.M. (2013): The Growth Implications of Oil Price Shock in Nigeria. *Journal of Emerging Trends in Economics and Management Sciences (JETEMS)*, **4(3)**, 343-349

Schwarz, G. (1978): Estimating the Dimension of a Model. *Annals of Statistics*, **6**, 461-464.

Taylor, S. J. (1986): Forecasting the Volatility of Currency Exchange Rates, *International Journal of Forecasting*, **3**, 159-170

Vinothini, V. and Varathan, N. (2015). Modeling and Forecasting of crude oil price. *International conference on multidisciplinary approaches at University of Sri Jayewardenepura*, Sri Lanka.

Wang, Y., Wu, C., and Yang, L. (2016): Forecasting energy market volatility using GARCH models: can multivariate models beat univariate models? *Energy Econ*. **34**, 2167-2181.

Yu L, Dai, W. and Tang, L. (2016): A novel decomposition ensemble model with extended extreme learning machine for crude oil forecasting. *Eng Appl Artif Intell*. **47**, 110-121

Zavadska, M., Morales, L., and Coughlan, J. (2020): Brent Crude Oil Prices Volatility during Major Crises. *Finance Research Letters*, **32**, 101078. https://doi.org/10.1016/j.frl.2018.12.026

Zhang, Y.-J. and Wang, Z.-Y. (2013): Investigating the price discovery and risk transfer functions in the crude oil and gasoline futures markets: some empirical evidence. Appl. *Energy*, **104**, 220-228.

Zhou J., Sun M., Han D. and Gao C. (2021): Analysis of oil price fluctuation under the influence of crude oil stocks and US dollar index – Based on time series network model. *Physica A: Statistical mechanics and its applications*, **582**, 126218. https://doi.org/10.1016/j.physa.2021.126218