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Simultaneous Equations System vis-aviz Classical Regression Estimators on Impacts of Some Economic Indicators in Nigeria

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Abstract

The choice of models for analyzing the relationship between Total Debt Services (TDS), Real Interest Rate (RIR), Inflation Consumer Price (ICP) and Gross Domestic Product (GDP) variables using recursive models, though produces non-spurious results since the coefficient of determination (R^2) is strictly less than the Durbin Watson (DW) statistic. However, the recursive model did not give a good fit for the relationship between TDS and GDP, since macroeconomic variables are usually prone to endogeneity, first order serial correlation, autocorrelation problems among others. This research work x-rays the TDS-GDP relationship on one hand and TDS versus GDP, RIR and ICP indices on the other hand in Nigeria. The TDS, GDP and RIR are endogenous variables, while the ICP, TRA, and GDP_{t-1} are exogenous variables. Pre-tests analyses using time series datasets extracted from the repository of World Governance Index showed level stationary series I (0) and a causal relationship between TDS and GDP variables. Furthermore, the results from the estimation techniques showed that the Three-Stage Least Square (3SLS) and Seemingly Unrelated Regression (SUR) outperformed the Ordinary Least Squares (OLS) and Two-Stage Least Squares (2SLS) estimators when applied to both the exactly and over-identified structural equations in the simultaneous equation system (SES).

Keyword: Estimator, Indicators, Regression, Gross, Debt, Rate, Exogenous, Endogenous

1. Introduction

The impact of foreign debt service on economic growth index remains very contentious, particularly for externally-indebted developing countries of the world (Adepoju et al.,2007; Onafowora & Owoye, 2019). Meanwhile, external debt servicing has been tagged as a major constraint of economic growth in many developing countries, especially in Sub-Saharan Africa (SSA) where Nigeria is one of the major blocs. Statistics have shown that Total Debt Service

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(TDS) in Nigeria has decreased by 78% from year 2000 to 2019 whereas Gross Domestic Product (GDP) Per Capita has increased exponentially by 99.9% from year 1970 to 2019

(World Governance Index, 2019). Consequently, Debt service therefore, negates economic growth through reduction in amount of available capital (Udeh *et al.*,2016*).*

Hellmann et al. (2000) contributed that government borrowing can crowd out investment, which will reduce future output and wages thereby making the welfare of the citizens more vulnerable.

Since 1992, the World Bank has designated Nigeria along with the majority of other less developed countries (LDCs) as a seriously indebted low-income country. One of the major impediments to the influx of external resources into the economy is the country's failure to satisfy all of its debt service repayments. The buildup of debt service arrears, exacerbated by excessive interest payments, has pushed the external debt stock to exceptionally high level despite all efforts to dwindle it.

Furthermore, no government can function efficiently and successfully on its own; it requires assistance. Foreign borrowing often called external debt, is an important source of assistance (Panizza, 2008). The reason for external debt is that an emerging country like Nigeria, lacks sufficient internal financial resources which necessitates the need for external aid. External debt develops as a result of several factors. However, it is a significant source of government revenue (Zohaib, 2020). External debt buildups should not be interpreted as an indication of a groggy economy. The incapacity of a government to satisfy its debt obligations has exacerbated by a lack of knowledge on the type, structure, and quantity of external debt (Antonio et al., 2007).

According to Soludo (2003), countries borrow for two reasons: macroeconomic reasons to finance higher investment or consumption and to avoid hard budget constraints. By implication, an economy borrows to boost economic growth and alleviate poverty.

According to Pattillo *et al*. (2002), a developing country reasonable borrowing levels are likely to boost its economic growth. When economic growth is being increased by at least 5%, the poverty situation is likely to improve. A country in early stages of development like Nigeria, borrows to stimulate economic growth. This becomes effective as long as the borrowed funds and some internally ploughed back funds are properly utilized for productive investments provided that they do not suffer macroeconomic instability, policies that distort economic incentives, or sizeable adverse shocks. Growth will increase, not only that it will enhance timely debt repayments. This growth will affect per capita income positively as a prerequisite for poverty reduction. Although, the debt overhang models do not analyze the effects of debt on growth explicitly, the implication still remains that large debt stocks lower growth partially by reducing investment with a resultant negative effect on poverty. But the incentive effects associated with debt stocks tend to reduce the benefits expected from policy reforms that would enhance efficiency and economic growth, such as trade liberalization and fiscal adjustment. When this happens, the government will be less willing to incur debt, if it perceives that the future benefits in terms of higher output will accrue partly to foreign lenders.

Also, Ayadi and Ayadi (2008) examined the impacts of the huge external debt with its attendant debt servicing requirements on Nigerian and South African economies. The Neoclassical growth model which incorporates external debt, debt indicators, and some macroeconomic variables was analyzed using both OLS and Generalized Least Square (GLS) methods. Their findings revealed negative impacts of debt and its servicing requirement on the economic growth index of Nigeria and South Africa.

Many researchers have worked on the relationship that exists between TDS and GDP in Nigeria. Sulaiman and Azeez (2012) studied the relationship between External Debt and gross domestic product (GDP) in Nigeria. They utilized annual time series data extracted from the Central Bank of Nigeria Statistical Bulleting and Debt Management Office from 1970 to 2010. Econometric techniques such as ordinary least squares (OLS), Augmented Dickey-Fuller (ADF) Unit Root testing, Johansen Co-integration test and Error Correction Method (ECM) were employed. The results showed that long-run equilibrium relationship exists between external debt and gross domestic product (GDP), which implies that external debt service has positive impact on Nigerian economy.

Furthermore, Ajayi and Oke (2012) examined the effect of the external debt on economic growth index of Nigeria using regression analysis. It was revealed that external debt burden had negative impact on the national and per capital income of the nation. Also, huge external debt led to continuous industrial strike, poor educational system, increase in retrenchment of workers and devaluation of the nation currency.

Uma *et al.* (2013) argued that when debt reaches a certain level, it begins to have adverse effect and debt servicing becomes a huge burden. This has negative impact on Nigeria's rapid economic development and worsened the social problems.

Udeh *et al*. (2016) used Autoregressive Distributed Lag Model (ARDL) approaches in a study to understand better the Nigeria's external debt and economic growth. They discovered that, in the short run, external debt had a positive association with GDP, but that in the long run, it had a negative relationship.

Onakoya and Ogunade (2017) investigated the impact of external debt on economic growth index of Nigeria between 1981 and 2014, the study utilized Autoregressive Distributed Lag (ARDL) and OLS techniques. The results revealed that external debt had negative impact on economic growth.

In the same vein, Ndubuisi (2017) examined the impact of external debt on economic growth index of Nigeria between 1985 and 2015 using Johansen Co-integration and error correction estimation technique. Their Findings showed that debt service payment has an adverse significant impact on economic growth while external debt stock had positive impact on economic growth. In addition, the causality test revealed that there is unidirectional causality from external debt to GDP.

Furthermore, Muhammad (2018) analyzed the effects of external debt management on the economic growth index of Nigeria for a period of 1962 to 2006 using time series data of the various bilateral and multilateral arrangements. Their study concluded that accumulation of external debt adversely affected Nigeria’s economic growth index.

Mohamed (2018) studied the effect of external debt on economic growth index of Sudan from 1969 to 2015 using Johansen cointegration and the Vector Error Correction Method (VECM) estimation technique. The study showed that external debt had positive impact on economic growth of Sudan whereas exchange rate and foreign direct investment had adverse effects on the economy.

Shkolnyk and Koilo (2018) investigated the nexus between external debt and economic growth in emerging economies between 2006 and 2016 and made use of ADL model and correlation analysis. The study revealed that external debt had no impact on the economic growth of the countries that were examined.

2. Materials and Methods

Data Description

Annual time series data were sourced on Nigeria's Gross Domestic Product (GDP), Total Debt Service (TDS), Real Interest Rate (RIR), Trade (TRA) and Inflation Consumer Prices (ICP), spanning 1970 to 2019 (50 years). The data were extracted from the repository of World Governance Index (WGI) via the URL https://databank.worldbank.org/source/worldwide-governance-indicators.

In this study the following estimators were employed viz: OLS, SUR, 2SLS, and 3SLS Estimators respectively.

Model Specification

Model specification defines the hypothesized relationships among the variables in the SEM based on a prior knowledge. It is the process of determining which independent variable(s) to include and exclude from a structural equation (LeSage and Fischer, 2008). Consider the following structural three-equation model in the system of equation that follows:

$$TDS_{1t} = \beta_{1o} + \beta_{12}GDP_{2t} + u_{1t} \quad (1)$$

$$GDP_{2t} = \beta_{2o} + \beta_{31}RIR_{3t} + u_{2t} \quad (2)$$

$$RIR_{3t} = \beta_{3o} + \gamma_{31}GDP_{t-1} + \gamma_{32}TRA_{2t} + \gamma_{33}ICP_{3t} + u_{3t} \quad (3)$$

where TDS, GDP and RIR are endogenous variables. Also, ICP, TRA, and $GDP_{t-1}$are exogenous variables, where. u_{1t}, u_{2t} and u_{3t} are the correlated disturbance terms.

Model Identification

Model identification is to check the status of the structural equations in Simultaneous Equation System, whether the model is over-identified, just-identified, or under-identified. Where model coefficients can be uniquely estimated is the just-identified or over-identified model. Here, we consider the following conditions for identification.

Order Condition of Identification Status of structural equations in SES

To identify the status of a structural equation,

We, let G = the total number of equations or endogenous variable,

K = number of variables in the model (endogenous and predetermined), and

M =number of variables, endogenous and exogenous included a particular equation

If K – M = G – 1, the equation is exactly identified, but if K – M < G – 1, it is under-identified and if K – M > G – 1, the equation is over-identified.

Identification status of equation (1)

K = 6, M = 2 and G = 3; K – M ≥ G – 1; 6 – 2 ≥ 3 – 1; 4 > 2, therefore

Equation (1) is over-identified

Identification status of equation (2)

K = 6, M = 2 and G = 3; K – M ≥ G – 1; 6 – 2 ≥ 3 – 1; 4 > 2, therefore

Equation (2) is over-identified

Identification status of equation (3)

K = 6, M = 4 and G = 3; K – M ≥ G – 1; 6 – 4 ≥ 3 – 1; 2 = 2, therefore

Equation (3) is just-identified (exactly identified)

Rank Condition of Identification Status of the structural equations in SES

An equation with G - endogenous variable under rank condition is said to be identified if and only if at least one nonzero determinant of order $(G-1)$ can be constructed from the coefficients of the variables (both endogenous and predetermined variables) excluded from that equation but included in the other equations in the model (Damodar, 2004).

In order to obtain the rank condition for each structural equation in the model, we re-arrange equations (1), (2) and (3) as follows:

$$u_{1t} = TDS_{1t} - \beta_{1o} - \beta_{12}GDP_{2t} \quad (4)$$

$$u_{2t} = GDP_{2t} - \beta_{2o} - \beta_{31}RIR_{3t} \quad (5)$$

$$u_{3t} = RIR_{3t} - \beta_{3o} - \gamma_{31}GDP_{t-1} - \gamma_{32}TRA_{2t} - \gamma_{33}ICP_{3t} \quad (6)$$

Reduced-form Equation

The reduced form of a model expresses each Y variable only in terms of the exogenous variables X.

Reduced-form Equation for Structural Equation 1

From the given equations 1, 2 and 3 respectively. Put (2) in (1), we have;

$$TDS_{1t} = \beta_{1o} + \beta_{12}(\beta_{2o} + \beta_{31}RIR_{3t} + u_{2t}) + u_{1t}$$

$$TDS_{1t} = \beta_{1o} + \beta_{12}\beta_{2o} + \beta_{12}\beta_{31}RIR_{3t} + \beta_{12}u_{2t} + u_{1t} \quad (7)$$

By putting (3) in (7), we have;

$TDS_{1t} = \beta_{1o} + \beta_{12}\beta_{2o} + \beta_{12}\beta_{31}(\beta_{3o} + \gamma_{31}GDP_{t-1} + \gamma_{32}TRA_{2t} + \gamma_{33}ICP_{3t} + u_{3t}) + \beta_{12}\beta_{31}u_{3t} + \beta_{12}u_{2t} + u_{1t}$

$TDS_{1t} = \beta_{1o} + \beta_{12}\beta_{2o} + \beta_{12}\beta_{31}\beta_{3o} + \beta_{12}\beta_{31}\gamma_{31}GDP_{t-1} + \beta_{12}\beta_{31}\gamma_{32}TRA_{2t} + \beta_{12}\beta_{31}\gamma_{33}ICP_{3t} + \beta_{12}\beta_{31}u_{3t} + \beta_{12}u_{2t} + u_{1t}$

$TDS_{1t} = (\beta_{1o} + \beta_{12}\beta_{2o} + \beta_{12}\beta_{31}\beta_{3o}) + \beta_{12}\beta_{31}\gamma_{31}GDP_{t-1} + \beta_{12}\beta_{31}\gamma_{32}TRA_{2t} + \beta_{12}\beta_{31}\gamma_{33}ICP_{3t} + (\beta_{12}\beta_{31}u_{3t} + \beta_{12}u_{2t} + u_{1t})$

$$TDS_{1t} = \pi_{1o} + \pi_{11}GDP_{t-1} + \pi_{12}TRA_{2t} + \pi_{13}ICP_{3t} + V_{1t} \quad (8)$$

where:

$\pi_{1o} = \beta_{1o} + \beta_{12}\beta_{2o} + \beta_{12}\beta_{31}\beta_{3o}$, $\pi_{11} = \beta_{12}\beta_{31}\gamma_{31}$, $\pi_{12} = \beta_{12}\beta_{31}\gamma_{32}$, $\pi_{12} = \beta_{12}\beta_{31}\gamma_{33}$ and $V_{1t} = \beta_{12}\beta_{31}u_{3t} + \beta_{12}u_{2t} + u_{1t}$.

Reduced Form Equation of Structural Equation (2)

By putting (3.3) in (3.2), we have;

$GDP_{2t} = \beta_{2o} + \beta_{31}(\beta_{3o} + \gamma_{31}GDP_{t-1} + \gamma_{32}TRA_{2t} + \gamma_{33}ICP_{3t} + u_{3t}) + u_{2t}$

$GDP_{2t} = \beta_{2o} + \beta_{31}\beta_{3o} + \beta_{31}\gamma_{31}GDP_{t-1} + \beta_{31}\gamma_{32}TRA_{2t} + \beta_{31}\gamma_{33}ICP_{3t} + \beta_{31}u_{3t} + u_{2t}$

$GDP_{2t} = (\beta_{2o} + \beta_{31}\beta_{3o}) + \beta_{31}\gamma_{31}GDP_{t-1} + \beta_{31}\gamma_{32}TRA_{2t} + \beta_{31}\gamma_{33}ICP_{3t} + (\beta_{31}u_{3t} + u_{2t})$

$$GDP_{2t} = \pi_{2o} + \pi_{21}GDP_{t-1} + \pi_{22}TRA_{2t} + \pi_{23}ICP_{3t} + V_{2t} \quad (9)$$

Where:

$\pi_{2o} = \beta_{2o} + \beta_{31}\beta_{3o}, \pi_{21} = \beta_{31}\gamma_{31}, \pi_{22} = \beta_{31}\gamma_{32}, \pi_{23} = \beta_{31}\gamma_{33}, V_{2t} = \beta_{31}u_{3t} + u_{2t}$ respectively.

Reduced Form Equation of Structural Equation (3)

Since equation (3) contains only exogenous variables, then reduced form equation follows

$$RIR_{3t} = \pi_{3o} + \pi_{31}GDP_{t-1} + \pi_{32}TRA_{2t} + \pi_{32}ICP_{3t} + V_{3t} \quad (10)$$

where

$\pi_{3o} = \beta_{3o}, \pi_{31} = \gamma_{31}, \pi_{22} = \gamma_{32}, \pi_{23} = \gamma_{33}$ and $V_{2t} = u_{3t}$ respectively.

Hausman test for endogeneity (Hausman Specification Test)

The Hausman test detects endogeneity problems in a model. It enables endogenous variables to have values that are determined by other variables in the system of over-identified equations. Having endogenous variables in a model will cause estimators to fail. However, there is need to figure out which variables are endogenous using Hausman test (Hausman, 1978).

Hausman Test for Endogeneity in the Over-Identified Equation (1)

H_o: $\rho_{(GDP_{2t},u_{1t})}$ vs H_1: Not H_o

Recall from equation (9)

GDP_{2t} = $(\hat{\pi}_{2o} + \hat{\pi}_{21}GDP_{t-1} + \hat{\pi}_{22}TRA_{2t} + \hat{\pi}_{23}ICP_{3t}) + \hat{V}_{2t}$

Where $\hat{V}_{2t}$ = OLS residuals

The estimate of equation (9) yields:

$$\widehat{GDP}_{2t} = \hat{\pi}_{2o} + \hat{\pi}_{21}GDP_{t-1} + \hat{\pi}_{22}TRA_{2t} + \hat{\pi}_{23}ICP_{3t} \quad (11)$$

Thus,

$$GDP_{2t} = \widehat{GDP}_{2t} + \hat{V}_{2t} \quad (12)$$

Now substitute equation (12) into the over-identified equation (3)

$TDS_{1t} = \beta_{1o} + \beta_{12}(\widehat{GDP}_{2t} + \hat{V}_{2t}) + u_{1t}$

$TDS_{1t} = \beta_{1o} + \beta_{12}\widehat{GDP}_{2t} + (\beta_{12}\hat{V}_{2t} + u_{1t})$

$$TDS_{1t} = \beta_{1o} + \beta_{12}\widehat{GDP}_{2t} + \hat{V}_{1t}* \quad (13)$$

Where $\hat{V}_{1t}* + \beta_{12}\hat{V}_{2t} + u_{1t}$

Hausman Test for Endogeneity in the Over-Identified Equation (2)

H_o: $\rho_{(RIR_{3t},u_{2t})}$vs H_1: Not H_o

Recall from equation (9):

$$RIR_{3t} = (\hat{\pi}_{3o} + \hat{\pi}_{31}GDP_{t-1} + \hat{\pi}_{32}TRA_{2t} + \hat{\pi}_{32}ICP_{3t}) + \hat{V}_{3t} \quad (14)$$

Estimate of (9*) is given by:

$$\widehat{RIR}_{3t} = \hat{\pi}_{3o} + \hat{\pi}_{31}GDP_{t-1} + \hat{\pi}_{32}TRA_{2t} + \hat{\pi}_{33}ICP_{3t} \quad (15)$$

Now, equation (9*) becomes:

$$RIR_{3t} = \widehat{RIR}_{3t} + \hat{V}_{3t} \quad (16)$$

Now, put (9) into (2), we get:

$GDP_{2t} = \beta_{2o} + \beta_{31}(\widehat{RIR}_{3t} + \hat{V}_{3t}) + u_{2t}$

$GDP_{2t} = \beta_{2o} + \beta_{31}\widehat{RIR}_{3t} + (\beta_{31}\hat{V}_{3t} + u_{2t})$

$$GDP_{2t} = \beta_{2o} + \beta_{31}\widehat{RIR}_{3t} + \hat{V}_{2t}* \quad (17)$$

where $\hat{V}_{2t}* = \beta_{31}\hat{V}_{3t} + u_{2t}$

Estimation of Model Parameters.

Estimation of Model parameters using Two Stage least squares (2SLS)

Two-stage least-squares regression uses instrumental variables that are uncorrelated with the error terms to compute estimated values of the problematic predictor(s) (the first stage), and then uses those computed values to estimate a linear regression model of the dependent variable (the second stage).

Estimation using Three Stage least squares (3SLS)

The procedure for estimating the 3SLS is as follows;

1. We first estimate each of the equations in the system equations 1 to 3 separately and from each estimate, we determine the error vectors $\hat{\varepsilon}_{i(n')} = y_i - X_i\hat{\beta}_i$. For $i = 1,2,\ldots,m$ and $n' = 1,2,\ldots,n$.
2. From the estimated errors $\hat{\varepsilon}_{i(n')}$, we compute the initial estimates of the elements of contemporaneous variance-covariance Ω denoted by $\widehat{\Omega}$. By this procedure, the estimate of the diagonal elements of Ω (the variance of the error term for each equation, σ_{ii}^2) can be estimated by the expression $\sigma_{ii}^2 = \frac{1}{n-p_i-1}\sum_{n'}^{n} \varepsilon_{i(n')}^2$ and for $i,j = 1,2,\ldots,m.\, i \neq j,$ the estimate of the off-diagonal elements of Ω(the covariances between i^{th} and j^{th} equations σ_{ij}^2) can also be estimated by the expression:

$$\sigma_{ij}^2 = \frac{1}{n - \max(p_i, p_j) - 1}\sum_{n'}^{n} \varepsilon_{i(n')}\varepsilon_{j(n')}$$

Estimation of model parameters using Seemingly Unrelated Regression (SUR)

The SUR is a generalization of a linear regression model that consists of system of regression equations, each having its own dependent variable and different sets of exogenous (explanatory) variables. Each equation is linear in parameters and can be estimated separately, hence the system is called seemingly unrelated regression models. The model can be estimated using the classical OLS). Such estimates are consistent, however generally not as efficient as the SUR method, which transformed to feasible generalized least squares with a specific form of the variance-covariance matrix.

The SUR model can be viewed as either the simplification of the general linear model where certain coefficients in matrix β are restricted to be equal to zero, or as the generalization of the general linear model where the regressors on the right-hand-side are not the same in each equation. The SUR model can be further generalized into the simultaneous equations model, where the right-hand side regressors are perceived to be the endogenous variables as well.

Assuming we have g- regression equations;

$y_{ir} = {x_{ir}}^T\beta_i + \varepsilon_{ir}$, i=1..., g

Where *i* represents the number of observations(n) and r is the number of regressors. The number of observations *(n)* is assumed to be large, so that in the analysis we take $n \rightarrow \infty$, whereas the number of equations *m* remains fixed. Each equation *i* has a single response variabley_{ir}, and k_i dimensional vector of regressors x_{ir}. If we stack observations corresponding to the i^{th} equation into *R*-dimensional vectors and matrices, then the model can be written in vector form as:

$y_i = X_i\beta_i + \varepsilon_i$, r=1…g. Where y_i and ε_i are n×1 vector, X_i is a $R \times k_i$ matrix, and β_i is a k_i×1 vector. Finally, we stack these *g* vector equations on top of each other to form the system:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_g \end{pmatrix} = \begin{pmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & X_g \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_g \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_g \end{pmatrix} = X\beta + \varepsilon.$$

3. Result and Discussion

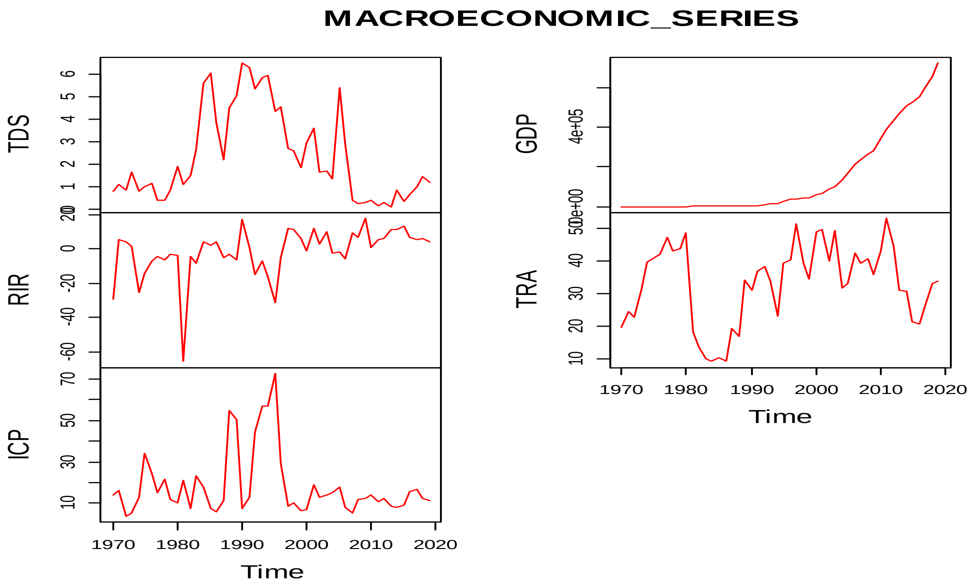

Figure 1: Time plot of Total Debt Service (TDS), Real Interest Rate (RIR), Inflation Consumer Prices (ICP) Gross Domestic Product (GDP) and Trade (TRA).

It was shown from Figure 1 that all the variables displayed periodic movement (i.e. upward and downward trends) which are indications of non-stationary series.

Table 1: Summary Results for Identification of Equations (1), (2) and (3)

Equation	Excluded variable (K-M)	Included variable (G-1)	Remarks
1	4	2	over-identified
2	4	2	over-identified
3	2	2	just identified

It can be established from the results of order condition of identification presented in Table 1 that, equations (1) and (2) were over-identified while only equation (3) was exactly identified respectively.

Table 2: Estimates of Reduced-form Equation (7) from structural equation (1)

Variable	*Coefficient*	*Std. Error*	*t-ratio*	*p-value*	
Constant	2.77875	0.802258	3.4637	0.00118	***
GDP_{t-1}	-2.48889e-05	1.32126e-05	-1.8837	0.06607	*
TRA	-0.0309451	0.0209891	-1.4743	0.14735	
ICP	0.0533956	0.0159907	3.3392	0.00170	***

Summary of other Statistics

Mean dependent var	2.354028	S.D. dependent var	2.020992
Sum squared resid	130.5747	S.E. of regression	1.703426
R-squared	0.333978	Adjusted R-squared	0.289576
F(3, 45)	7.521768	P-value(F)	0.000349
Log-likelihood	-93.54106	Akaike criterion	195.0821
Schwarz criterion	202.6494	Hannan-Quinn	197.9531
Rho	0.594461	Durbin-Watson	0.787525

Table 2 presents the estimates of reduced-form equation (4) from a structural equation (1) and the fitted reduced-form equation is given as: $\widehat{TDS_{1t}} = 2.77875 - 2.48889e^{-05}GDP_{t-1} - 0.0309451TRA_{2t} + 0.0533956ICP_{3t}$ which will later be used to get the predicted value of $\widehat{TDS}_{1t}$ and residuals$\hat{v}_{1t}$. The result from the predicted value will be used in testing the presence of simultaneity between TDS and the stochastic disturbance ($\hat{v}_{1t}$).

Table 3: Estimates of Reduced-form Equation (3.8) from structural equation (2)

Variables	*Coefficient*	*Std. Error*	*t-ratio*	*p-value*	
Constant	106141	48183.1	2.2029	0.03277	**
GDP_{t-1}	9.56553	0.793543	12.0542	<0.00001	***
TRA	-2501.28	1260.59	-1.9842	0.05335	*
ICP	-1027.88	960.393	-1.0703	0.29021	

Summary of other Statistics

Mean dependent var	144740.3	S.D. dependent var	211498.0
Sum squared resid	4.71e+11	S.E. of regression	102306.7
R-squared	0.780635	Adjusted R-squared	0.766011
F (3, 45)	53.37923	P-value(F)	7.26e-15
Log-likelihood	-632.6924	Akaike criterion	1273.385
Schwarz criterion	1280.952	Hannan-Quinn	1276.256
Rho	0.497759	Durbin-Watson	1.002681

Table 3 shows the estimates of reduced-form equation (5) from a structural equation (2) and the fitted reduced-form equation is given as: $GDP_{2t} = 106141 + 9.56553\,GDP_{t-1} - 2501.28TRA_{2t} - 1027.88ICP_{3t}$ which is used to further get the predicted value of $\widehat{GDP}_{2t}$ and residuals, $\hat{v}_{2t}$. The result from the predicted value will be used in testing the presence of endogeneity between GDP and the stochastic disturbance ($\hat{v}_{2t}$) as described by.

Table 4: Estimates of Reduced-form Equation (9) from structural equation (3)

Variables	*Coefficient*	*Std. Error*	*t-ratio*	*p-value*	
Constant	2.12512	5.7601	0.3689	0.71390	
GDP_{t-1}	0.000107458	9.48648e-05	1.1328	0.26332	
TRA	0.0770577	0.150698	0.5113	0.61161	
ICP	-0.387531	0.114811	-3.3754	0.00153	***

Summary of other Statistics

Mean dependent variable	-0.825639	S.D. dependent variable	13.77454
Sum squared residual	6731.165	S.E. of regression	12.23035
R-squared	0.260914	Adjusted R-squared	0.211642
F (3, 45)	5.295351	P-value(F)	0.003264
Log-likelihood	-190.1337	Akaike criterion	388.2675
Schwarz criterion	395.8347	Hannan-Quinn	391.1385
Rho	0.185023	Durbin-Watson	1.620229

Table 4 reveals the estimated reduced-form equation (6) from a structural equation (3) and the fitted reduced-form equation is given as: $GDP_{2t} = 2.12512 + 0.000107458\, GDP_{t-1} + 0.0770577\, TRA_{2t} - 0.387531\, ICP_{3t}$ which is used to get the predicted value of $\widehat{RIR}_{3t}$ and residuals$\hat{v}_{3t}$. The result from the predicted value will be used in testing the presence of simultaneity between RIR and the stochastic disturbance ($\hat{v}_{3t}$) as described by.

Table 5 indicates the results of simultaneity tests conducted on the over-identified equation (1) and the following hypothesis would be tested.

Hypothesis 1

H_o: $\rho_{(GDP_{2t}, u_{1t})}$ vs H_1: Not H_o

Decision Rule: Reject H_o if p-value < level of significance (i.e. 0.05). Otherwise, do not reject H_o.

Table 5: Hausman Test for Endogeneity of Over-identified Structural Equation (1)

Variable	*Coefficient*	*Std. Error*	*t-ratio*	*p-value*	
Constant	2.93663	0.332247	8.8387	<0.00001	***
$\widehat{GDP}_{2t}$	-4.02515e-06	1.4147e-06	-2.8452	0.00660	***
$\hat{V}_{1t}$	-5.56518e-06	2.66873e-06	-2.0853	0.04262	**

Summary of other Statistics

Mean dependent var	2.354028	S.D. dependent var	2.020992
Sum squared resid	154.3082	S.E. of regression	1.831536
R-squared	0.212921	Adjusted R-squared	0.178700
F(2, 46)	6.221954	P-value(F)	0.004059
Log-likelihood	-97.63271	Akaike criterion	201.2654
Schwarz criterion	206.9409	Hannan-Quinn	203.4187
Rho	0.752874	Durbin-Watson	0.477098

It was shown that the fitted $\widehat{GDP}_{2t}$ statistically significantly affected by the total debt service (TDS) at 5% level of significance (p-value = 0.00660). Furthermore, estimate $\hat{v}_{1t}$ was statistically significant having its p-value = 0.04262 < 0.05. H_o is therefore, rejected in favour of H_1 and conclude that endogeneity exist between TDS and GDP. Similarly, Durbin Watson statistic (DW=0.477098) is significantly less than two which indicates the presence of a positive serial correlation. From the same Table 6, the estimated model of the Hausman specification tests is given below:

$$\widehat{\mathrm{TDS}_{1t}} = 2.93663 - 4.02515e^{-06}\widehat{\mathrm{GDP}}_{2t} \qquad (6)$$

Table 6 indicates the results of simultaneity tests conducted on the over-identified equation (2) and the following hypothesis would be tested.

Hypothesis 2

H_o: $\rho_{(RIR_{3t}, u_{2t})}$ vs H_1: Not H_o

Decision Rule: Reject H_o if p-value < level of significance (i.e. 0.05). Otherwise, do not reject H_o.

Table 6: Hausman Test for Endogeneity of Over-identified Structural Equation (2)

Variables	*Coefficient*	*Std. Error*	*t-ratio*	*p-value*	
Constant	157065	26610.2	5.9024	<0.00001	***
$\widehat{RIR}_{3t}$	14927.8	3794.62	3.9339	0.00028	***
$\hat{V}_{2t}$	2546.34	2254.6	1.1294	0.26459	

Summary of other Statistics

Mean dependent var	144740.3	S.D. dependent var	211498.0
Sum squared resid	1.57e+12	S.E. of regression	184975.9
R-squared	0.266949	Adjusted R-squared	0.235077
F(2, 46)	8.375696	P-value(F)	0.000791
Log-likelihood	-662.2512	Akaike criterion	1330.502
Schwarz criterion	1336.178	Hannan-Quinn	1332.656
Rho	0.912035	Durbin-Watson	0.275796

Table 6 shows the fitted $\widehat{RIR}_{3t}$ is statistically significantly affects gross domestic product (GDP) at 5% level of significance (p-value = 0.00028). More so, estimate $\hat{v}_{2t}$ was statistically insignificant with its p-value = 0.26459 > 0.05. H_o is therefore not rejected in favour of H_1 and conclude that simultaneity does not exist between GDP and RIR. In the same vein, Durbin Watson statistic (DW = 0.275796) is significantly less than two which reveals the presence of a positive serial correlation. It was as well shown from Table7 that, the estimated model of the Hausman specification tests is given below:

$$\widehat{GDP}_{2t} = 157065 + 14927.8\,\widehat{RIR}_{3t} \quad (7)$$

From Table 6, it was shown that there is problem of endogeneity since the t-value of $\hat{v}_{1t}$ is statistically significant (p-value = 0.04262 < 0.05). To solve the problem of simultaneity that exist between TDS and GDP, two stage least squares, three stage least squares and seemingly unrelated regression (2SLS, 3SLS and SUR) will be employed and their results are as follow respectively.

Table 7: Unique Estimates of the Parameters of the Over-identified Structural Equation (1) using single and system equation estimators

	Single Equation Estimators				
	Variable	Coeff.	Std. Error	t-ratio/Z	p-value
OLS	Constant	2.921	**0.3123***	9.3548	0.0001***
2SLS	Constant	2.872	0.3294	8.7196	0.0001***
OLS	GDP	-4.220	**1.2395e^{-06}***	-3.4042	0.0014***
2SLS	GDP	-3.58e^{-06}	1.39e^{-06}	3.257	0.0011***
OLS		**R^2= 0.194479, DW= 0.460535 and P-value (F) = 0.001348**			
2SLS		**R^2= 0.208473, DW= 0.469324 and P-value = 0.010065**			
	System Equation Estimators				
3SLS	Constant	2.8716	0.3228	8.896	5.78e^{-019} ***
SUR	Constant	3.031	**0.3079***	9.842	5.35e-013 ***
3SLS	GDP	-3.58e^{-06}	1.37e^{-06}	3.547	0.0004 ***
SUR	GDP	-4.68e^{-06}	**1.2011e^{-06}***	-3.894	0.0003 ***
3SLS		**R^2= 0.208473**			
SUR		**R^2= 0.208473**			

Asterisked (*) indicates the best result reported by standard errors (S.E) of the regression parameters.

Results shown in Table 7 revealed that the OLS is more appropriate estimator for estimating an over-identified equation (1) having lower standard error = 1.2395e^{-06}. The OLS and 2SLS conveyed different R^2 (= 0.194479 and 0.208473), this shows that 19.5% and 20.8% variations respectively in TDS was explained by GDP. Similarly, SUR estimator as shown in the same Table outperformed the 3SLS, OLS as well as 2SLS estimators in estimating the structural coefficients of the over-identified equation (1) having reported the least standard error value for the exogenous variable. The value of R^2 (= 0.208473) reported by the SUR showed that 20.8% variation in TDS was explained by GDP.

Table 8: Unique Estimates of the Parameters of the Over-identified Structural Equation (2) using single and system equation estimators.

	Single Equation Estimators				
	Variable	Coeff.	Std. Error	t-ratio/Z	p-value
OLS	Constant	14983	**27851***	5.3796	0.0001***
2SLS	Constant	157065	33873.3	4.6368	0.0001***
OLS	RIR	5721.27	**1969.61***	2.9048	0.0055***
2SLS	RIR	14927.8	4830.34	3.0904	0.0020***
OLS		**R^2= 0.149505, DW= 0.269661 and P-value (F) = 0.005542**			
2SLS		**R^2= 0.141554, DW= 1.107874 and P-value = 0.001999**			
	System Equation Estimators				
3SLS	Constant	156128	33173.7	4.706	$2.52e^{-06}$ ***
SUR	Constant	151367	**27756.1***	5.453	$1.79e^{-06}$ ***
3SLS	RIR	13792.3	4719.79	2.922	0.0035 ***
SUR	RIR	8025.58	**2002.17***	4.008	0.0002 ***
3SLS		**R^2= 0.141554**			
SUR		**R^2= 0.141554**			

Asterisked (*) indicates the best result reported by standard errors (S.E) of the regression parameters.

Furthermore, Table 8 revealed that the OLS is the best estimator for estimating an over-identified equation (2) having the least standard error = 1969.61. The OLS and 2SLS reported different R^2 (= 0.149505 and 0.141554), this indicated that 14.9% and 14.2% variations respectively in GDP was explained by RIR. In the same vein, SUR estimator as shown in the same Table 8 outperformed the 3SLS, OLS as well as 2SLS estimators respectively in estimating the structural coefficients of an over-identified equation (2) with the least standard error value for the exogenous variable. More so, both 3SLS and SUR reported the same value of R^2 (= 0.141554) which shows that both estimators (3SLS and SUR) can be used to estimate the structural coefficients of an over-identified equation (2) but SUR is more efficient than 3SLS.

Table 9: Unique Estimates of the Parameters of the exactly-identified Structural Equation (3) using single and system equation estimators.

	Single Equation Estimators				
Estimator	Variable	Coeff.	Std. Error	t-ratio/Z	p-value
OLS	Constant	2.1251	5.7601*	0.3689	0.7139
2SLS	Constant	2.12512	5.7601*	0.3689	0.71217
OLS	GDP_{2t-1}	0.0001	9.4865e^{-05}*	1.1328	0.2633
OLS	TRA	0.0771	0.1507*	0.5113	0.6116
OLS	ICP	-0.3875	0.1148*	-3.3754	0.0015***
2SLS	GDP_{2t-1}	0.0001	9.4865e^{-05}*	1.1328	0.25732
2SLS	TRA	0.0771	0.1507*	0.5113	0.60911
2SLS	ICP	-0.3875	0.1148*	-3.3754	0.0007***
OLS		R2= 0.260914, DW= 1.620229 and P-value (F) = 0.003264			
2SLS		R2= 0.260914, DW= 1.620229 and P-value = 0.003264			
3SLS	Constant	-0.7895	4.2519*	-0.1857	0.8527
SUR	Constant	0.9476	5.1998	0.1822	0.8562
3SLS	GDP_{2t-1}	0.0003	7.2284e^{-05}*	4.743	2.10e-06 ***
3SLS	TRA	0.0305	0.1073*	0.2845	0.776
3SLS	ICP	-0.3334	0.0991*	-3.364	0.0008 ***
SUR	GDP_{2t-1}	0.0002	8.80e^{-05}	1.904	0.0633 *
SUR	TRA	0.1244	0.1353	0.9194	0.3628
SUR	ICP	-0.4581	0.1034	-4.432	5.92e-05 ***
3SLS		R2= 0.210489			
SUR		R2= 0.258696			

Asterisked (*) indicates the best result reported by standard errors (S.E) of the regression parameters.

Results reported in Table 9 showed that the OLS and 2SLS estimates are the same; which means that either of the two estimators is appropriate for estimating the exactly identified-equation (3) since it contains only exogenous variables. The OLS and 2SLS reported the same R^2 (0.260914) i.e. 26.1% variation in RIR was explained by GDP_{2t-1}, TRA and ICP respectively. More so, the standard errors of the structural parameters for these estimators are also the same. Similarly, 3SLS estimator as indicated in Table 9 outperformed SUR, OLS and 2SLS estimators respectively in estimating the structural coefficients of the exactly-identified equation (3) with the least standard error value for the exogenous variables. Furthermore, both 3SLS and SUR reported different value of R^2 (= 0.210489 and 0.258696) which showed that 21.1% and 25.9% variations in RIR were explained by GDP_{t-1}, TRA and ICP respectively.

Generally, this research work utilized SUR technique to examine the relationship between Total Debt Service (TDS) and Gross Domestic Product (GDP) in Nigeria. The three-equation model ((1), (2), and (3)) have been identified by both the order and rank conditions. The results for model identification are shown in Table 1 Equations (1) and (2) were determined to be over-identified by the order condition, whereas, equation (3) was determined to be exactly identified. The fitted GDP (i.e., $\widehat{GDP}_{2t}$)) was significantly influenced by TDS at all levels of significance, $\alpha = 1\%, 5\%\ and\ 10\%$ with p-value of 0.00660.

The null hypothesis of endogeneity between TDS and GDP in the over-identified equation (1) was equally accepted by the Hausman specification test, however the model is more affected by positive serial correlation leading to endogeneity problem. This was evident from the Durbin Watson statistic, DW=0.477098 < 2.0., which is significant. The fitted $\widehat{RIR}_{3t}$ was substantially influenced by TDS at levels of significance, $\alpha = 1\%, 5\%\ and\ 10\%$ with p-value of 0.00028. Although the Hausman specification test rejects the null hypothesis of endogeneity between TDS and RIR in the over-identified equation (2). However, the model is more affected by positive serial correlation. This was evident from the value of Durbin Watson statistic, DW=0.275796 < 2.0. Tables 5 displayed the estimated results of the over-identified equation (1), it demonstrated that the single-equation estimator (OLS) performed better than the 2SLS estimator by having the least standard error for each of the regression coefficients in the model. Although the SUR estimator gave the overall least values of the standard error for each of the regression coefficients, it is nevertheless favored over OLS, 2SLS, and 3SLS in estimating the structural parameters of equation (1). Furthermore, the single-equation estimator (OLS) outperformed 2SLS estimator by having the least standard error for each of the regression coefficients in the model, as evidenced by the estimated results of the over-identified equation (2) as shown in Tables 6. The SUR estimator is preferred to all other estimators in the model for estimating the structural parameters of equation (2) because it provided the overall least value of the standard error for each of the regression coefficients. Similarly, estimated results of the exactly identified equation (3) was shown in Tables 7, demonstrating that the single-equation estimators (OLS and 2SLS) yield identical estimates for each of the regression coefficients in the model as well as equal standard errors. However, the 3SLS estimator is preferred to any other estimators in the model in estimating the structural parameters of equation (3) because it reported the overall least values of the standard error for each of the regression coefficients.

4. Conclusion

This research work used a system of simultaneous equation modeling and seemingly unrelated regression techniques to explore the relationship between TDS and GDP of Nigeria economy. It was shown that GDP was significantly affected by the total debt service (TDS) at 5% level of significance (p-value = 0.00660). Consequently, if debt servicing continues, the higher the debt-to-GDP ratio, the less likely the country pays back its debt and the higher the risk of default which could cause a financial panic in the domestic and international market.

Furthermore, in estimating the structural parameters of the exactly and over-identified equations incorporated in the three-equation model, System estimators such as the 3SLS and SUR outperformed the single-equation estimators (OLS and 2SLS) based on the results of the analyses and the summary of findings. The SUR is more efficient at estimating the parameters of the over-identified equations, whereas the 3SLS is more efficient at estimating the regression coefficients of the exactly identified equation. The variance of the model parameters offered by SUR and 3SLS were the least for the respective equations.

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