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Variable Thermo-Physical and Electrical Field Influence on Nonlinear Convective Flow of non-Newtonian Fluid through an Inclined Annular Micro-Channel with Porosity

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Abstract

An investigation of variable thermo-electrical field, viscosity, and thermal conductivity influence on blood rheological (Casson) fluid flow through micro annular channel subjected to suction/injection, slip, jump, and the nonlinear convective process is presented. The assumed steady, fully developed, and magnetized flow through the annular cylinder is modeled and non-dimensionalized under; Slip, jump, suction/injection conditions. Employing the Chebyshev collocation method, an approximate solution of the flow distributions is obtained. Parameters of interest indicate a declination of the flow field to a hike in variable thermal and electrical field parameters, appreciation to the risen value of viscosity parameter. Meanwhile, both momentum and energy distributions were promoted to a wider curvature radius.

Keywords: Chebyshev Collocation Method; Annular channel; Casson fluid; nonlinear convection, non-dimensionalized.

1. Introduction

In recent times, considerable attention of researchers drawn to the study of transport process in porous media, natural convection with an electrically conducting fluid encompassing the fluid thermophysical condition is being monitored due to their wide application in applied science and engineering. Fluid flow through microchannel is not left out since the need to promote healthy level internal temperature, production of computer chips, maintaining stable transport phenomenon, improve the technological and thermal performance of any fluid transport is highly essential. Moreover, as knowledge in flow and heat transfer issues in micro/micro-channel areas consistently improves it is of great importance to familiarize with fundamental phenomena.

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Hence, it is imperative to examine the transport characteristic of magnetized blood rheological fluid with temperature common cause-effect, porosity, and nonlinear convective in the annular region. Micro-channels are utilized in the transportation of biological material such as cells, DNA, protein, embryos, also in the transportation of analytes and chemical samples, the reason why the blood rheological model is considered in the present investigation. Gireesha and Sindhu (2020a) investigated the entropy generated by Casson fluid flow through a vertical microchannel. Gireesha *et al*. (2019) highlighted the impact of ohmic heating on magnetized Casson fluid flow with mixed convection, however, reported that the Casson parameter negates the flow velocity. Idowu *et al*. (2020a) presented the analysis of Casson fluid flowing through the annular channel. Second law analysis with Casson rheology investigation in annular microchannel justified by Gireesha and Roja (2020) and Gireesha and Sindhu (2020b). Thriveni and Mahanthesh (2020) enlightened the audience on the nonlinear Boussinesq approximation and heat transport of nano liquid through the annular surface. Thriveni *et al*. (2019) explored the nonlinear convective influence on Casson fluid flowing through micro-channel. Other recent geometrical investigations of Casson fluid include; Akolade *et al*. (2020) thermophysical investigation over a squeezed parallel plate, gyrotactic micro-organisms with Casson nanofluid movement by Ansari *et al*. (2021) an experiment past a slendering sheet, and Idowu *et al*. (2020b), investigation on modified flux model with Casson rheology, Khan *et al*. (2020) Casson fluid investigation through a Y-shaped fin, and Hamid *et al*. (2019) demonstrated the wavelet solution approach to Casson fluid stagnation point flow, just to mention few. Gbadeyan *et al*. (2020) and Akolade *et al*. (2021) highlighted the effects of thermal conductivity as well as plastic dynamic viscosity on the flow of Casson rheology. They reported an appreciation in fluid momentum and a declination in energy to a hike in both viscosity and thermal conductivity parameters. Conclusively, the authors reveal that Casson fluid parameter assisted both the fluid temperature and heat transfer rate along the flow surfaces and its continuous injection of the parameter tends the model from non-Newtonian to Newtonian fluid.

Likewise, appreciable progress has been made in studies related to the MHD phenomenon as a result of prompt importance in electrolytes, liquid metal, and ionized gases. The dealing between a magnetic field and an electrically conducting fluid affects much industrial equipment like an MHD generator, pumps, and bearing among others, as the size of the applied magnetic field is believed to contribute greatly to the flow process. In chemical energy technology, the use of an MHD pump is embraced for the pumping of electrically conducting fluid at a certain atomic energy center. For electrically conducting fluid, an imposed magnetic field affects the free convection flow conventionally (2015). A theoretical analysis of mixed convection and MHD effect was discussed by Jha and Malgwi (2020). Nagaraju *et al*. (2019a and 2019b) presented the behavior, entropy generation, and heat transfer in different base fluids through a circular pipe. The free convection motion with the impact of micro rotational velocity was investigated by Panigrahi *et al*. (2020). Also, Jha and Aina (2018a) highlighted the impact of the MHD convection flow micro-channel study. Roja and Gireesha (2020) numerically analyzed the hall effects on a couple stress fluid with heat generation impact.

Recently, a series of literature had discussed the phenomena “porous media” due to its wide possible application in applied science and engineering, which includes; utilization in underground water resources in the field of agriculture, exploration of natural gas movement in petroleum technology, seepage of water in river-beds, water and oil through oil reservoirs, filtration and purification process in chemical engineering, among others Sharma *et al*. (2017) and Magaji (2016). Girish *et al*. (2020) studied buoyant convection development in double-passage vertical annuli with porosity subjected to unheated entry and unheated exit. Sankar *et al*. (2018) examined the magneto-convective heat transfer in vertical porous annuli by considering viscous dissipation numerically. The development of natural convection in vertical porous annuli subjected to isothermal and adiabatic thermal conditions by Girish *et al.* (2019a, 2019b and 2018). Ranjit and Shit (2019) presented the electro magnetohydrodynamic flow with irreversible analysis of a porous asymmetric microchannel under the influence of Joule heating. They stated that zeta potential is significantly important in the control of velocity and thermal response in the system. Jha and Yusuf (2018) analyzed the unsteady liquid flow through annular porous walls with heat generation/absorption. They stated that heat generation is functional for the optimal flow rate in the annular region most especially when the convective flow tends to increase by a regular heat flux.

The physics of thermophysical properties in the flow of non-Newtonian fluid play a significant role in determining the performance parameter, such as heat transfer coefficient (HTC), the energy efficiency of a thermal system, and pressure drop. Hence, for effective evaluation of pure substance, minimization of computational time and input data, a clear estimate of likely error machine error, an adequate evaluation of a given thermophysical property must be specified. On this note, Raju and Ojjela (2018) investigated the influence of variable thermal property in the motion of an unsteady channel flow of an incompressible fluid. Amirsom *et al*. (2019) presented second-order slip and variable thermophysical in bioconvection flow. A full analysis of micropolar fluid flow with MHD over an upper horizontal surface with variable thermal viscosity was presented by Sarojamma *et al*. (2019). Their investigation revealed that the material parameter intensity negates the surface drag coefficient while the reversed effect is shown by the magnetic field parameter. Mjankwi *et al*. (2019) analyzed unsteady nanofluid flow over an inclined stretching sheet with variable properties. They discovered that as the thermal conductivity and radiative heat flux increases, the heat transfer rate decreases while the skin friction and mass transfer rate increases. They further observed that there is a reduction in the skin friction, heat, and mass transfer rate as the porosity parameter increases. Rahman *et a*l. (2019) analyzed MHD micropolar flow motion with the effects of variable viscosity and thermal conductivity. Their investigation shows that an increase in thermal conductivity and slip increases the energy boundary layer thickness. In the analysis of Singh *et al*. (2019), mixed convection in boundary layer flow of water over a non-stationary vertical plate with Prandtl number and variable viscosity was discussed. They revealed that the free-stream velocity rate to the composite reference velocity has a significant impact on the velocity profile. Hasona *et al*. (2019) elaborated the effects of thermal radiation and variable electrical conductivity of Carreeau nanofluids on MHD peristaltic motion. While the novel work of Jha *et al*. (2016) and Jha and Aina (2018b) on variable viscosity highlighted the impact of temperature-dependent viscosity in the flow of Newtonian fluid through annular micro-channel, Recent literature on flow through a microchannel with different fluid properties includes; Jha *et al*. (2018), Jha and Aina (2018c), Agboola *et al*. (2018), Oni and Jha (2019), Boniface and Ajibade (2019), Yusuf and Gambo (2019), Ajibade and Gambo (2020), Taiwo and Dauda (2019).

The present examination is motivated by the work of Idowu *et al*. (2020a), hence, this study considered the assumption that the fluid properties all variable. Thus, we then implement the numerical technique of Chebyshev based Collocation Method on the Casson fluid motion in an inclined annular medium with variable thermo-physical, variable electrical field and suction/injection influences, as the properties impact on the flow process.

2. Model Formulation

The thermo-physical/electrical properties were assumed variable in the flow assumption of steady, MHD conducting, fully developed, incompressible, and dissipative fluid flow transfer of heat and nonlinear convective process of Blood rheological fluid motion in an inclined micro annular porous channel with slip/jump effect. The annular walls are subjected differently to heat with the outer surface of the inner porous cylinder sustained at a temperature T_1 while the inner surface of the outer porous cylinder at a temperature, T_2. Due to the temperature diversity, natural convection takes place. Figure 1, displayed the micro-channel suspended at an angle α, d_1 and d_0 are the outer and inner cylinder radius respectively, then $y -$ axis is taken towards the flow direction while the $\xi -$ axis is taken perpendicular to it. The governing systems of the magnetized annular flow are:

$$\nabla \cdot \vec{F} = 0 \tag{1}$$

$$\rho([\vec{F} \cdot \nabla]\vec{F}) = -\nabla P + \mu\nabla^2\vec{F} - \rho_K \mathrm{g}\cos[\alpha] + \mathrm{J} \times \mathrm{B}_0 - \frac{\mu}{k_p}\vec{F} \tag{2}$$

$$\rho c_p([\vec{F} \cdot \nabla]T) = \kappa\nabla^2 T + \mu(\nabla\vec{F} \cdot \nabla\vec{F}) + Q \tag{3}$$

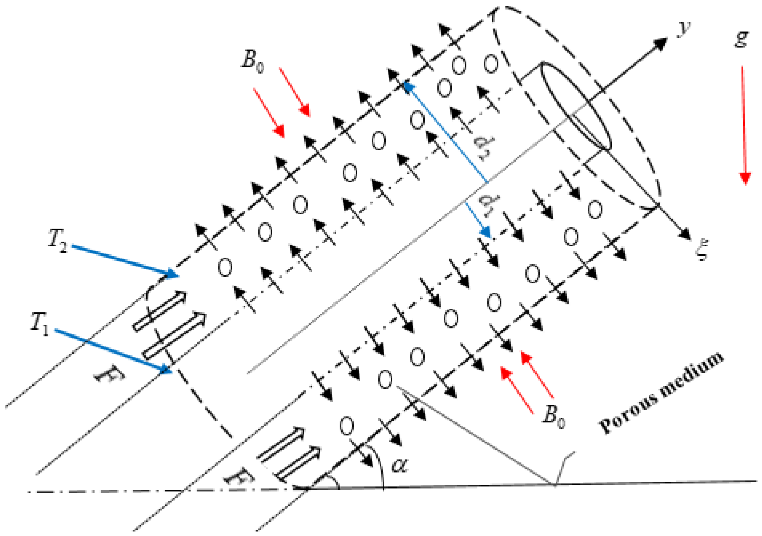


Figure. 1. Physical model coordinate system

For an isentropic and incompressible Casson fluid, we present the kinematic viscosity, variable viscosity, and thermal conductivity as an exponential function of temperature, a variable electrical conductivity as linear temperature function, and the buoyancy force nonlinear representation respectively as follows;

$$\begin{gathered} \nu = \frac{\mu}{\rho}\left(1 + \frac{1}{\beta}\right), \quad \mu(\mathrm{T}) = \mu_0 e^{-\gamma_1(\mathrm{T}-\mathrm{T}_0)}, \quad \kappa(\mathrm{T}) = \kappa_0 e^{-\gamma_2(\mathrm{T}-\mathrm{T}_0)}, \\ \sigma^* = \sigma_0[1 + \gamma_0(\mathrm{T} - T_0)], \quad \rho_K = -[k_a(T - T_0) + k_b(T - T_0)^2], \end{gathered} \tag{4}$$

To estimate the suction/Injection rate at the cylinder surfaces, we assume the suction/injection at the outer surface of the inner cylinder to as $F = F_0$ while, injection/suction rate at the inner surface of the outer cylinder is taken to be, $F = F_0 \frac{d_1}{d_0}$. Integrating equation (1) we obtain

$$F = F_0 \frac{d_1}{d} \tag{5}$$

The governing systems of electrically conducting, MHD, fully developed, Casson fluid flow through an inclined porous medium subject to all the assumptions of fluid properties, variable electric field, steady, absence of pressure, suction/ injection influence defined above are as follows; (Jha *et al*. (2015), Gireesha and Roja (2020), Idowu *et al*. (2020a).

$$F\frac{du}{d\xi} = \left(1+\frac{1}{\beta}\right)\frac{1}{\xi}\frac{d}{d\xi}\left[\mu(\mathrm{T})\,\xi\frac{du}{d\xi}\right] + g\rho_0\cos\alpha[k_a(\mathrm{T}-T_0)+k_b(\mathrm{T}-T_0)^2] - \left[\frac{\sigma^* B_0^2}{\rho_0}+\frac{\mu(\mathrm{T})}{k_p}\left(1+\frac{1}{\beta}\right)\right]u, \tag{6}$$

$$F\frac{dT}{d\xi} = \frac{1}{\rho_0 c_p}\frac{1}{\xi}\frac{d}{d\xi}\left[\kappa(\mathrm{T})\xi\frac{dT}{d\xi}\right] + \left(1+\frac{1}{\beta}\right)\frac{\mu(\mathrm{T})}{\rho_0 c_p}\left(\frac{du}{d\xi}\right)^2 + \frac{Q^*}{\rho_0 c_p}(\mathrm{T}-T_0). \tag{7}$$

Subject to the boundary conditions

$$\begin{aligned} u &= \left(1+\frac{1}{\beta}\right)\beta_v\lambda\frac{du}{d\xi}, \quad T = T_1 + \beta_t\frac{2\gamma}{\gamma+1}\frac{\lambda}{\mathrm{Pr}}\frac{dT}{d\xi}, \quad \text{at} \quad \xi = d_0, \\ u &= -\left(1+\frac{1}{\beta}\right)\beta_v\lambda\frac{du}{d\xi}, \quad T = T_0 - \beta_t\frac{2\gamma}{\gamma+1}\frac{\lambda}{\mathrm{Pr}}\frac{dT}{d\xi}, \quad \text{at} \quad \xi = d_1, \end{aligned} \tag{8}$$

where $\beta_v = \frac{2-\sigma_v}{\sigma_v}$, $\beta_t = \frac{2-\sigma_t}{\sigma_t}$, $\lambda = \frac{\sqrt{\pi R T_0/2\mu_0}}{\rho_0}$

Invoking equations (4) and (5) then utilizing the following dimensionless variables in equation (10) on the governing system of equations. (6) to (9);

$$\xi = \frac{t-d_0}{h}, \qquad h = d_1 - d_0, \quad f = \frac{u}{h_0}, \quad \eta = \frac{d_0}{d_1}, \quad h_0 = \frac{\rho_0 g k_a(\mathrm{T}_1 - T_0)}{\mu_0^2}h^2, \quad \theta = \frac{T-T_0}{T_1-T_0}. \tag{9}$$

The resulting system of equations. (6) - (9) are reduced to;

$$\begin{aligned} \frac{\delta}{\eta+(1-\eta)\mathrm{t}}\frac{df}{dt} &= \left(1+\frac{1}{\beta}\right)\frac{1}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\left[[\eta+(1-\eta)\mathrm{t}]e^{-B_1\theta}\frac{df}{dt}\right] + \cos[\alpha](\theta+\varepsilon\theta^2) \\ &\quad - \left\{Ha^2(1+B_3\,\theta) + \frac{1}{Da}\left(1+\frac{1}{\beta}\right)e^{-B_1\theta}\right\}f, \end{aligned} \tag{10}$$

$$\frac{\delta \mathrm{Pr}}{\eta+(1-\eta)\mathrm{t}}\frac{d\theta}{dt}=\frac{1}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\left[[\eta+(1-\eta)\mathrm{t}]e^{-B_2\,\theta}\frac{d\theta}{dt}\right]+\mathrm{Pr}\left\{\chi\theta+Ec\,e^{-B_1\,\theta}\left(1+\frac{1}{\beta}\right)\left(\frac{df}{dt}\right)^2\right\} \tag{11}$$

while boundary conditions become;

$$\begin{aligned} f&=\left(1+\frac{1}{\beta}\right)\beta_v Kn\frac{df}{dt},\quad \theta=1+G\beta_v Kn\frac{d\theta}{dt},\qquad \text{at}\quad t=0,\\ f&=-\left(1+\frac{1}{\beta}\right)\beta_v Kn\frac{df}{dt},\quad \theta(\mathrm{t})=-G\beta_v Kn\frac{d\theta}{dt},\qquad \text{at}\quad t=1. \end{aligned} \tag{12}$$

where

$$\left.\begin{aligned} &Ha^2=\frac{\sigma_0 B_0^2(1-\eta)^2}{\mu_0\rho},\quad \varepsilon=\ (T_1-T_0)\frac{k_b}{k_a},\quad Da=\frac{k_p}{(1-\eta)^2},\ \beta_t=\frac{2-\sigma_t}{\sigma_t}\frac{\gamma}{\gamma+1}\frac{1}{Pr},\\ &Ec=\frac{h^0}{\rho c_p(\mathrm{T}_1-T_2)},\quad \delta=\frac{k_0(1-\eta)}{\mu_0},\quad G=\frac{\beta_t}{\beta_v},\quad \eta=\frac{d_0}{d_1},\quad Kn=\frac{\lambda}{h},\\ &B_3=\gamma_0(\mathrm{T}_1-T_0),\ \mathrm{B}_2=\gamma_2(\ \mathrm{T}_1-T_0),\quad \chi=\frac{Q^*}{\mu_0\,c_p}\eta^2,\quad Pr=\frac{c_p\mu_0}{k_0},\ \mathrm{B}_1=\gamma_1(\mathrm{T}_1-T_0), \end{aligned}\right\} \tag{13}$$

We present the flow volumetric rate as follows;

$$Q(\mathrm{t})=2\pi\int_0^1 t\,f(t)dt \tag{14}$$

and the Skin friction along with the Nusselt number accordingly;

$$\tau_{0,1}=\left(1+\frac{1}{\beta}\right)\frac{df}{dt}\bigg|_{t=0,1},\quad Nu_{0,1}=-\frac{d\theta}{dt}\bigg|_{t=0,1}. \tag{15}$$

3. Numerical Solution

To come up with the closed-form solution of the governing systems (10) – (12) will turn out to be a complicated task due to its nonlinearity form. Therefore, a numerical technique of the Chebyshev-based Collocation Method (CCM) was implemented due to its wide capability in handling linear/nonlinear systems of equations (Idowu *et al*. (2020a), (2020b] and *Akolade et al*. (2020). The technique is centered on the expansion by the efficacy of Chebyshev polynomial. At the initial stage, we assume a trial solution to depend on the unknown coefficient to the Chebyshev base function, implement the trial solution on the boundary conditions, then on the governing system to generate the residual error which is aimed at minimizing closed to zero using the collocation technique. (For detail see the references above).

3.1 Application of the method (CCM)

We assume $f(t)$and $\theta(t)$ as Chebyshev base trial functions, defined by

$$f(t) = \sum_{k=0}^{R} a_k Q_k(2t-1), \text{ and } \theta(t) = \sum_{k=0}^{R} b_k Q_k(2t-1), \tag{16}$$

where a_k and b_k are the constants to be determined and $Q_k(2t-1)$ is the shifted Chebyshev function from $[-1,1]$ to $[0,1]$. Substituting equation (16) in the boundary conditions in equation (12) we have

$$\begin{aligned} &\left[\sum_{k=0}^{R} a_k Q_k(2t-1) - \left(1+\frac{1}{\beta}\right)\beta_v Kn \frac{d}{dt}\sum_{k=0}^{R} a_k Q_k(2t-1)\right]_{t=0} = 0, \quad \left[\sum_{k=0}^{R} b_k Q_k(2t-1) - 1 - G\beta_v Kn \frac{d}{dt}\sum_{k=0}^{R} b_k Q_k(2t-1)\right]_{t=0} = 0, \\ &\left[\sum_{k=0}^{R} a_k Q_k(2t-1) + \left(1+\frac{1}{\beta}\right)\beta_v Kn \frac{d}{dt}\sum_{k=0}^{R} a_k Q_k(2t-1)\right]_{t=1} = 0, \quad \left[\sum_{k=0}^{R} b_k Q_k(2t-1) + G\beta_v Kn \frac{d}{dt}\sum_{k=0}^{R} b_k Q_k(2t-1)\right]_{t=1} = 0 \end{aligned} \tag{17}$$

Also substituting equation (16) into the governing equations (10) and (11) produced

$$\begin{aligned} D_f := &\left(1+\frac{1}{\beta}\right)\frac{1}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\left([\eta+(1-\eta)\mathrm{t}]e^{-B_1\sum_{k=0}^{R} b_k Q_k(2t-1)}\frac{d}{dt}\sum_{k=0}^{R} a_k Q_k(2t-1)\right) + \cos[\alpha]\left\{\sum_{k=0}^{R} b_k Q_k(2t-1) + \varepsilon\left(\sum_{k=0}^{R} b_k Q_k(2t-1)\right)^2\right\} - \\ &\left\{Ha^2\left(1+A\sum_{k=0}^{R} b_k Q_k(2t-1)\right) + \frac{1}{Da}\left(1+\frac{1}{\beta}\right)e^{-B_1\sum_{k=0}^{R} b_k Q_k(2t-1)}\right\}\sum_{k=0}^{R} a_k Q_k(2t-1) - \frac{\delta}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\sum_{k=0}^{R} a_k Q_k(2t-1) \end{aligned} \tag{18}$$

And

$$\begin{aligned} D_\theta := &\frac{1}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\left([\eta+(1-\eta)\mathrm{t}]e^{-B_2\sum_{k=0}^{R} b_k Q_k(2t-1)}\frac{d}{dt}\sum_{k=0}^{R} b_k Q_k(2t-1)\right) \\ &+ \Pr\left\{\chi\sum_{k=0}^{R} b_k Q_k(2t-1) + Ec\left(1+\frac{1}{\beta}\right)e^{-B_2\sum_{k=0}^{R} b_k Q_k(2t-1)}\left(\sum_{k=0}^{R} a_k Q_k(2t-1)\right)^2\right\} \\ &- \frac{\Pr\delta}{\eta+(1-\eta)\mathrm{t}}\frac{d}{dt}\sum_{k=0}^{R} b_k Q_k(2t-1) \end{aligned} \tag{19}$$

residues $D_f(\mathrm{t}, \mathrm{a}_k, b_k)$ and $D_\theta(\mathrm{t}, \mathrm{a}_k, b_k)$, are derived from the above Eqs (18) and (19) accordingly. Implementing the shifted Gauss Lobato collocation technique $t_k = \frac{1}{2}\left(1 - cos\left(\frac{j\pi}{R}\right)\right), \quad for \quad j = 0,1 \cdots, R.$, the residues are minimized close to zero (See Akolade *et al*. (2020), Idowu *et al*. (2020a)). Conclusively, the unknown constants a_k, and b_k are sought for from the system of 2N+2 algebraic equations with 2N+2 unknown coefficients using the Newton method, and the approximate solutions$f(\mathrm{t})$and $\theta(\mathrm{t})$are evaluated.

4. Results and Discussion

The goal of this study is to examine combined variable thermophysical and electrical field influence on the nonlinear convective flow of blood rheological motion in annular micro-channel with porosity, by employing Chebyshev based Collocation Method, a numerical technique. For a clear understanding of the physical problem, we set $Ha = 1, \eta = 0.5,\ \beta = 0.2,$ $Da = 1, \varepsilon = 1.5, B_1 = B_2 = B_3 = 0.2, \mathrm{Pr} = 0.71,\ S = \pm 0.5,\ \alpha = \frac{\pi}{3},\ \chi = 0.1,\ G = 5,\ \beta_v kn =$ $0.05,$ unchanged in the study unless otherwise stated. To ascertain the accuracy of the used method, the combined residual error analysis of the distributions is presented in Figure 2 and it is found in line with the work of Idowu *et al*. (2020a).

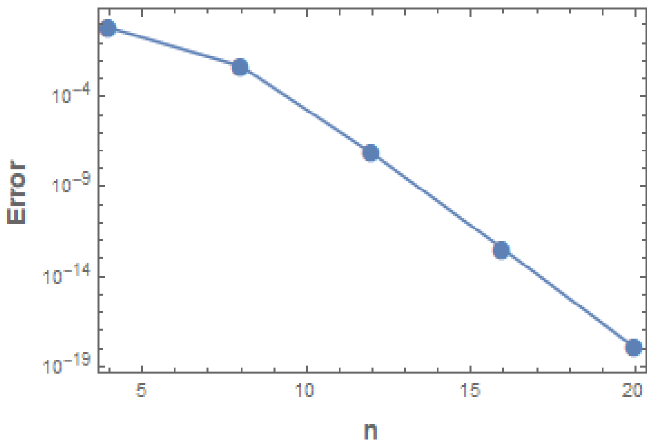
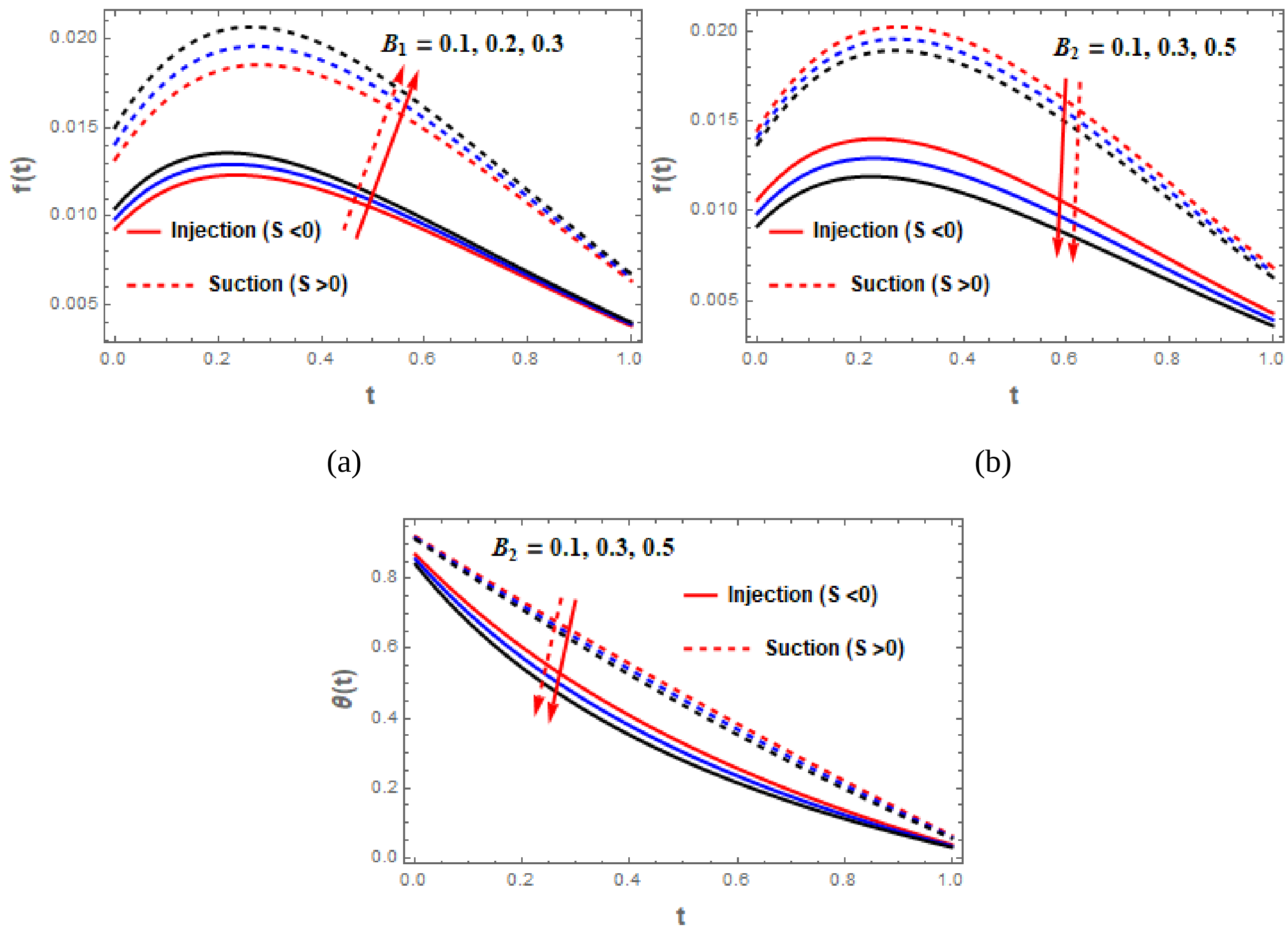


Figure 2. Minimized residual error

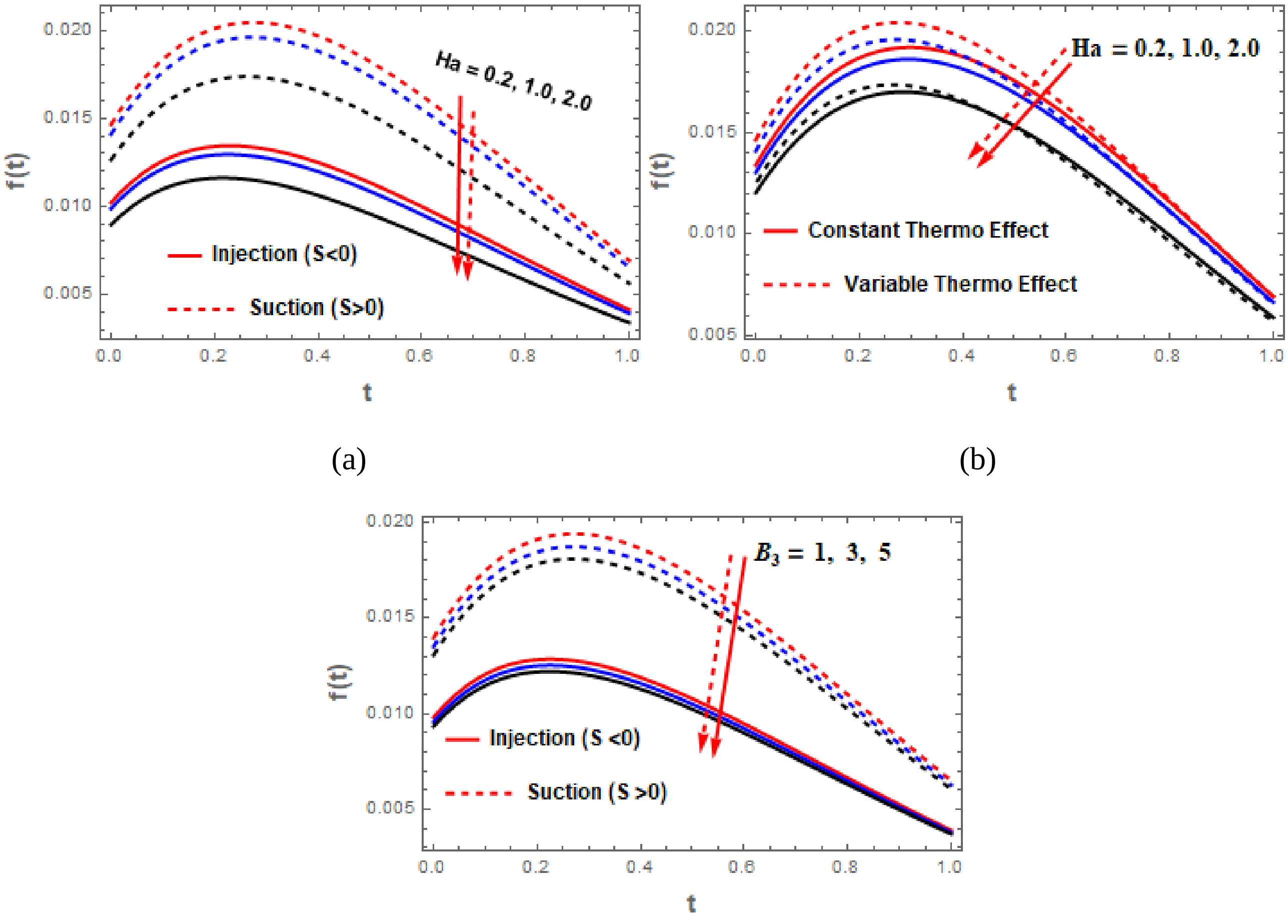
Figure 3(a-c) depicts the impact of variable thermophysical features under the influence of suction/injection on the flow velocity and energy fields accordingly. In the flow distributions, the force exerted by a partial vacuum on the fluid particles called suction dominates the flow fields when compared to injection (insertion of fluid particles) instance. Physically, suction effect moves the fluid particles near the cylinder surfaces, while injection influence showcase the continuous accumulation of fluid particle. It is observed that variable viscosity impacted the flow field positively, which is however attributed to fluid resistance as B_1 appreciates, while the thermal conductivity downsized both velocity and temperature profiles accordingly (see Figure 3 b and c).



(a) (b)

(c)

Figure 3. Impact of variable properties under Suction/Injection condition on the flow velocity and temperature

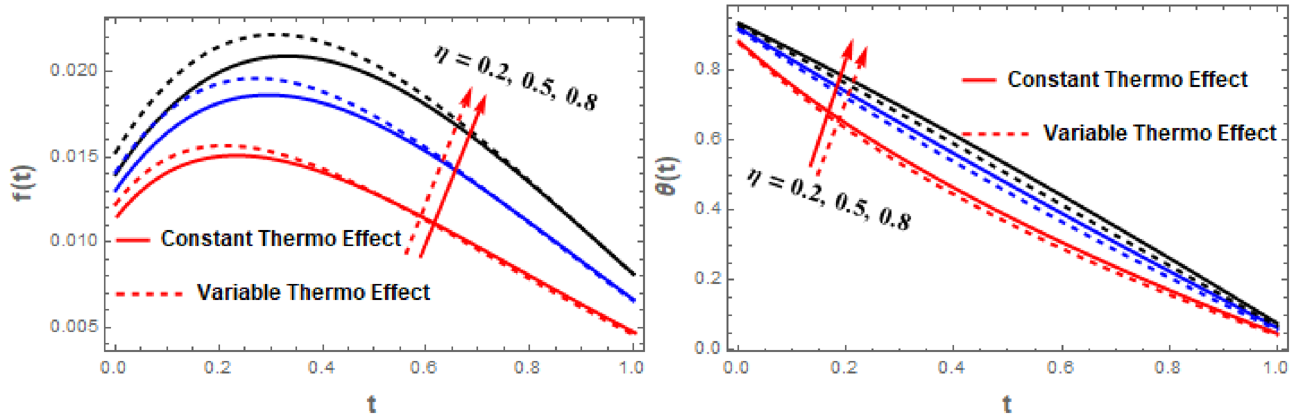


(a) (b)

(c)

Figure 4. Influence of (a) Suction/Injection and Hartmann number, (b) variable properties, and Hartmann number (c) variable electrical field influence on the flow velocity.

Figure 4(a-c) shows the impact of the Hartmann number and variable electrical field impact on the velocity field. It is observed that as the Hartmann number increases, it leads to a decrease in fluid velocity, variable properties impact appreciate the flow velocity within the region $t \leq 0$, but declined the flow field in the region $t = 1$. Also, the impact of the variable electrical field impacted the flow field negatively (see Figure 4c). This is physically true as the impact of the magnetic field in the presence of an electrically conducting fluid generates the Lorentz force which leads to retardation of the fluid flow that results in a decrease in the velocity of the fluid passing through the annular channel.

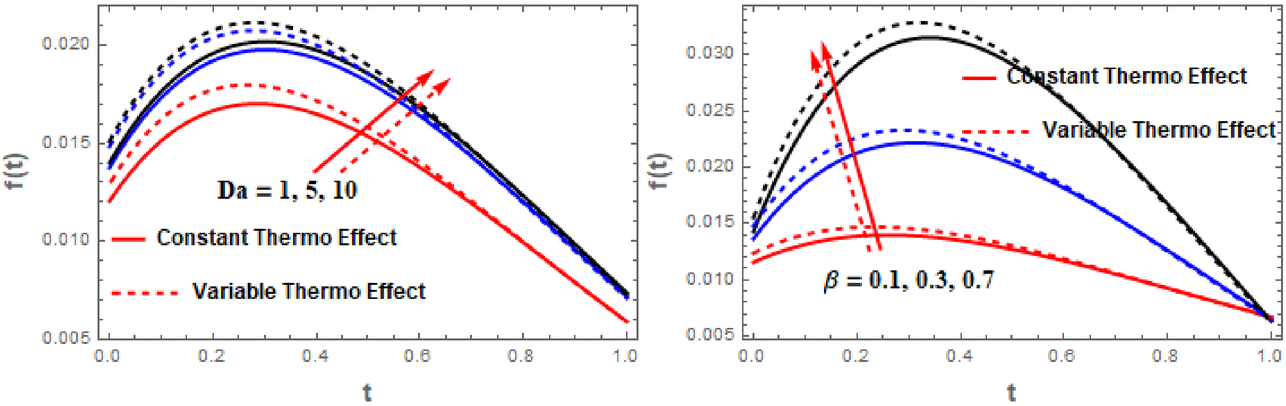


(a) (b)

Figure 5. Influence of curvature radius ratio on (a) velocity and (b) temperature field

Figure 5 explained graphically, the influence of curvature radius ratio(η) on the flow and energy profiles. Physically, boarding fluid flow region brings about free movement of the fluid particles, resulting in enhancement of velocity and energy due to the medium and fluid particles collisions which reduce the internal binding force between the fluid particles. Fig 5a-b indicates that a broad annular gap ratio with both constant thermo effect and variable thermo effect. It is important to know that, an increase in the curvature radius ratio leads to a rise in temperature at both surfaces.

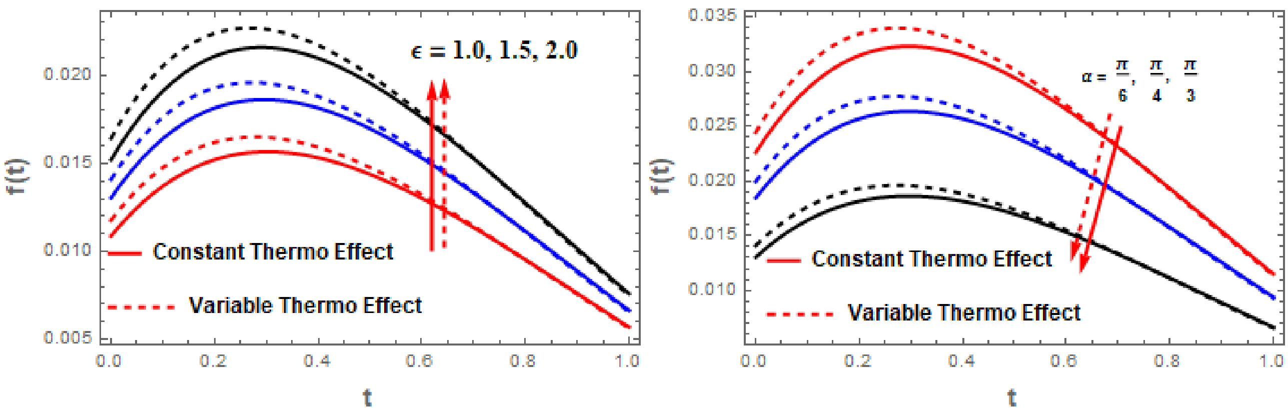
In the same manner, a hike in Darcy number and continuous injection of Casson fluid promotes the velocity field accordingly (Figure 6 (a and b)). Figure 6a shows that there is a corresponding increase in the velocity of the permeability of the fluid material when the Darcy number increases. Figure 6b vividly indicates that the fluid velocity increases with a perpetual rise β. Physically, $\beta \to \infty$, the neutralization of the fluid descends from a non-Newtonian fluid to Newtonian. Hence, the bulkiness of β results to initial layer enhancement, thereby arriving in the respective results (see Figure 6a-b).



(a) (b)

Figure 6:. Influence of (a) Darcy, (b) Casson parameter on the flow velocity profiles

In Figure 7a-b, the influence of nonlinear convection parameter and inclination angle on the flow velocity is presented. It is observed that the rise in nonlinear convection term leads to an increase in the flow velocity. It is clearly visible in both cases (Figure 7a-b) of the velocity that variable thermo effects are higher than constant thermo effects. While a rise in inclination angle reduces the flow velocity.



(a) (b)

Figure 7: Influence of (a) nonlinear convection parameter, (b) inclination angle on the flow velocity

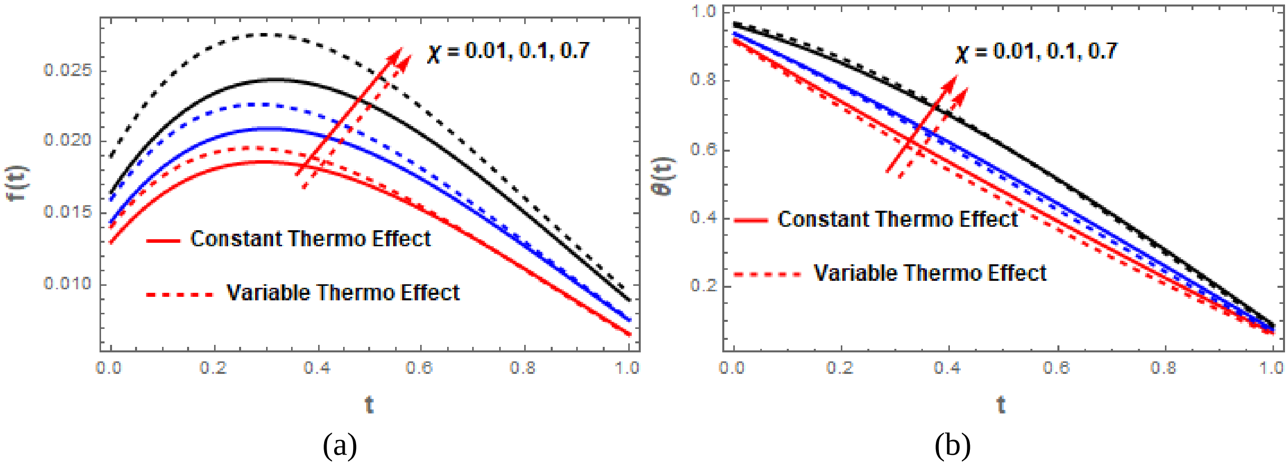


Figure 8: Influence of heat source parameter on (a) velocity and (b) temperature

The influence of the heat source parameter (χ) on the velocity and temperature is presented in Figure 8a-b. It shows that as the heat source parameter increases, the velocity and temperature of the fluid tends to increases in both cases of constant thermo effects and variable thermo effect. On this note, the fluid turns warmer that improves the velocity and temperature field. Figure 9a-b illustrates the impact of fluid-wall interaction parameter (G) on flow and energy field respectively, clearly a decrease in fluid velocity and temperature which leads to an increase in slip velocity close to the exterior surface of the interior cylinder while the opposite behavior is observed at the interior surface of the exterior cylinder.

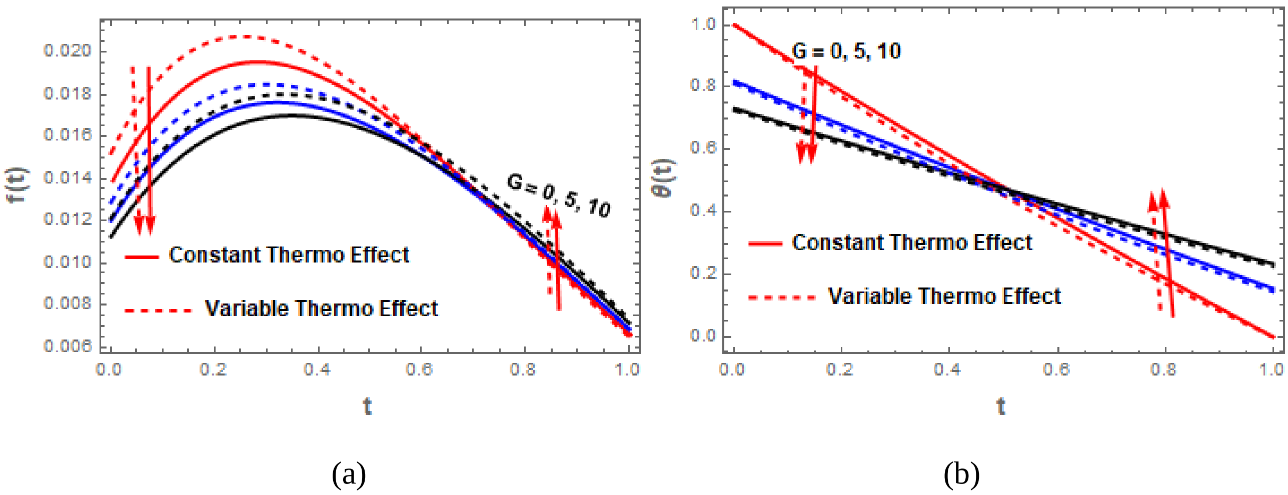


Figure 9: Influence of thermo properties on (a) velocity and (b) temperature for different values of the fluid-wall interaction parameter (G)

5. Conclusion

Examinations of combined thermophysical impacts, nonlinear thermal convection process on the motion of fluid in an annular inclined cylinder with porous medium are presented. The nondimensionalized controlling systems are solved numerically by Chebyshev based Collocation method, thus, we deduced that; a hike in variable thermal and electrical field parameters declined the flow field, while viscosity parameter gave an appreciation pattern. Both momentum and energy distributions were promoted to a wider curvature radius, a corresponding rise in Darcy number increases corresponds to a higher flow rate and the force exerted by a partial vacuum on the fluid particles dominates the flow fields when compared to the insertion of fluid particles instance.

Nomenclature

B_0 magnetic flux density
Cp specific heat at constant pressure
βt, βv dimensionless parameter
γ ratio of specific heat
d_0 radius of inner porous cylinder
d_1 radius of the outer porous cylinder
β Casson parameter
χ heat source parameter
λ molecular mean free path
To temperature at the inlet
T_1 temperature at the outlet
Kn Knudsen number
α inclination angle
Ha Hartman number
ε nonlinear thermal convection
G wall–fluid interaction parameter
Q volume flow rate
μ_0 ynamic viscosity
δ suction/injection parameter
ρ_0 fluid mixture density
Pr Prandtl number
η radii ratio parameter
Da Darcy Parameter
k_0 constant suction/injection velocity
g gravitational acceleration
R specific gas constant
K_a thermal conductivity
ρ fluid density
B_1 variable viscosity
B_2 variable thermal conductivity
B_3 variable electrical conductivity
σ_v thermal accommodation

σ_t tangential momentum accommodation
θ dimensionless temperature
Ec Eckert number
T fluid temperature
F dimensionless axial velocity
U dimensional axial velocity
ξ dimensional radial coordinates
T dimensionless radial coordinate
H dimensionless gap between the cylinders

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