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An Application of Picard Iteration Method to Fractional Quadratic Ricccati Differential Equations

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Abstract

In this paper, we present an algorithm of the Picard's Iteration method to solve Fractional Quadratic Riccati differential equations with Caputo-derivatives. The non-linear terms are easily and simply expanded using the traditional method of expansion. Four (4) illustrative examples are given to verify the reliability and efficiency of the method. The approximate solutions obtained compare favorably with the exact solutions and the approximate solutions obtained by other numerical methods in the literature.

Keywords: Caputo-Derivatives, Fractional Quadratic Riccati Differential Equations, Picard's Iteration Method

1. Introduction

The application of Fractional Differential equations, in numerous diverse field of science and engineering and particularly in the modeling of various physical phenomena, very accurately, has made its appearance so frequent in these different areas of study. Many physical process also appear to exhibit fractional order behavior that may vary with time or space Podlubny (1999). These lines subsequently necessitated the need to seek for its solution be it analytical or numerical unfortunately most fractional differential equations do not have exact solution, so approximate and numerical techniques thus required. Significant efforts have been made by many authors and current researchers studying the analytical and approximate solution of these problems in this very particular class (Taiwo and Bello, 2014).

The Riccati differential equation which is named after the Italian Nobleman Count Jacapo Francesco Riccati (1676-1754) concerns with applications in pattern formation in dynamic games, linear systems with Markovian jumps, riverflows, econometric model, stochastic systems, control theory, and diffusion problems (Arikoglu & Ozkol, 2007). The fundamental theories of Riccati equation, with applications to random processes, optimal control and diffusion problems. Numerical treatment of Fractional Riccati Quadratic equation has been

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concern of many erudite due to their appearance in any applications such as in formation of dynamic games, linear systems with Markorian jumps, River Rows, econometric models, stochastic control theory, diffusion problem, and in variant embedding (Odetunde and Taiwo, 2013).

Among the different approximate numerical solutions are Khan *et al*. (2011), who solved this problem using homotopy analysis Method, Jafari & Tajadodi (2010) also solved the same problem using variational iteration Method, Momani & Shawagfeh (2006) solved it using the Adomians decomposition method. A decomposition of algorithm by Odetunde and Taiwo (2013).

The Picard iteration method, or successive approximations method, is a direct and convenient technique for the resolution of differential equations. This method solves any problem by finding successive approximations to the solution by starting with the zeroth approximation. Scholars have used PIM to solve both linear and non-linear, differential, integral, integro - differential equations. Among the scholars includes: Wazwaz (2011), Chen *et al*. (2015) just to mention a few. The success of the method for integer-order calculus has motivated the researcher to extend the application of the scheme to approximate the Fractional Quadratic Riccati Differential equations.

The rest of the paper is organised a follows: In section 2, we give briefly definitions related to the theory of fractional calculus. In section 3, we present the solution procedure of the Iterative Decomposition Method (IDM). Numerical examples are presented in section 4 to illustrate the efficiency and accuracy of the IDM. The conclusions are then given in the last section 5.

2. Basic Definitions

In the section, we give some definitions and properties of the fractional calculus.

Definition 2.1: A real function $f(x), x > 0$, is said to be in the space $C_\mu, \mu \in R$ if there exists a real number $p > \mu$, such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C \in (0, \infty)$ and it is said to be in space C_μ^n if and only if $f^{(n)} \in C_\mu, n \in N$.Clearly $C_\mu \subset C_\beta$if $\beta \leq \mu$.

Definition 2.2:

Let $\alpha \geq 0\ and\ n < \alpha \leq n+1, n \in N$. The operator ${}_\alpha D_t^\alpha$, defined by

$$D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \frac{d^n}{dt^n} \int_\alpha^t (t-x)^{n-\alpha-1} f(x)\, dx, a \leq t \leq b \qquad (1)$$

$${}_\alpha D_t^\alpha f(t) = f(t)$$

is called the Reimann-Liouville Fractional derivative operator of orderα.

Definition 2.3:

The Reimann-Liouville fractional integral operator defined on $L_1[a,b]$ of order $\alpha \geq 0$ of a function $f \in C_\mu, \mu \geq -1$ is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} dt, \alpha >, x > 0 \tag{2}$$

Definition 2.4:

Let $n < \alpha \leq n+1, n \in N$, and $f^{(n)}(x) \in L_1[a,]$. The operatorD_*^α defined by

$$D_*^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_\alpha^t (t-x)^{n-\alpha-1} f^{(n)}(x) dx \tag{3}$$

is called the Caputo Fractional Derivative Operator of order α.

Properties of the operator J^α is found in [2] and included the following

$$J^\alpha f(x) = f(x) \tag{4}$$

$$J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+a)} x^{\alpha+\gamma} \tag{5}$$

$$J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x) \tag{6}$$

$$J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x) \tag{7}$$

Also, if $m-1 < \alpha < m, m \in N$and $f \in C_\mu^m, \mu \geq -1$ then

$$D_*^\alpha J^\alpha f(x) = f(x)$$

$$J^\alpha D_*^\alpha f(x) = f(x) - \sum_{n=0}^{m-1} \frac{x^{\mathrm{n}}}{\mathrm{n}!} f^n(0)$$

$${}_a^bJ^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}(b^{\alpha+\gamma} - a^{\alpha+y})$$

2.1 Picard's Iteration Method for Fractional Riccati Differential Equations

Here, we consider

$$D^\alpha y(x) = A(x) + B(x)y(x) + C(x)y^2(x), \tag{8}$$

$x \in R, 0 < \alpha \leq 1, x > 0$

Subject to the initial condition

$$y^k(0) = y_k, k = 0,1,2, \dots, n-1 \tag{9}$$

where α is the order of the fractional derivatives, x is an integers, $A(x), B(x)$ and $C(x)$ are known functions, and y_k is a constant. The derivative is the Caputo-type derivatives. ${}_0^xJ^\alpha$ is introduced to both sides of (8) and simplified to obtain

$$y(x) = y_0(x) + J^\alpha\{A(x) + B(x)y(x) + C(x)y^2(x)\} \tag{10}$$

Picard's iteration scheme suggests that

$$y_n(x) = y_0(x) + J^{\alpha}\{A(x) + B(x)y_{n-1}(x) + C(x)y_{n-1}^2(x)\} \quad (11)$$

where the zeroth approximation y_0 is our initial condition (9)

several successive approximations $y_k, k \geq 1$ is determined as

$$y_1(x) = y_0(x) + J^{\alpha}\{A(x) + B(x)y_0(x) + C(x)y_0^2(x)\} \quad (12)$$

$$y_2(x) = y_0(x) + J^{\alpha}\{A(x) + B(x)y_1(x) + C(x)y_1^2(x)\} \quad (13)$$

$$y_3(x) = y_0(x) + J^{\alpha}\{A(x) + B(x)y_2(x) + C(x)y_2^2(x)\} \quad (14)$$

$$\vdots \quad (15)$$

$$y_n(x) = y_0(x) + J^{\alpha}\{A(x) + B(x)y_{n-1}(x) + C(x)y_{n-1}^2(x)\} \quad (16)$$

(17)

3. Numerical Examples

We now apply our proposed method described in section 3 to solve four (4) numerical examples.

Example 4.1: consider the Riccati Differential equation

$$D_*^{\alpha}(x) = 1 + y^2, 0 < \alpha < 1 \quad (18)$$

Subject to the condition $y(0) = 0$

The exact solution for the case $\alpha = 1\ is\ y(x) = \tan(x)$

Introduce ${}_0^xJ^{\alpha}$to both sides of (18) and by Picard's method we obtain

$$y_n(x) = y_0 + {}_0^xJ^{\alpha}\{1 + y_{n-1}^2\} \quad (19)$$

Taking our zeroth approximation to be our initial condition $y = 0$ from 19 we therefore obtain

$$y_1(x) = \frac{x^{\alpha}}{\Gamma(\alpha+1)}$$

$$y_2(x) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha)\Gamma(3\alpha+1)}$$

$$y_3(x) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha)\Gamma(3\alpha+1)} + \frac{2\Gamma(1+2\alpha)\Gamma(1+4\alpha)x^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{\Gamma(1+6\alpha)\Gamma^2(1+2\alpha)x^{7\alpha}}{\Gamma^4(1+\alpha)\Gamma^2(1+3\alpha)\Gamma(1+7\alpha)}$$

$$y_4(x) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha)\Gamma(3\alpha+1)} + \frac{2\Gamma(1+2\alpha)\Gamma(1+4\alpha)x^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{\Gamma(1+6\alpha)\Gamma^2(1+2\alpha)x^{7\alpha}}{\Gamma^4(1+\alpha)\Gamma^2(1+3\alpha)\Gamma(1+7\alpha)}$$
$$+ \frac{4\Gamma^2(2\alpha+1)\Gamma(4\alpha+1)\Gamma(8\alpha+1)x^{9\alpha}}{\Gamma^5(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(5\alpha+1)\Gamma(9\alpha+1)} + \frac{2\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)\Gamma(8\alpha+1)x^{9\alpha}}{\Gamma^5(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)\Gamma(9\alpha+1)}$$
$$+ \frac{2\Gamma^3(2\alpha+1)\Gamma(6\alpha+1)\Gamma(10\alpha+1)x^{11\alpha}}{\Gamma^6(\alpha+1)\Gamma^3(3\alpha+1)\Gamma(7\alpha+1)\Gamma(11\alpha+1)} + \frac{4\Gamma^2(2\alpha+1)\Gamma(4\alpha+1)\Gamma(8\alpha+1)x^{11\alpha}}{\Gamma^6(\alpha+1)\Gamma^2(3\alpha+1)\Gamma^2(5\alpha+1)\Gamma(11\alpha+1)}$$

$$+\frac{2\Gamma^3(2\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)\Gamma(12\alpha+1)x^{13\alpha}}{\Gamma^7(\alpha+1)\Gamma^3(3\alpha+1)\Gamma(5\alpha+1)\Gamma(7\alpha+1)\Gamma(13\alpha+1)}$$

$$+\frac{2\Gamma^3(2\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)\Gamma(12\alpha+1)x^{13\alpha}}{\Gamma^7(\alpha+1)\Gamma^3(1+3\alpha)\Gamma(1+5\alpha)\Gamma(1+7\alpha)\Gamma(1+13\alpha)}$$

$$+\frac{\Gamma^2(1+2\alpha)\Gamma(1+14\alpha)}{\Gamma^8(\alpha+1)\Gamma^4(1+3\alpha)\Gamma^2(1+7\alpha)\Gamma^2\Gamma(1+15\alpha)}$$

For the case $\alpha = 1$ we have,

$$y(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{29x^7}{630} + \frac{157x^9}{11340} + \frac{134x^{11}}{51975} + \frac{2x^{13}}{12285} + \frac{x^{15}}{59535} + \cdots$$

Table 1: Comparison of solutions for Example 1 for $\alpha = 1$

x	Exact	Approx. Soln By PIM	Error
0	0.000000000	0.000000000	0.000000000
0.1	0.1003346721	0.103346713	$8.162E-10$
0.2	0.2027100355	0.2027099297	$1.0580E-7$
0.3	0.3093362496	0.3093343442	$1.9050E-6$
0.4	0.4227932187	0.427777155	$1.5500E-5$
0.5	0.5463024898	0.5462212762	$8.1210E-5$
0.6	0.6841368083	0.6838056920	$3.31103E-4$
0.7	0.8422883805	0.8411449022	$1.1430E-3$
0.8	1.0296385570	1.0261001110	$3.5380E-3$
0.9	1.2601582180	1.2499664940	$1.1090E-3$
1.0	1.5574077250	1.5293009690	$2.8110E-3$

Example 2:

Consider the fractional Riccati Differential Equation

$$D_*^\alpha = -y^2(x) + 1, \quad 0 < \alpha \leq 1$$

With the initial condition $y(0) = 0$.

The exact solution for the case $\alpha = 1$ is $y(x) = \frac{e^{2x}-1}{e^2+1}$

Solution

$$y_4(x) = \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha)\Gamma(3\alpha+1)} + \frac{2\Gamma(1+2\alpha)\Gamma(1+4\alpha)x^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{\Gamma(1+6\alpha)\Gamma^2(1+2\alpha)x^{7\alpha}}{\Gamma^4(1+\alpha)\Gamma^2(1+3\alpha)\Gamma(1+7\alpha)}$$

$$+\frac{4\Gamma^2(2\alpha+1)\Gamma(4\alpha+1)\Gamma(8\alpha+1)x^{9\alpha}}{\Gamma^5(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(5\alpha+1)\Gamma(9\alpha+1)} + \frac{2\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)\Gamma(8\alpha+1)x^{9\alpha}}{\Gamma^5(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)\Gamma(9\alpha+1)}$$

$$+\frac{2\Gamma^3(2\alpha+1)\Gamma(6\alpha+1)\Gamma(10\alpha+1)x^{11\alpha}}{\Gamma^6(\alpha+1)\Gamma^3(3\alpha+1)\Gamma(7\alpha+1)\Gamma(11\alpha+1)} + \frac{4\Gamma^2(2\alpha+1)\Gamma^2(4\alpha+1)\Gamma(10\alpha+1)x^{11\alpha}}{\Gamma^6(\alpha+1)\Gamma^2(3\alpha+1)\Gamma^2(5\alpha+1)\Gamma(11\alpha+1)}$$

$$+\frac{2\Gamma^3(2\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)\Gamma(12\alpha+1)x^{13\alpha}}{\Gamma^7(\alpha+1)\Gamma^3(3\alpha+1)\Gamma(5\alpha+1)\Gamma(7\alpha+1)\Gamma(13\alpha+1)} + \frac{2\Gamma^3(1+2\alpha)\Gamma(1+6\alpha)\Gamma(1+4\alpha)\Gamma(1+2\alpha)x^{13\alpha}}{\Gamma^7(\alpha+1)\Gamma^3(1+3\alpha)\Gamma(1+5\alpha)\Gamma(1+7\alpha)\Gamma(1+3\alpha)}$$

For the case $\alpha = 1$

$$y(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{29x^7}{630} + \frac{157x^9}{11340} + \frac{134x^{11}}{51975} + \frac{2x^{13}}{12285} + \frac{x^{15}}{59535} + \cdots$$

Table 2: Approximate Solutions for Example 4.2 for case α0.5 and 1

x	Exact $\alpha = 1$	Approx. by IDM $\alpha = 1$	Approx. VIM[8] $\alpha = 0.5$	Approx. by IDM $\alpha = 0.5$	Approx. VIM [8] $\alpha = 0.5$
0.0	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.099667	0.09968	0.099667	0.083613	0.086513
0.2	0.197375	0.197373	0.197375	0.152754	0.161584
0.3	0.291312	0.291295	0.291320	0.217856	0.2338356
0.4	0.379948	0.379933	0.380005	0.300562	0.321523
0.5	0.462117	0.462417	0.462375	0.400376	0.413682
0.6	0.537049	0.536867	0.537923	0.509746	0.515445
0.6	0.537049	0.536867	0.537923	0.509746	0.515445
0.7	0.604367	0.606782	0.606768	0.619649	0.626403
0.8	0.664036	0.667654	0.669695	0.737509	0.745278
0.9	0.716297	0.712766	0.72139	0.859919	0.870074
1.0	0.761594	0.774428	0.784126	0.979442	0.998176

Example 3: Consider the Fractional Riccatti Differential Equation

$$D^{\alpha} y(x) = 2y(x) - y^2(x) + 1, \quad 0 < \alpha \leq 1$$

With initial condition $y(0) = 0$

The exact solution for the case $\alpha = 1$ is

$$y(x) = 1 + \sqrt{2}\tanh\left\{\sqrt{2x} + \frac{1}{2}\log\left[\frac{(\sqrt{2}-1)}{\sqrt{2}+1}\right]\right\}$$

Then, $y_4(x)$ can be approximated as

$$y_4(x)\frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{2\alpha}}{(2\alpha+1)} - \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}$$
$$-\left\{\frac{2\Gamma^2(2\alpha+1) + 4\Gamma(\alpha+1)\Gamma(3\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}\right\}x^{4\alpha} + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)(1-2\Gamma(3\alpha+1))x^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}$$
$$+\frac{4\Gamma(5\alpha+1)x^{6\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\Gamma+1)} - \frac{\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)x^{7\alpha}}{\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha)}$$

For the particular case $\alpha = 1$,we have

$$y_4(x) = x + x^2 + \frac{x^3}{3} - \frac{2x^4}{3} - \frac{22x^5}{15} + \frac{x^6}{9} - \frac{x^7}{63}$$

Table 3: Approximate Solutions of Example 4.3 for value α

x	$y(x)$Exact $\alpha = 1$	$y(x)$Approx PIM $\alpha = 1$	$y(x)$PIM $\alpha = 0.5$	$y(x)\alpha = 0.5[10]$	$y(x)$PIM $\alpha = 0.75$	$y(x)\alpha = 0.75[10]$
0.0	0	0	0	0	0	0
0.1	0.110295	0.110267	0.494672	0.577431	0.233598	0.244460
0.2	0.241976	0.241778	0.904529	0.912654	0.447051	0.469709
0.3	0.395104	0.395002	1.154389	1.166253	0.654329	0.698718
0.4	0.567812	0.566839	1.353673	1.353549	0.899065	0.924319
0.5	0.756014	0.755839	1.483765	1.482633	1.134787	1.137952
0.6	0.953566	0.953409	1.558548	1.559656	1.330745	1.331462
0.7	1.152946	1.152645	1.579990	1.589984	1.470879	1.497600
0.8	1.346363	1.344702	1.600439	1.5578559	1.619521	1.630234
0.9	1.526911	1.524688	1.671108	1.530028	1.699098	1.724439
1.0	1.689498	1.683289	1.779976	1.448805	1.769830	1.776542

Example 4:

Consider the Fractional Quadratic Riccati Differential Equation

$$D^{*}y(x) = x^2 + y^2(x), \quad 0 < \alpha \leq 1$$

With the initial condition $y(0) = 1$

The exact solution for the case $\alpha = 1$ is

$$\begin{aligned} y(x) = 1 &+ \frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{2x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{4x^{2\alpha+2}}{\Gamma 2\alpha+3} \\ &+ \frac{\Gamma(2\alpha+1)x^{3\alpha}}{\Gamma^2(\alpha+3)\Gamma(3\alpha+1)} + \frac{8x^{3\alpha+2}}{\Gamma(3\alpha+3)} + \frac{4\Gamma(2\alpha+3)x^{3\alpha+2}}{\Gamma(3\alpha+3)} + \\ &+ \frac{4\Gamma(2\alpha+5)x^{3\alpha+4}}{\Gamma^2(\alpha+3)\Gamma(3\alpha+3)} + \frac{8\Gamma(3\alpha+3)x^{4\alpha+2}}{\Gamma(\alpha+1)\Gamma(2\alpha+3)\Gamma(4\alpha+3)} + \frac{16\Gamma(3\alpha+5)x^{4\alpha+4}}{\Gamma(\alpha+3)\Gamma(2\alpha+3)\Gamma(4\alpha+5)} + \frac{8\Gamma(2\alpha+5)x^{4\alpha+2}}{\Gamma^2(\alpha+3)\Gamma(4\alpha+5)} \\ &+ \frac{16\Gamma(3\alpha+3)x^{4\alpha+4}}{\Gamma(\alpha+1)\Gamma(2\alpha+3)\Gamma(4\alpha+5)} + \frac{8\Gamma(2\alpha+5)\Gamma(4\alpha+5)x^{5\alpha+4}}{\Gamma(\alpha+1)\Gamma^2(\alpha+3)\Gamma(3\alpha+5)\Gamma(5\Gamma+5)} \\ &+ \frac{16\Gamma(2\alpha+5)\Gamma(4\alpha+7)x^{5\alpha+6}}{\Gamma^2(\alpha+3)\Gamma(3\alpha+5)\Gamma(5\alpha+7)} + \frac{32\Gamma(2\alpha+5)\Gamma(5\alpha+7)x^{6\alpha+6}}{\Gamma^2\Gamma(2\alpha+3)\Gamma(3\alpha+5)\Gamma(6\alpha+7)} + \frac{16\Gamma^2(2\alpha+5)\Gamma(6\alpha+9)x^{7\alpha+8}}{\Gamma^4(\alpha+3)\Gamma^2(3\alpha+5)\Gamma(7\alpha+9)} \end{aligned}$$

Table 4: Comparison of solutions for Example 4 for alpha=1

x	Exact	Approx. Solution by PIM
1	0.110295	0.110265
1	0.241976	0.241564
3	0.395104	0.393354
4	0.567812	0.563330
5	0.756014	0.747445
6	0.953566	0.939972
7	1.152946	1.133622
8	1.346363	1.319708
9	1.526911	1.488325
10	1.689498	1.628571

Conclusions

In this paper, the Picards iteration Method has been successfully applied to find approximate solutions of fractional quadratic Riccati Differential Equations.

The methods is effective for Riccati differential equations, and hold very great promise for its applicability to other nonlinear fractional differential equations.

The four examples used indicate the efficiency and accuracy of the method for fractional quadratic Riccati differential equations. The results obtained are in very acceptable agreement with those obtained by other known methods.

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