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Mathematical Modelling of Two Layered Blood Flow through a Tapered Artery with Overlapping Stenotic Condition

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Abstract

This study deals with the effects of overlapping stenosis on flow rate, resistance to flow and wall shear stress in a tapered artery for different parameters. A two layered mathematical model has been included here by considering the peripheral layer as Newtonian fluid and the central core layer as Bingham plastic type non-Newtonian fluid. The numerical results are presented in graphical form and discussed. It was found that volumetric flow flux reduces as overlapping stenosis height increases and resistance to flow rises with increase in stenosis height and values of artery shape φ.

Keywords: Overlapping stenosis, Bingham-Plastic Fluid, Resistance to Flow, Wall Shear Stress, Yield stress, Flux

1. Introduction

Blood is the familiar red fluid in the body that contains white and red blood cells, platelets, proteins, and other elements. It is transported throughout the body by the circulatory system. Blood functions in two directions: arterial and venous. Blood in the artery transports oxygen and nutrients to tissues while venous blood carries carbon dioxide and metabolic by-products to the lungs and kidneys, respectively, for removal from the body. The blood vessels could be affected by conditions that subsequently affect the blood flow. Stenosis is a condition where the artery become narrowed and hardened due to a buildup of plague around the artery wall. It is also known as arteriosclerotic vascular disease. The disease disrupts the flow of blood around the body thereby posing the risk of serious complications such as stroke, heart attacks and peripheral vascular disease. Blood flow disruption has been a global issue, since it is the major causes of health problems and death, so it is very important to study the blood flow in a stenotic condition.

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Several investigators such as Majumdar *et al*. (1995), Sanyal and Maiti (1997) and Sanyal *et al*. (2009) have studied mathematical model of two layered blood flow, taking blood as non-Newtonian. Many researchers Tu and Deville (1986), Lee and Fung (1970), Jung *et al*. (1995), Krumholz *et al*. (1995), Peterson *et al*. (2008), Rathod and Tanveer (2009), Shukla *et al*. (1980) and Maiti (2014) have examined experimental outcomes in studying different forms of blood flow through atherosclerotic condition. Padma *et al*. (2009) has studied a two-layered model of blood flow through composite stenosed artery. They observed that resistance to flow and wall shear stress increases as the peripheral layer viscosity increases and that the existence of peripheral layer is useful in representation of diseased arterial system.

Many mathematician have investigated the behaviour of blood flow by considering both the layers either Newtonian or non-Newtonian fluid. Pontrelli (2001), Sankar and Hemalatha (2000) and Shalman (2002) have proposed different kinds of mathematical models by considering the peripheral layer as Newtonian fluid and the core layer as non-Newtonian fluid. They observed that stenosis is important in micro circulation, where peripheral layer thickness and viscosity effects dominate the blood flow properties. Srivastava and Rastogi (2010) have shown the effects of overlapping stenosis on blood flow by considering two-layered fluid model. Singh *et al.* (2010) have studied the effect of magnetic field on blood flow by presenting two dimensional of blood flow with variable viscosity. Shit (2014) studied Mathematical modelling of blood flow through a tapered overlapping stenosed artery with variable viscosity. They observed that the shape of artery have important impact on the velocity profile, pressure gradient and wall shear stress and the effect of primary stenosis on the secondary one has been observed.

Furthermore, Chakravarty and Mandal (1994) studied Mathematical modelling of blood flow through an overlapping Arterial stenosis and observed that the flow rate becomes inversely proportional to the resistive impedance arising out of the stenotic flow in vivo, the severity of the overlapping stenosis affects the resistive impedance significantly and that the wall shear stress becomes inversely proportional to the amplitude of the pressure gradient. In addition Maiti (2016) have studied mathematical modelling on blood flow under atherosclerotic condition.

In all the discussed studies, mathematical modelling of two layered blood flow through a tapered artery with overlapping stenotic condition has not been addressed studied. Therefore, the focus of this work is to investigate the effects of stenotic condition on resistance of blood flow and wall shear stress in presence of tapered and overlapping artery when the blood flow is considering peripheral layer as Newtonian fluid and the core layer as Bingham plastic type non-Newtonian fluid in presence of arterial stenosis. The problems were solved analytically.

2. Formulation of the problem

The laminar, incompressible and non-Newtonian two-layered flow of blood, consisting of a central core layer of red cell suspensions in plasma of radius R_1 and a peripheral plasma layer of thickness ($R-R_1$), through axisymmetric one-dimensional tapered and overlapping stenosed artery is being investigated. The cylindrical polar coordinate (r,θ,z), is used for any point in the fluid, where z is measured along the axis of the artery and that of r and θ along the radial and circumferential directions respectively. The mathematical expression that corresponds to the geometrical representation (Figure 1) of this problem is given by

$$\frac{R(z)}{R_0}=\begin{cases}\left(\frac{mz}{R_0}+1\right)-\frac{\delta cos\emptyset}{R_0L_0}(z-d)\left\{11-\frac{94}{3L_0}+\frac{32}{L_0^2(z-d)^2}-\frac{32}{3L_0^3}(z-d)^3\right\}, d\leq z\leq d+\frac{3L_0}{2}\\ \left(\frac{mz}{R_0}+1\right), \quad otherwise,\end{cases} \tag{1}$$

$$\frac{R_1(z)}{R_0}=\begin{cases}\left(\frac{mz}{R_0}+1\right)\beta-\frac{\delta cos\emptyset}{R_0L_0}(z-d)\left\{11-\frac{94}{3L_0}+\frac{32}{L_0^2(z-d)^2}-\frac{32}{3L_0^3}(z-d)^3\right\}, d\leq z\leq d+\frac{3L_0}{2}\\ \left(\frac{mz}{R_0}+1\right)\beta, \quad otherwise,\end{cases} \tag{2}$$

where R(z) denotes the radius of the tapered arterial segment in the constricted region, R_0 is the constant radius of the normal artery in the non-stenotic region, φ is the angle of tapering, $\frac{3L_0}{2}$ is the length of overlapping stenosis, d is the location of the stenosis, δcosφ is taken to be the critical height of the overlapping stenosis, β is the ratio of the central core radius to the tube radius outside the stenotic region and m(tanφ) represents the slope of the tapered vessel. Exploring the possibility of the different shapes of the artery, it can be categorized as converging tapering (φ<0), non-tapered artery (φ = 0) and the diverging tapering (φ>0).

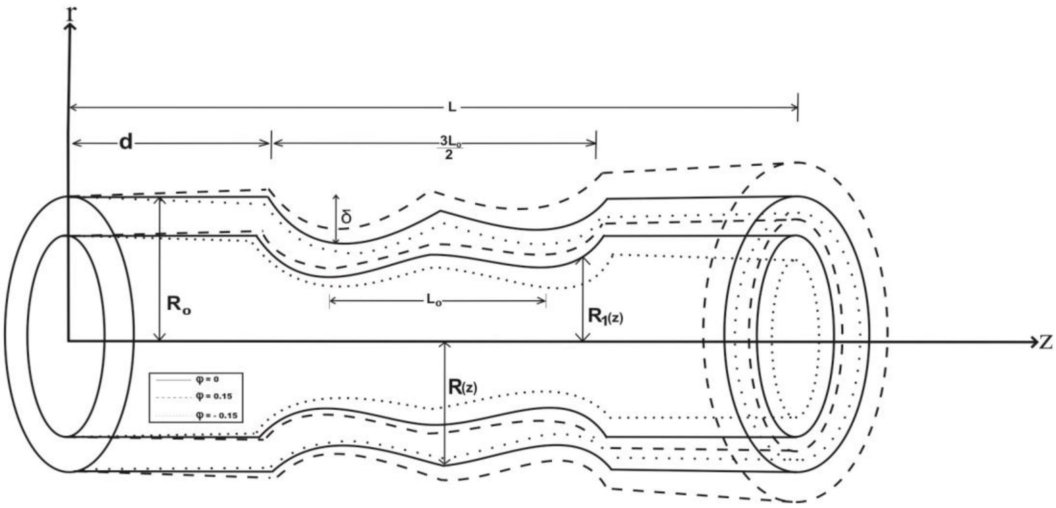

Figure 1: Schematic diagram of two layered overlapping stenosed artery.

3. Mathematical Formulation

The equation of motion for laminar, incompressible steady fully developed flow is given by

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial(r\tau)}{\partial r}\,, \tag{3}$$

where (r, z) are the axial co-ordinates and $\frac{dp}{dz}$ is the pressure gradient. The relationship between shear stress and shear rate for Bingham plastic fluid was by Maiti (2016) as follows:

$$\tau = \mu\left(-\frac{\partial u}{\partial r}\right) + \tau_0, \tag{4}$$

where τ_0 is the yield stress.

The Boundary Conditions: The following boundary conditions are considered:

1. τ is finite at r=0 (regularity condition),

2. u = 0 at r = R(z) (no-slip condition).

Integrating equation (3) with respect to r and using the boundary condition (1), we obtain

$$\tau = \frac{r}{2}\frac{dp}{dz}. \tag{5}$$

The volumetric flow flux, Q_1 at the core layer is given by:

$$Q_1 = \int_0^{R_1(z)} \pi r^2 du, \tag{6}$$

$$= \int_0^{R_1(z)} \pi r^2 \left(-\frac{\partial u}{\partial r}\right) dr$$

$$= \pi \int_0^{R_1(z)} \frac{r^2}{\mu_c}\left(\frac{dp}{dz}\frac{r}{2} - \tau_0\right) dr = \frac{\pi}{\mu_c}\left[\frac{dp}{dz}\frac{R_1^4}{8} - \frac{\tau_0 R_1^3}{3}\right] dr, \tag{7}$$

using the boundary condition for no-slip. The relationship between shear stress and shear rate of the peripheral layer as a Newtonian fluid is

$$\tau = \mu_p \left(\frac{\partial u}{\partial r}\right), \tag{8}$$

where μ_p is the viscosity of the blood in perpheral layer.

The volumetric flow flux Q_2 at the plasma layer is given by

$$Q_2 = \int_{R_1(z)}^{R(z)} \pi r^2 \frac{dp}{dz}\frac{r}{2\mu_p} dr, \tag{9}$$

and using Equations (5) and (8) in Equation (9), one obtains

$$Q_2 = \frac{\pi}{8\mu_p}\frac{dp}{dz}[R^4 - R_1^4]. \tag{10}$$

Therefore, the total flux is given as $Q = Q_1 + Q_2$, where

$$Q = \frac{\pi}{\mu_p}\left[\frac{dp}{dz}(R^4 - (1-\mu)R_1^4) - \frac{8\tau_0\mu}{3}R_1^3\right], \tag{11}$$

$\mu = \frac{\mu_p}{\mu_c}$ and μ_c is the viscosity of the blood in central core layer. After simplifying the Equation (11), one obtains

$$Q = \frac{\pi}{\mu_p}\left\{\frac{dp}{dz}\left[R_0^4\left(\frac{R(z)}{R_0}\right)^4 - (1-\mu)R_0^4\left(\frac{R_1(z)}{R_0}\right)^4\right] - \frac{8\tau_0\mu}{3}R_0^3\left(\frac{R_1(z)}{R_0}\right)^4\right\} \tag{12}$$

Therefore, the pressure gradient from Equation (12) is

$$\frac{dp}{dz} = \frac{\frac{8\mu_p Q}{\pi} + \frac{8\tau_0\mu}{3}R_0^3\left(\frac{R_1(z)}{R_0}\right)^3}{R_0^4\left(\frac{R(z)}{R_0}\right)^4 - (1-\mu)R_0^4\left(\frac{R_1(z)}{R_0}\right)^4}. \tag{13}$$

Integrating Equation (13) from p = p_1 at z = 0 and p = p_2 at z = L, we have

$$p_2 - p_{1=} \int_0^L \left\{ \frac{\frac{8\mu_p Q}{\pi} + \frac{8\tau_0 \mu}{3} R_0^3 \left(\frac{R_1(z)}{R_0}\right)^3}{R_0^4 \left(\frac{R(z)}{R_0}\right)^4 - (1-\mu) R_0^4 \left(\frac{R_1(z)}{R_0}\right)^4} \right\} dz \,.$$

The resistance of flow λ is given by Maiti (2016) as $\lambda = \frac{p_2 - p_1}{Q}$

Hence

$$\lambda = \int_0^L \left\{ \frac{\frac{8\mu_p}{\pi} + \frac{8\tau_0 \mu}{3Q} R_0^3 \left(\frac{R_1(z)}{R_0}\right)^3}{R_0^4 \left(\frac{R(z)}{R_0}\right)^4 - (1-\mu) R_0^4 \left(\frac{R_1(z)}{R_0}\right)^4} \right\} dz \tag{14}$$

and

$$\lambda = \frac{8\mu_p}{\pi R_0^4} \left\{ \int_0^d \frac{dz}{\rho} + \int_d^{d+\frac{3L_0}{2}} \frac{dz}{\rho} + \int_{d+\frac{3L_0}{2}}^L \frac{dz}{\rho} \right\} + \frac{8\tau_0 \mu}{3R_0 Q} \left\{ \int_0^d \frac{\omega^4}{\rho} dz + \int_d^{d+\frac{3L_0}{2}} \frac{\omega^4}{\rho} dz + \int_{d+\frac{3L_0}{2}}^L \frac{\omega^4}{\rho} dz \right\}, \tag{15}$$

where $\omega = \left(\frac{R_1(z)}{R_0}\right)$ and $\rho = \left(\frac{R(z)}{R_0}\right)^4 - (1-\mu)\omega$.

The stenosis is present in the region $d \le z \le d + \frac{3L_0}{2}$. If there is no stenosis $\frac{\mathrm{R(z)}}{\mathrm{R_0}} = (\frac{mz}{R_0} + 1)$ and $\frac{R_1(z)}{R_0} = \left[(\frac{mz}{R_0} + 1)\beta \right]$ from Equations (1) and (2) respectively. Therefore

$$\lambda = \frac{8\mu_p}{\pi R_0^4} \left\{ \int_0^d \frac{dz}{\vartheta} + \int_d^{d+\frac{3L_0}{2}} \frac{dz}{\rho} + \int_{d+\frac{3L_0}{2}}^L \frac{dz}{\vartheta} \right\} + \frac{8\tau_0 \mu}{3R_0 Q} \left\{ \int_0^d \frac{\varepsilon^4}{\vartheta} dz + \int_d^{d+\frac{3L_0}{2}} \frac{\omega^4}{\rho} dz + \int_{d+\frac{3L_0}{2}}^L \frac{\varepsilon^4}{\vartheta} dz \right\}, \tag{16}$$

where $\varepsilon = \left[\left(\frac{mz}{R_0} + 1\right)\beta \right]$ and $\vartheta = \left[\left(\frac{mz}{R_0} + 1\right)^4 - (1-\mu)\varepsilon^4 \right]$.

Thus, the non-dimensional resistance to flow may be expressed as

$$\bar{\lambda} = \frac{\lambda}{\lambda_0}. \tag{17}$$

The wall shear stress can be expressed as

$$\tau_s = \frac{\left[\frac{4\mu_p Q}{\pi R_0^3} + \frac{4\tau_0 \mu}{3} \left(\frac{R_1(z)}{R_0}\right)^3 \right] \left(\frac{R(z)}{R_0}\right)}{\left(\frac{R(z)}{R_0}\right)^4 - (1-\mu)\left(\frac{R_1(z)}{R_0}\right)^4} \,. \tag{18}$$

The wall shear stress for Newtonian fluid in the absence of stenosis is given by

$$\tau_N = \frac{4\mu_p Q}{\pi R_0^3\left(\frac{mz}{R_0}+1\right)^3}. \tag{19}$$

Thus, the wall shear stress in dimensionless form can be written as

$$\bar{\tau} = \frac{\tau_s}{\tau_N}. \tag{20}$$

The wall shear stress at the throat of the stenosis is given by

$$\tau_{wm} = \frac{\left[\frac{4\mu\mu_C Q}{\pi R_0^3}+\frac{4\tau_0\mu}{3}\left(\left(\frac{mz}{R_0}+1\right)\beta-\frac{\delta}{R_0}\right)^3\right]\left[\left(\frac{mz}{R_0}+1\right)-\frac{\delta}{R_0}\right]}{\left[\left(\frac{mz}{R_0}+1\right)-\frac{\delta}{R_0}\right]^4-(1-\mu)\left[\left(\frac{mz}{R_0}+1\right)\beta-\frac{\delta}{R_0}\right]^4}. \tag{21}$$

Thus, the non-dimensional wall shear stress at the throat of the stenosis can be written as

$$\bar{\tau}_m = \frac{\tau_{wm}}{\tau_N}. \tag{22}$$

4. Results and Discussions

To interpret the present analysis, the results are shown graphically with the help of Maple-18. In this section, calculation of the effects of stenotic condition on two-layer blood flow in a tapered artery had been shown. From Maiti (2016), the values for the parameters are taken. The values of the parameters are considered with its range as τ_0 = 0.00 − 0.04, $\frac{dp}{dz} = 0.9$, λ = 2, Q = 2, L = 2.5, L_0 = 1.0–1.4, d = 0.5, $R_0 = 1, \frac{\delta}{R_0} = 0.0 - 0.5$, β = 0.90 − 0.96, π = 3.142, $\lambda_0 = 30$, and μ = 0.1 − 0.2, φ = −0.15, 0.00 and 0.15.

The volumetric flow flux for both central core layer and peripheral plasma layer is given by Equation (12) analytically. Figure (2)-(4) show the variation of flux Q for different value of μ and β with artery shape φ. Figure (2) and (4). Revealed that the flux Q increases with the increase of μ but decreases with the increase of β for all stenosis heights. Figure (3) shows the variation of flux Q for different values of φ. It depicts that resistance to flow rises with an increase in values of artery shape φ. Hence, diverging tapering produces upper bound resistance and converging tapering results in lower bound resistance while non-tapered is between the two. Figure (5) - (9) show the variation of non-dimensional resistance to flow ($\bar{\lambda}$) for various parameters with respect to stenosis height. Figure (5) shows the variation of non-dimensional resistance to flow for different values of μ at an angle φ = 0.15. It is observed that resistance to flow decreases with the increases in the value of μ for all stenosis heights. Figure (6) shows the variation of non-dimensional resistance to flow for different values of φ. It behaves similar to that of Figure (3). Figure (7) shows the variation of non-dimensional resistance to flow for different values of β at an angle φ = 0.15. Resistance to flow increases with the increase in the value of β for all stenosis heights. Figure (8) and (9) shows the variation of non-dimensional resistance to flow for different values of τ_0 and L_0 at an angle φ = 0.15. As stenosis length (L_0) increases, resistance to flow decreases with for all value of stenosis heights and yield stress (τ_0) produces no significant changes.

Figure (10)-(12) represent the variation of non-dimensional wall shear stress ($\overline{\tau}$) with the variation of stenosis size, β, φ and μ. It is clear from the figures that wall shear stress ($\overline{\tau}$) increases as β is increasing and it reduces as μ increases. Figure (11) shows variation of non-dimensional wall shear stress ($\overline{\tau}$) for different values of φ. It depicts that non-dimensional wall shear stress ($\overline{\tau}$) rises with an increase in values of angle φ for stenosis height less or equal to 0.270 and reduces as value of stenosis height developed above 0.270.

Figure (13)-(15) represent the fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) for the variation of β, φ and μ with respect to stenosis height. As β increases, fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) increases but it decreases as μ increases while fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) decreases as stenosis height increases. Figure (14) shows the fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) for different values of φ. It depicts that fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) decreases with an increase in values of angle φ. Table 1 shows the resistance to flow with variation of yield stress and stenosis height $\frac{\delta}{R_0}$ when φ = 0.15. It depicts that resistance to flow slightly increases with an increase in values of yield stress τ_0 and rise as stenosis height developed. Table 2 represents the variation of non-dimensional wall shear stress $\bar{\tau}$ and stenosis height $\frac{\delta}{R_0}$ for different values of φ. Non-dimensional wall shear stress $\bar{\tau}$ increases with an increase in values of angle φ when stenosis height is less or equal to 0.2 and its decreases as the value of stenosis height greater than or equal to 0.3. Also, wall shear stress increases as stenosis height increases.

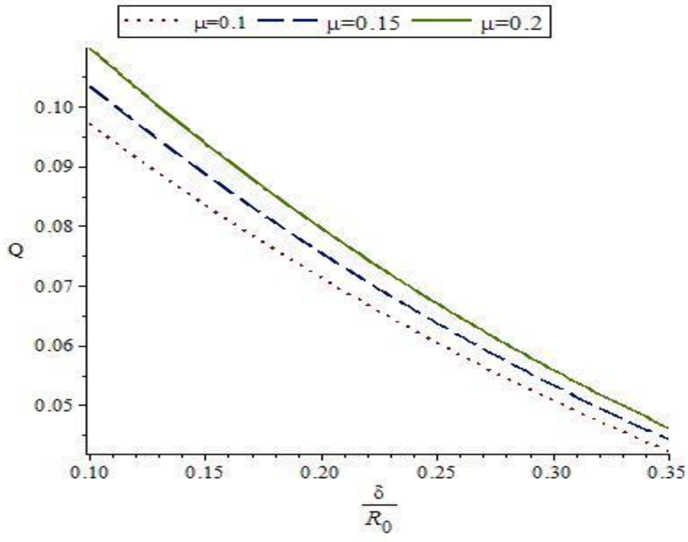

Figure 2: Variation of flux Q for different values of μ at an angle $\varphi = 0.15$

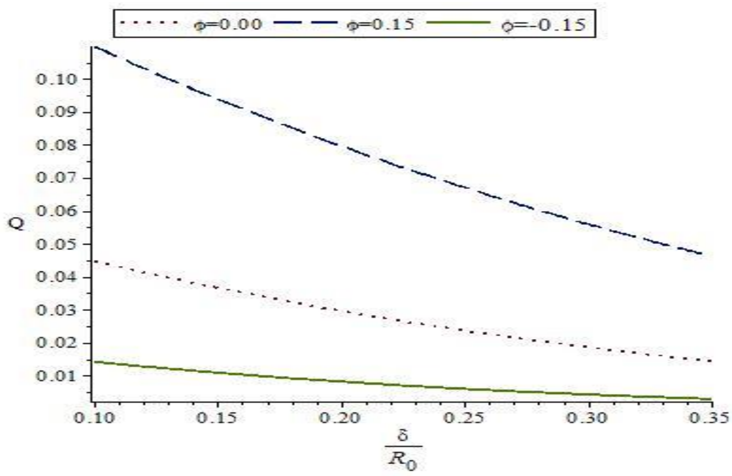

Figure 3: Variation of flux Q for different values of angle φ

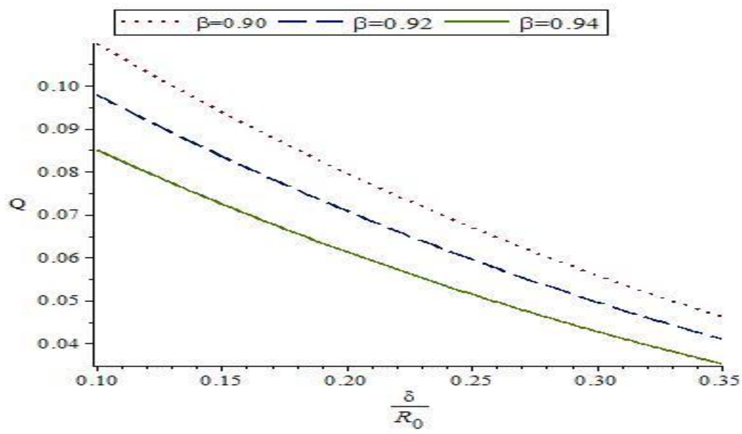

Figure 4: Variation of flux Q for different values of β at an angle $\varphi = 0.15$

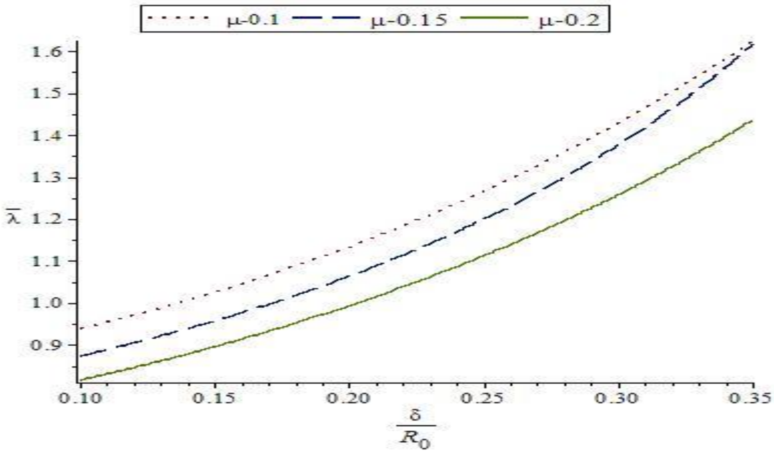

Figure 5: Variation of non-dimensional resistance to flow for different values of μ at an angle φ =0.15

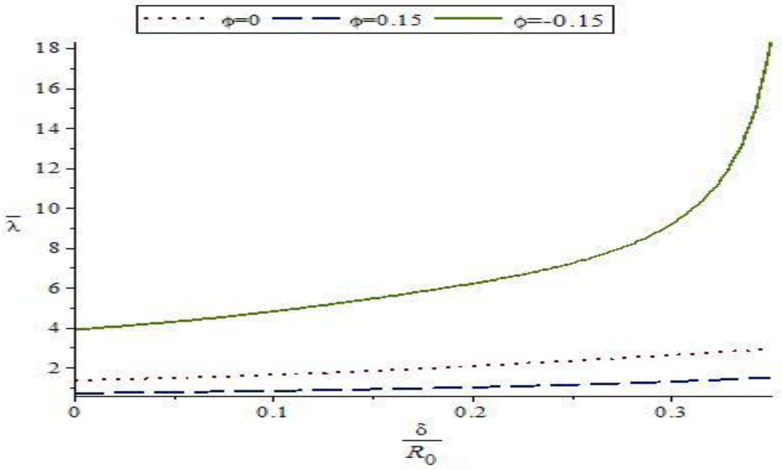

Fig. 6: Variation of non-dimensional resistance to flow for different values of angle φ

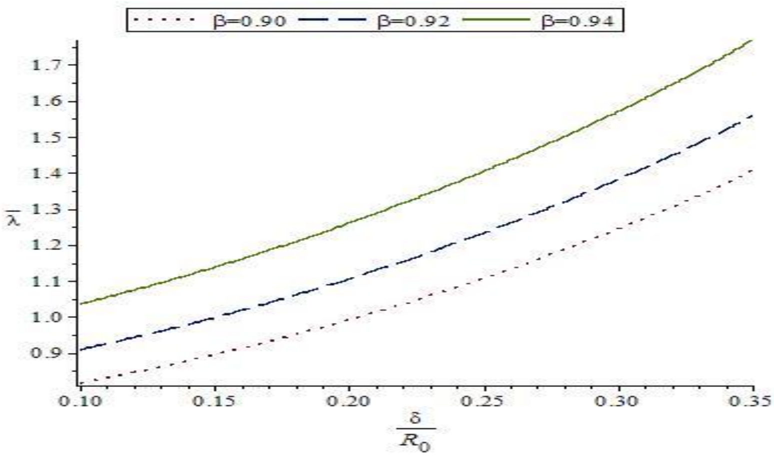

Figure 7: Variation of non-dimensional resistance to flow for different values of β at an angle φ =0.15

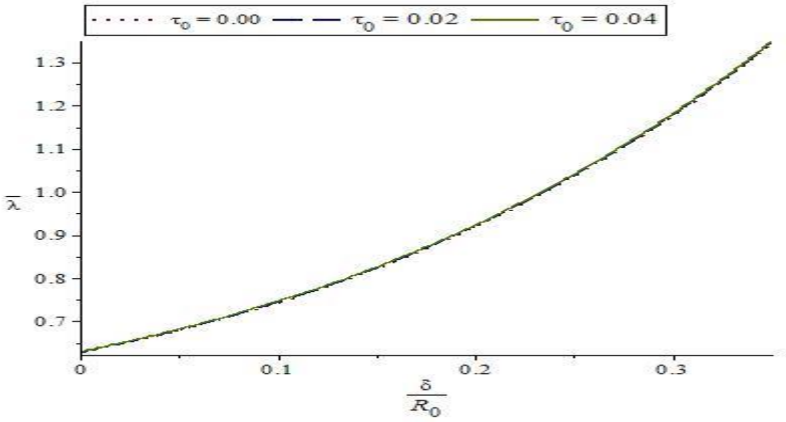

Figure 8: Variation of non-dimensional resistance to flow for different values of τ_0 at an angle φ=0.15.

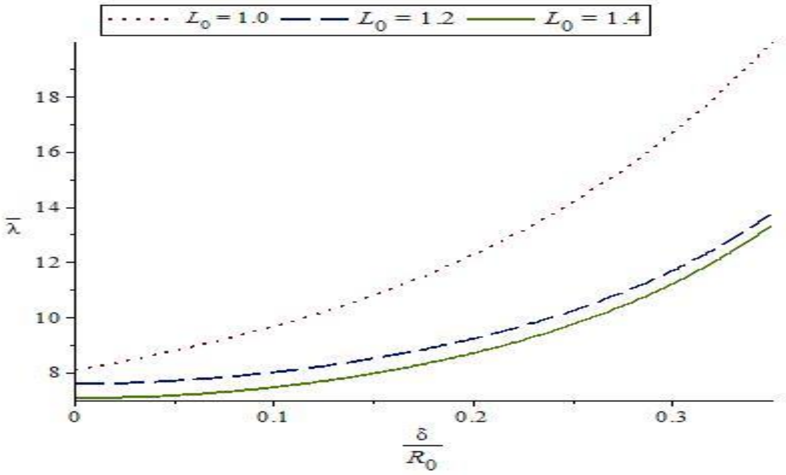

Figure 9: Variation of non-dimensional resistance to flow for different values of L_0 at an angle φ=0.15.

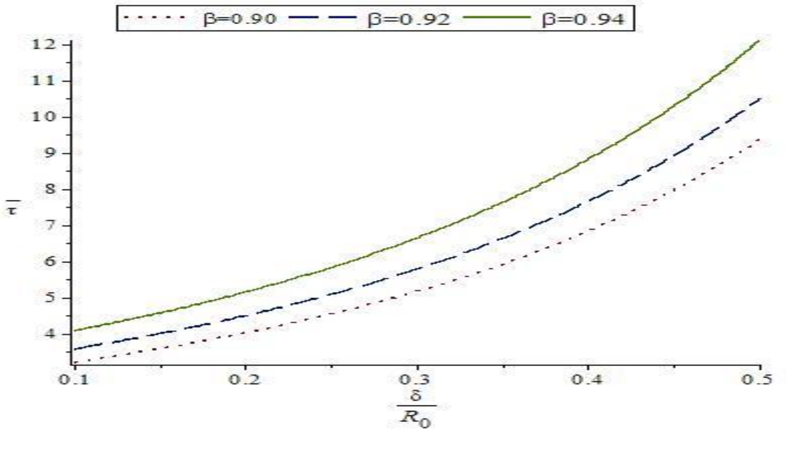

Figure 10: Variation of non-dimensional wall shear stress for different values of β at an angle φ=0.15.

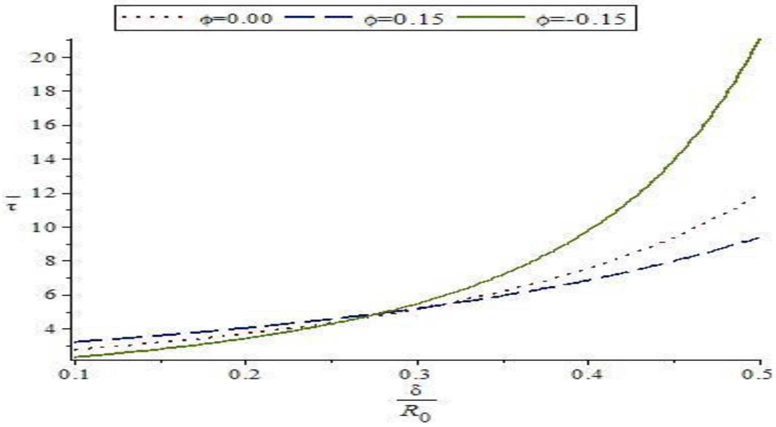

Figure 11: Variation of non-dimensional wall shear stress for different values of angle φ=0.15.

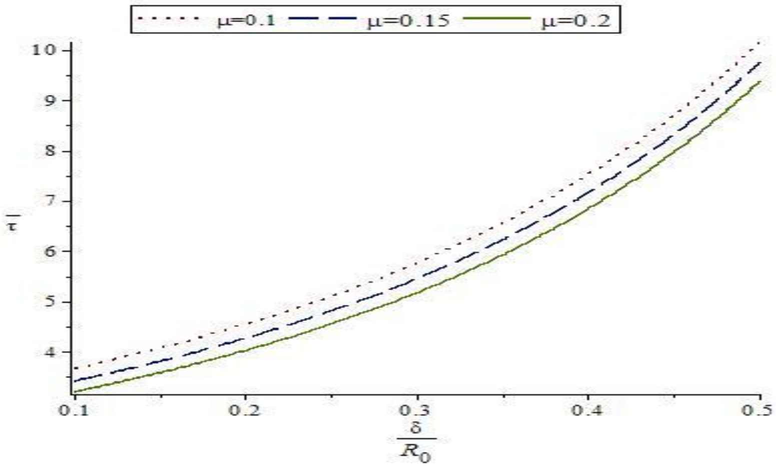

Figure 12: Variation of non-dimensional wall shear stress for different values of μ at an angle φ=0.15.

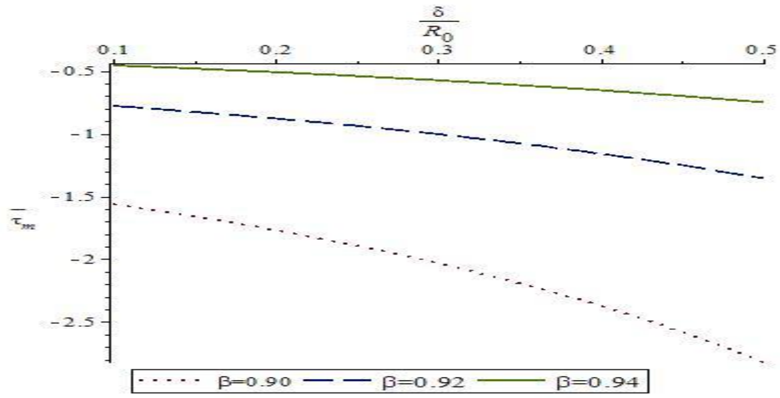

Figure 13: Fluctuation of non-dimensional wall shear stress at the throat of the stenosis for different values of β at an angle φ=0.15.

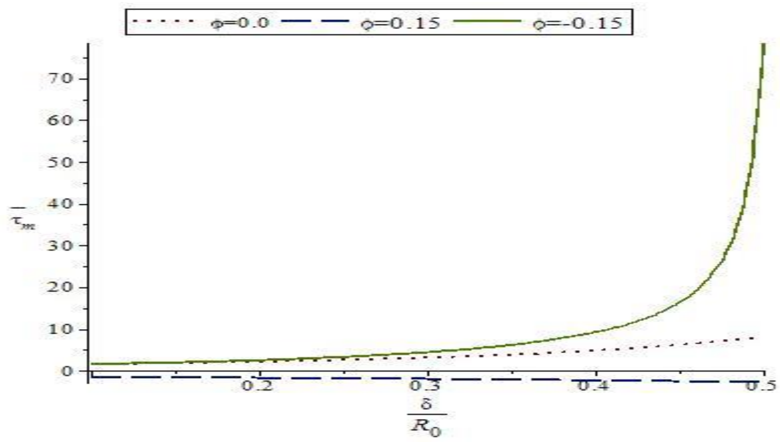

Figure 14: Fluctuation of non-dimensional wall shear stress at the throat of the stenosis for different values of angle φ

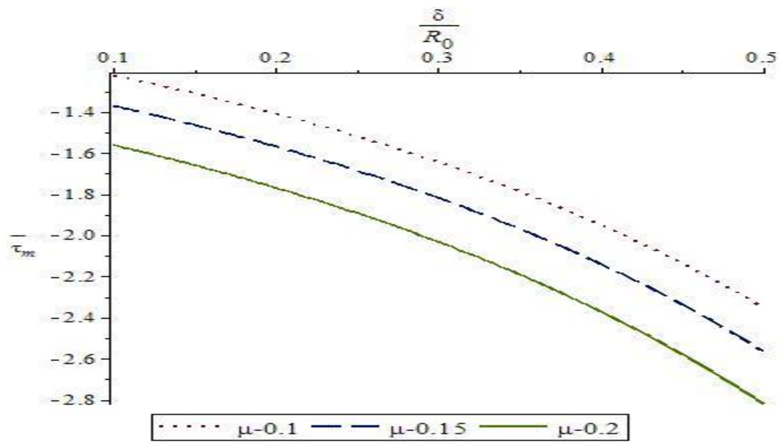

Figure 15: Fluctuation of non-dimensional wall shear stress at the throat of the stenosis for different values of μ at an angle φ=0.15.

Table 1: Resistance to flow with variation of yield stress τ_0 and stenosis height $\frac{\delta}{R_0}$ when φ =0.15.

Yield stress	$\frac{\delta}{R_0} = 0.1$	$\frac{\delta}{R_0} = 0.2$	$\frac{\delta}{R_0} = 0.3$	$\frac{\delta}{R_0} = 0.4$	$\frac{\delta}{R_0} = 0.5$
$\tau_0 = 0.00$	0.6962166800	0.8124096313	0.9876786743	1.1653181560	1.6059907580
$\tau_0 = 0.02$	0.6977940170	0.8142303540	0.9898665273	1.1678781040	1.6094737680
$\tau_0 = 0.04$	0.6993713543	0.8160510767	0.9920543800	1.1704380530	1.6129567790

Table 2: Non-dimensional wall shear stress $\bar{\tau}$ with variation of artery shape (φ) and stenosis height ($\frac{\delta}{R_0}$).

Artery shapes	$\frac{\delta}{R_0}=0.1$	$\frac{\delta}{R_0}=0.2$	$\frac{\delta}{R_0}=0.3$	$\frac{\delta}{R_0}=0.4$	$\frac{\delta}{R_0}=0.5$
$\varphi = 0.15$	3.200581896	4.021125696	5.169445499	6.836758492	9.372656766
$\varphi = 0.00$	2.746271376	3.683107546	5.135855646	7.541635679	11.90228602
$\varphi = -0.15$	2.294536612	3.407090951	5.450629358	9.754816018	21.08404299

5. Conclusion

The effects of stenotic condition on resistance to two layer blood flow through a tapered with overlapping stenosed artery and wall shear stress was determined in the present study. It was found that volumetric flow flux decreases as stenosis height increases. The resistance to flow increases with increasing of stenosis height while the influence of yield stress (τ_0) has slight increase. Furthermore, fluctuation of non-dimensional wall shear stress at the throat of the stenosis ($\bar{\tau}_m$) decreases as stenosis height increases. It is also observed that the resistances to flow decreases with increasing viscosity but increases with the increase in stenosis size. The present study is able to predict the main property of the physiological flows which have played a significant role in biomedical investigations.

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