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Magnetohydrodynamic Boundary Layer Flow of a Nanofluid past a Stretching/Shrinking Sheet in the Presence of Viscous Dissipation with Heat Generation/Absorption and Convective Boundary Condition

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Abstract

The problem of laminar fluid flow which results from the stretching/shrinking of a flat surface in a nanofluid has been obtained using the Adomian Decomposition Method. The model used for the nanofluid was presented in its rectangular form. The model is considered in the presence of viscous dissipation, heat generation/absorption with convective boundary condition and the effect of Brownian motion and thermophoresis. A similarity solution is presented which depends on magnetic parameter *(M)*, Eckert number *(E_c)*, heat generation and absorption, Biot number *(B_i)*, Lewis number *(L_e)*, Brownian motion *(N_b)* number and thermophoresis number *(N_t)*. In the results presented graphically, it is observed that the physical quantities such as the Magnetic parameter has the same effects on both stretching and shrinking sheets, ie as it increases the velocity profile reduces on both sheets.

Keywords: Adomian Decomposition Method, Nanofluid, Nanoparticles, Thermophoresis, Darcy number, Eckert number.

1. Introduction

The fluid flow over a stretching/shrinking sheet is important in applications such as extrusion, wire drawing, metal spinning, and hot rolling. It is crucial to understand the heat and flow characteristics of the process so that the end product meets the desired qualities. A wide variety of problems dealing with heat and fluid flow over a stretching/shrinking sheet have been studied with both Newtonian and non-Newtonian fluids and with the inclusion of magnetic fields, different thermal boundary conditions, and power law variation of the stretching/shrinking velocity. Both similarity as well as direct numerical solutions of the convective transport equations has been reported. The term "nanofluid" was coined by Choi (1995). The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda *et al.* (1993). This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems. A benchmark study on the thermal conductivity of nanofluids was made by Buongiorno *et al.* (2009). Venerus *et al.* (2010) have

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studied the viscosity measurements on colloidal dispersions (nanofluids) for heat transfer applications. Gharagozloo *et al*. (2008) have examined the diffusion, aggregation, and the thermal conductivity of nanofluids and Philip *et al.* (2008) have presented the nanofluid with tenable thermal properties.

Further, Kuznetsov and Nield (2010) have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. In this pioneering study they have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Nield and Kuznetsov (2009) have analysed the effect of nanoparticles on natural convection boundary-layer flow in a porous medium past a vertical plate and employed the Darcy model for the momentum equation. Bach et al. (2010) have studied theoretically the problem of steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream and it is found that dual solutions exist when the plate and the free stream flow move in the opposite directions. The problem of laminar fluid flow resulting from the stretching of a flat surface in a nanofluid has been investigated numerically by Khan and Pop (2010).

Aiyesimi *et al.* (2015) carried out an analytical investigation of a convective boundary-layer flow of a nanofluid past a stretching sheet with radiation using the Adomian Decomposition Method. Recently, Aiyesimi *et al.* (2015) carried out an analytical investigation of a nanofluid model in a porous medium with permeability and incorporates the magnetic effect, thermal radiation effect and the effect of Brownian motion and thermophoresis. A similarity solution was also presented which depends on Darcy number, magnetic effect, inertia coefficient, Prandtl number, Radiation, Lewis number, Brownian motion number and thermophoresis number and it was observed that the Darcy number enhances the velocity, temperature and concentration profile of the fluid.

We found it to be appropriate to consider the work of Khan and pop (2010) past a stretching/shrinking sheet in the presence of viscous dissipation with heat generation/absorption and convective boundary condition using Adomian Decomposition Method to obtain the analytical solution of the model. Aiyesimi *et al.* (2013) have previously used the Adomian Decomposition to obtain the analytical solution of hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non Darcian medium with variable permeability. A few examples are the research works of Aiyesimi *et al*. (2013), Jiya and Oyubu (2012).

This work is a new development in the literature in which an analytical solution of MHD boundary-layer flow of a nanofluid past a stretching/shrinking sheet in the presence of viscous dissipation with heat generation/absorption and convective boundary condition is proposed using the Adomian decomposition method.

2. Materials and Methods

We consider a steady, two dimensional boundary layer flow of a nanofluid over a continuously moving stretching/shrinking surface with the linear velocities $u(x,y)=ax$ and $u(x,y)=-ax$ for stretching and shrinking sheet, where a is constant and x is the coordinate measured along the stretching/shrinking sheet surface. A uniform magnetic field B_0 is applied along the $y-axis$. We assumed that the stretching surface, the nanoparticle fraction C have constants value C_w. Following the formulation of Khan and pop (2010) with magnetic field effect, viscous dissipation, and heat generation/absorption over a stretching/shrinking sheet with convective boundary condition, the problem is governed by the following equations:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B_0{}^2}{\rho}v \tag{3}$$

Energy equation:-

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+\frac{\upsilon}{c_p}(\frac{\partial u}{\partial y})^2+\frac{Q(x)}{\rho c_p}(T-T_\infty)$$
$$+\tau\left(D_B(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y})+\frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right)\right) \tag{4}$$

Nanofraction equation:-

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

Subject to the boundary conditions:

1. For stretching sheet:-

$$y=0: u=ax, \quad v=0, \quad -k\frac{\partial T}{\partial y}=h(T_f-T_\infty), \quad C=C_W,$$

$$y\to\infty: u\to 0, \quad T\to T_\infty, \quad C\to C_\infty, \tag{6}$$

2. For shrinking sheet:-

$$y=0: u=-ax, \quad v=0, \quad -k\frac{\partial T}{\partial y}=h(T_f-T_\infty), \quad C=C_W,$$

$$y\to\infty: u\to 0, \quad T\to T_\infty, \quad C\to C_\infty, \tag{7}$$

where u and v are the velocity components along the x and y axes respectively, p is the fluid pressure, k is the thermal conductivity, h is the convective heat transfer coefficient, B_0 is an external magnetic field, ρ_f is the density of the base fluid, σ is the electrical conductivity, α is the thermal diffusivity, υ is the kinematic viscousity, T_f is the convective fluid temperature below the moving sheet C_p is the specific heat capacity at constant pressure, D_B is the Brownian diffusion coefficient, D_T the thermopheric diffusion coefficient and $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the fluid with ρ being the density, ρ_P is the density of the particles, $Q(x)$ heat generation or absorption coefficient. Note that the temperature at the sheet surface (T_W) in this case, is not constant.

Defining the dimensional stream function $(\psi(x,y))$ in the usual way such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ and using the following dimensionless variables:-

$$\eta=\left(\frac{a}{\upsilon}\right)^{1/2}y, \quad \psi=(a\upsilon)^{1/2}xf(\eta), \; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \text{ and } \chi(\eta)=\frac{C-C_\infty}{C-C_\infty} \tag{8}$$

where η, $f(\eta)$, $\theta(\eta)$, $\chi(\eta)$ are the dimensionless fluid distance, velocity profile, temperature profile, and nanoparticle concentration with

$$\left.\begin{aligned}
&T = T_\infty + \left(T_f - T_\infty\right)\theta(\eta) \\
&\frac{\partial T}{\partial y} = \left(\frac{a}{\upsilon}\right)^{1/2}\left(T_f - T_\infty\right)\theta'(\eta) \\
&\frac{\partial T}{\partial y} = -\frac{h}{k_1}\left(T_f - T\right) \\
&\Rightarrow \left(\frac{a}{\upsilon}\right)^{1/2}\left(T_f - T_\infty\right)\theta'(\eta) = -\frac{h}{k_1}\left(T_f - T\right) \\
&\Rightarrow \left(\frac{a}{\upsilon}\right)^{1/2}\left(T_f - T_\infty\right)\theta'(\eta) = -\frac{h}{k_1}\left(T_f - T_\infty - \left(T_f - T_\infty\right)\theta(\eta)\right) \\
&\Rightarrow \theta'(\eta) = -\frac{h}{k_1}\left(\frac{\upsilon}{a}\right)^{1/2}\left(1 - \theta(\eta)\right)
\end{aligned}\right\} \tag{9}$$

An order of magnitude analysis of the y direction momentum equation (normal to the sheet) using the usual boundary layer approximations. Substituting the expressions in (8) and (9) into (1) to (7) and neglecting the pressure gradient the equations reduces to the following similarity solutions:-

$$\left.\begin{aligned}
&f''' + ff'' - f'^2 - Mf' = 0 \\
&\theta'' + \mathrm{P}_r f\theta' + \mathrm{P}_r N_b \chi'\theta' + \mathrm{P}_r N_t \theta'^2 + P_r E_C f''^2 + P_r \phi_0 \theta = 0 \\
&\chi'' + L_e f\chi' + \frac{N_t}{N_b}\theta' = 0 \\
&f(0) = 0,\ f'(0) = 1, \theta'(0) = -B_i\left(1 - \theta(0)\right), \chi(0) = 1, \\
&f'(\infty) = 0, \theta(\infty) = 0, \chi(\infty) = 0
\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}
&f''' + ff'' - f'^2 - Mf' = 0 \\
&\theta'' + \mathrm{P}_r f\theta' + \mathrm{P}_r N_b \chi'\theta' + \mathrm{P}_r N_t \theta'^2 + P_r E_C f''^2 + P_r \phi_0 \theta = 0 \\
&\chi'' + L_e f\chi' + \frac{N_t}{N_b}\theta' = 0 \\
&f(0) = 0,\ f'(0) = -1, \theta'(0) = -B_i\left(1 - \theta(0)\right), \chi(0) = 1, \\
&f'(\infty) = 0, \theta(\infty) = 0, \chi(\infty) = 0
\end{aligned}\right\} \tag{11}$$

in which :

$$M=\frac{\sigma B_0^2}{a\rho},\quad P_r=\frac{\upsilon}{\alpha},\quad L_e=\frac{\upsilon}{D_B},\quad N_b=\frac{(\rho c)_p D_B(C_W-C_\infty)}{(\rho c)_f\,\upsilon},\quad N_t=\frac{(\rho c)_p D_T(T_W-T_\infty)}{(\rho c)_f\,T_\infty\upsilon},$$

$$E_C=\frac{a^2x^2}{\left(T_W-T_\infty\right)C_P},\quad \phi_0=\frac{Q}{\rho C_P},\quad B_i=\frac{h}{k_1}\left(\frac{\upsilon}{a}\right)^{1/2} \tag{12}$$

are the Magnetic parameter, Prandtl number, Lewis number, Brownian motion parameter, thermophoresis parameter respectively, Eckert number, heat generation/absorption, and Biot number.

2.1 Adomian Decomposition Method

For the purpose (Adomian, 1989) of illustrating the method of Adomian decomposition we begin with the (deterministic) form $F(u)=g(t)$ where F is a nonlinear ordinary differential operator with linear and nonlinear items. We could represent the linear term Lu where L is a linear operator. We write the linear term $Lu+Ru$ where we choose L as the highest-ordered derivative. Now L^{-1} is simply n-fold integration for an n^{th} order. The remainder of the linear operator is R (in case where stochastic terms are present in linear operator, we can include a stochastic operator term Ru). The nonlinear term is represented by Nu .Thus, $Lu+Ru$ + Nu= g and we write

$L^{-1}\,L\,u$ = L^{-1} g$-L^{-1}\,Ru$ $-L^{-1}\,Nu$ for initial value problems, we conveniently define L^{-1} $=\frac{d^n}{dt^n}$ as the n- fold definite integration operator from 0 to t. For the operator $L=\frac{d^2}{dt^2}$, for example we have:

$L^{-1}\,L\,u = u - u(0) - tu'(0)$

$\therefore\quad u = u(0) + L^{-1}\,\mathrm{g} - L^{-1}\,Ru - L^{-1}\,Nu$

For the same operator equation but now considering a boundary value problem, we let L^{-1} be an indefinite integral and write $u=A+Bt$ for the first two terms and evaluate A , B from the given condition the first three terms are identified as u_0 in the assumed decomposition

u=$\sum_{n=0}^{\infty}u_n$

Finally, assuming Nu is analytic, we write Nu=$\sum_{n=0}^{\infty}A_n(u_0\ldots u_n)$ where the A_n are specially generated Adomian polynomials for the specific nonlinearity. The nonlinear coupled differential equations (10) is solved using the ADM methods. Thus, Equation (10) in operator form:

$$L_1[f]=-ff''+f'^2+Mf', \tag{13}$$

$$L_2[\theta] = -\mathrm{P}_r\,(f\theta^{/} + \mathrm{P}_r\,N_b\chi^{/}\theta^{/} + \mathrm{P}_r\,N_t\theta^{/2} + P_rE_C f^{//2} + P_r\phi_0\theta)\,, \tag{14}$$

$$L_2[\chi] = -L_e\,f\,\chi' - \frac{N_t}{N_b}\theta'' + KS_c\chi\,. \tag{15}$$

Applying the inverse operator, the ADM solution is obtained by:

$$\sum_{m=0}^{\infty} f_m(\eta) = 1 - be^{-\eta} - L_1^{-1}[\sum_{m=0}^{\infty} A_m] + L_1^{-1}[\sum_{m=0}^{\infty} B_m] + ML_1^{-1}[\sum_{m=0}^{\infty} f_m^{/}]\,, \tag{16}$$

$$\sum_{n=0}^{\infty}\theta_n(\eta) = ce^{-\eta} - P_r\left(L_2^{-1}[\sum_{n=0}^{\infty} C_n] + N_b L_2^{-1}[\sum_{n=0}^{\infty} D_n] + N_t L_2^{-1}[\sum_{n=0}^{\infty} E_n] + E_c L_2^{-1}\sum_{n=0}^{\infty}[F_n] + \phi_o L_n^{-1}\sum_{n=0}^{\infty}[\theta_n]\right)$$

(17)

$$\sum_{n=0}^{\infty}\chi_n(\eta) = he^{-\eta} - L_e L_2^{-1}[\sum_{n=0}^{\infty} G_n] - \frac{N_t}{N_b}L_2^{-1}[\sum_{n=0}^{\infty}\theta_n^{//}] + KS_c L_2^{-1}[\chi_n]\,, \tag{18}$$

where the ADM polynomials are defined as follows:

$$A_m = \sum_{v=0}^{m} f_{m-v}\,f^{//}{}_v\,, \tag{19}$$

$$B_n = \sum_{v=0}^{n} f^{/}{}_{n-v}\,f^{/}{}_v\,, \tag{20}$$

$$C_n = \sum_{v=0}^{n} f_{n-v}\theta^{/}{}_v\,, \tag{21}$$

$$D_n = \sum_{v=0}^{n}\theta^{/}{}_{n-v}\chi^{/}{}_v\,, \tag{22}$$

$$E_n = \sum_{v=0}^{n}\theta^{/}{}_{n-v}\theta^{/}{}_v\,, \tag{23}$$

$$F_n = \sum_{v=0}^{n} f^{//}{}_{n-v}\,f^{//}{}_v\,, \tag{24}$$

$$G_n = \sum_{v=0}^{n} f_{n-v}\chi^{/}{}_v\,. \tag{25}$$

For determination of other components of $f(\eta)$, $\theta(\eta)$ and $\chi(\eta)$, we have:

$$\sum_{m=0}^{\infty} f_{m+1}(\eta) = -L_1^{-1}[\sum_{m=0}^{\infty} A_m] + L_1^{-1}[\sum_{m=0}^{\infty} B_m] + ML_1^{-1}[\sum_{m=0}^{\infty} f_m^{/}]\,, \tag{26}$$

$$\sum_{n=0}^{\infty}\theta_{n+1}(\eta) = -P_r\left(L_2^{-1}[\sum_{n=0}^{\infty} C_n] + N_b L_2^{-1}[\sum_{n=0}^{\infty} D_n] + N_t L_2^{-1}[\sum_{n=0}^{\infty} E_n] + E_c L_2^{-1}\sum_{n=0}^{\infty}[F_n] + \phi_o L_n^{-1}\sum_{n=0}^{\infty}[\theta_n]\right)$$

(27)

$$\sum_{n=0}^{\infty} \chi_{n+1}(\eta) = -L_e L_2^{-1}[\sum_{n=0}^{\infty} F_n] - \frac{N_t}{N_b} L_2^{-1}[\sum_{n=0}^{\infty} \theta_n^{"}] + KS_c L_2^{-1}[\chi_n]. \quad (28)$$

The general solutions are:

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta) = f_0 + f_1 + f_2 \cdots \ , \quad (29)$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta) = \theta_0 + \theta_1 + \theta_2 \cdots \ , \quad (30)$$

$$\chi(\eta) = \sum_{m=0}^{\infty} \chi_m(\eta) = \chi_0 + \chi_1 + \chi_2 \cdots \quad (31)$$

for conveniences, we used Maple-18 to compute the integrals.

Table 1: Comparison of Results for Skin Friction – $f''(0)$

M	Numerical	Present work
5	2.449430407411	2.449489743
10	3.316642264001	3.316624790
60	7.810249675907	7.810249676
100	10.04987562112	10.04987562

3. Results and Discussion

The nonlinear coupled differential equations (10) and (11) with are solved using the Adomian Decomposition Methods. In order to assess the accuracy of the present method, we have compared our solution for the skin friction $f''(0)$ for different values of Magnetic parameter (*M*), with the numerical method (Runge-Kutta's method) as shown in Table 1. It was observed that the present method is in good agreement with the numerical method. Figures 1 to 3 shows the effect of magnetic parameter on velocity, temperature, and concentration profiles over both the stretching and shrinking sheets. It is observed that as the magnetic parameter increases, the velocity and temperature profiles boundary thicknesses reduces while concentration profile boundary thickness increases on both sheets. It was further observed that the temperature and concentration profiles on shrinking sheet are higher than

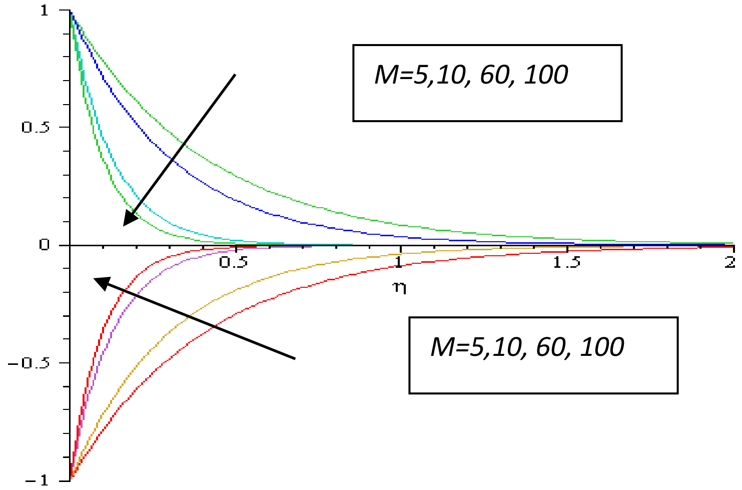

Figure 1: Effect of Magnetic Parameter *M* on the Velocity Profile over a Stretching and Shrinking Sheets.

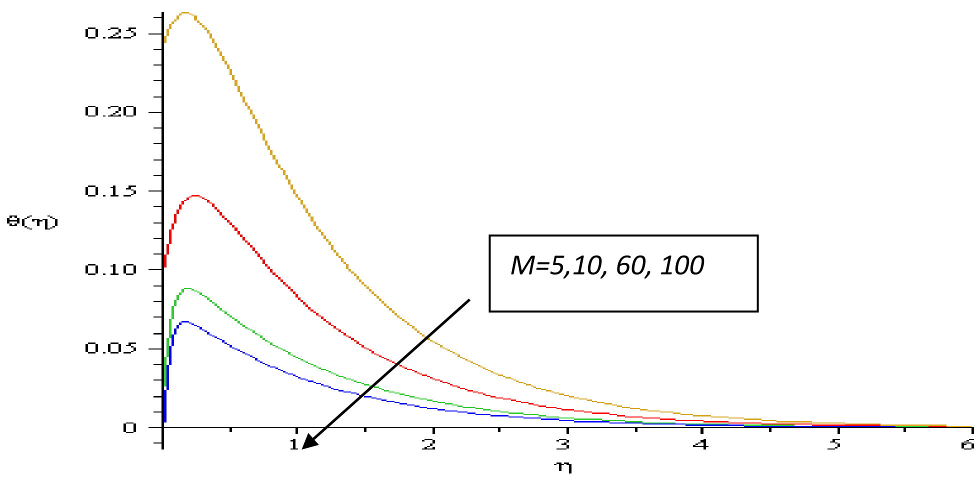

Figure 2a: Effect of Magnetic Parameter *M* on the Temperature Profile over a Stretching Sheet.

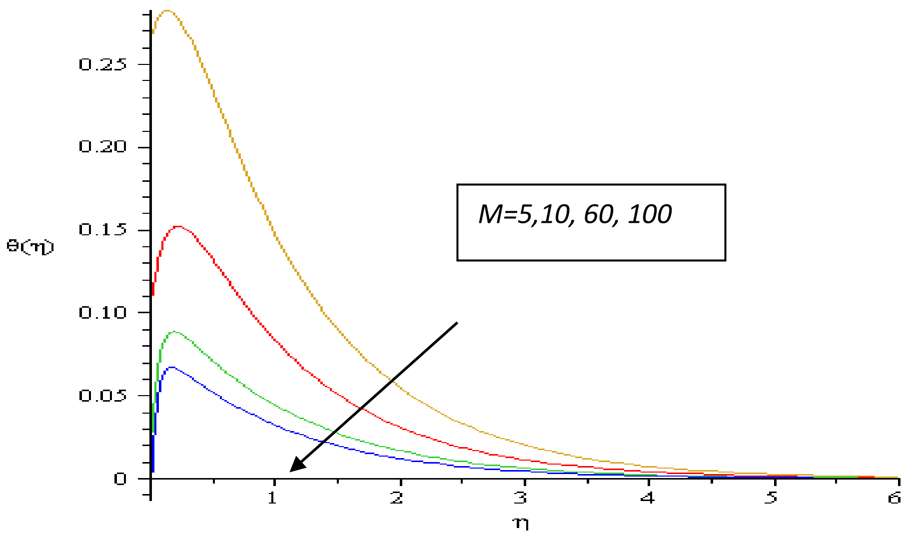

Figure 2b: Effect of Magnetic Parameter *M* on the Temperature Profile over a Shrinking Sheet.

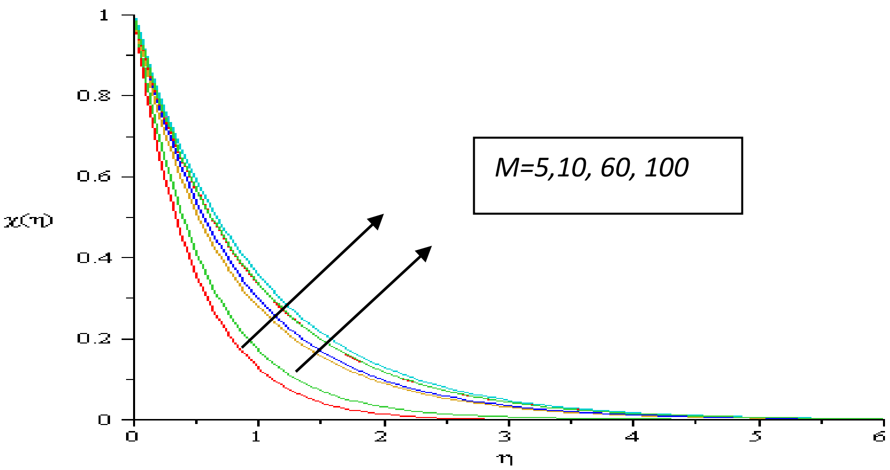

Figure 3: Effect of Magnetic Parameter *M* on the Temperature Profile over a Stretching and Shrinking Sheet.

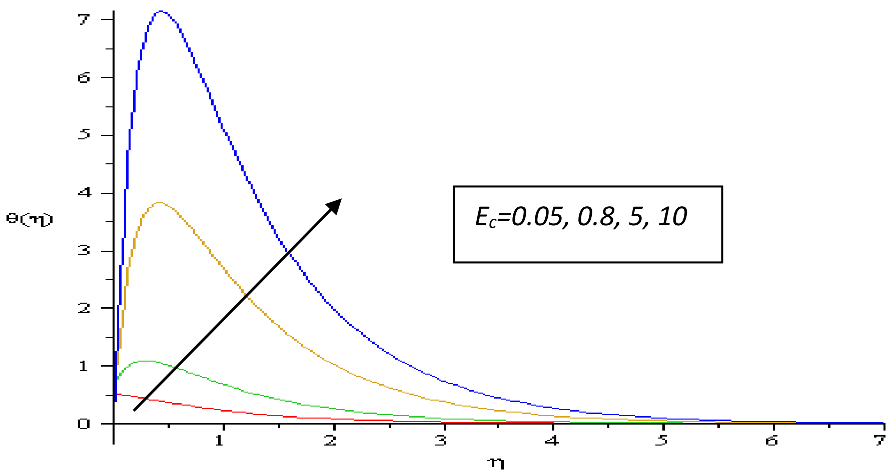

Figure 4: Effect of Eckert Number *Ec* on the Temperature Profile over a Stretching Sheet.

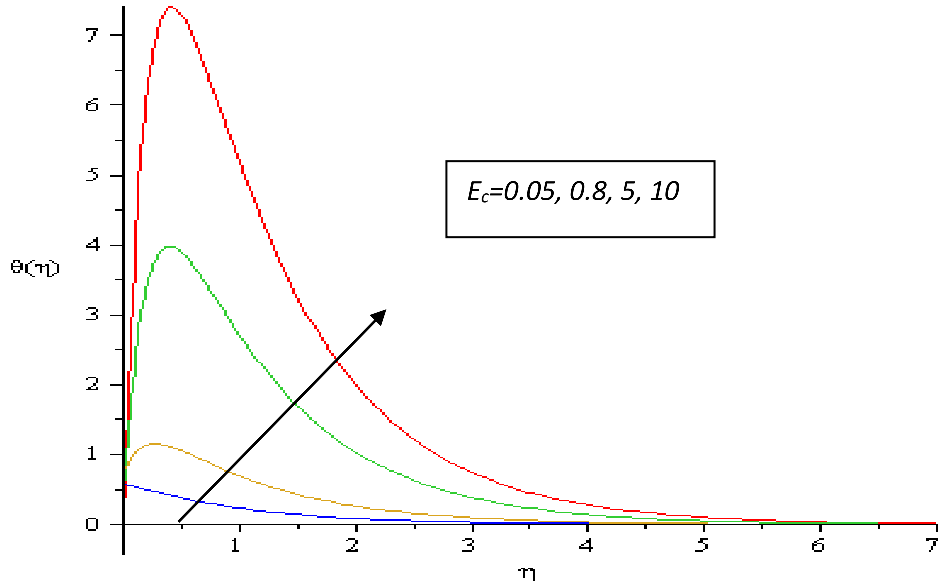

Figure 4b: Effect of Eckert Number *Ec* on the Temperature Profile over a Shrinking Sheet.

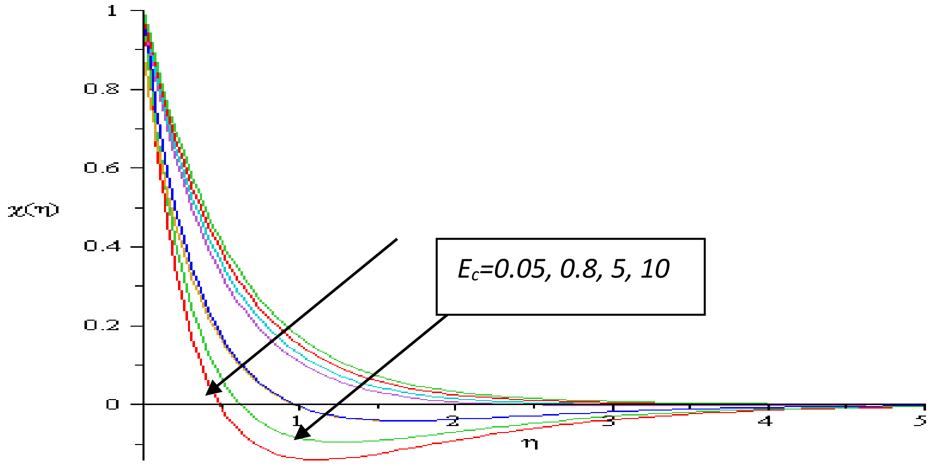

Figure 5: Effect of Eckert Number *Ec* on the Concentration Profile over a Stretching and Shrinking Sheet.

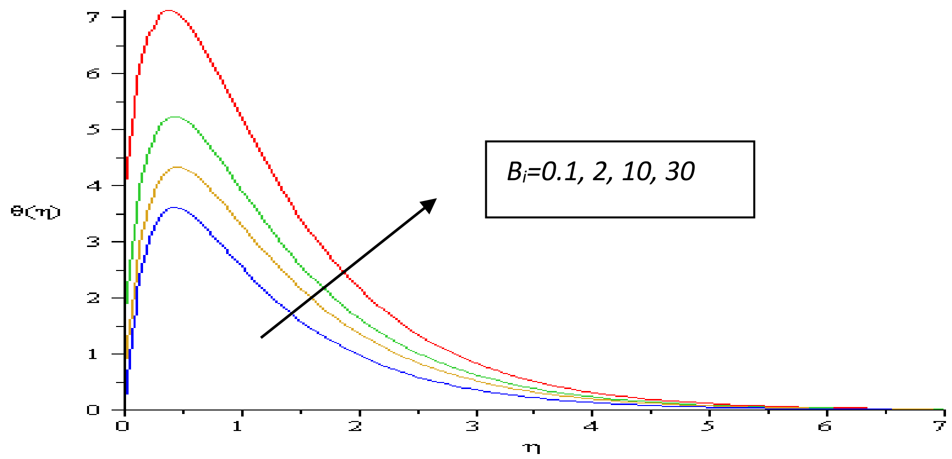

Figure 6a: Effect of Biot Number B_i on the Temperature Profile over a Stretching Sheet.

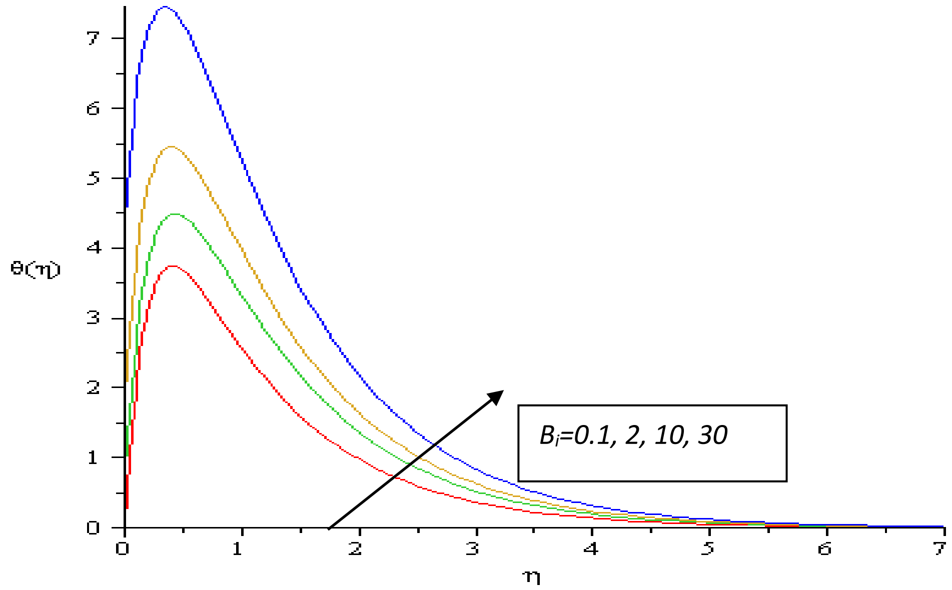

Figure 6b: Effect of Biot Number B_i on the Temperature Profile over a Shrinking Sheet.

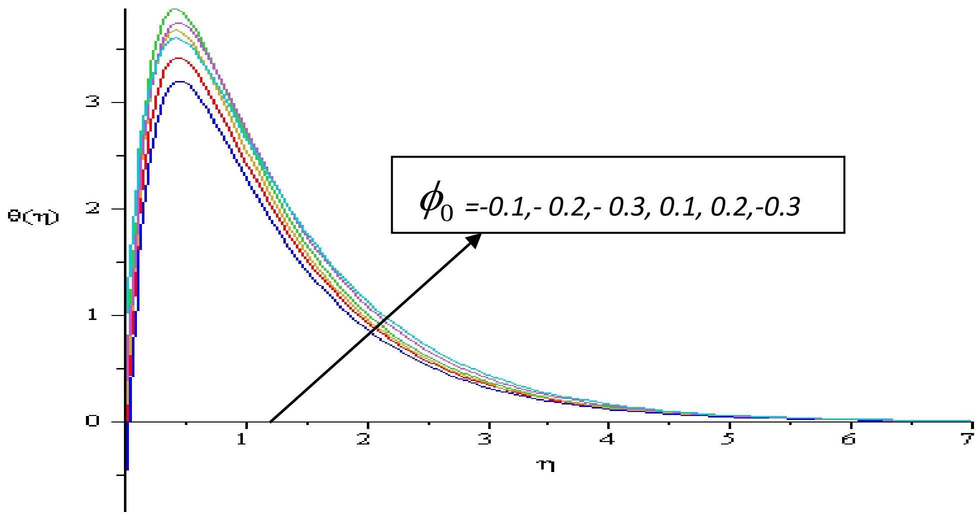

Figure 7: Effect of Generation and Absorption ϕ_0 on the Temperature Profile over a Stretching.

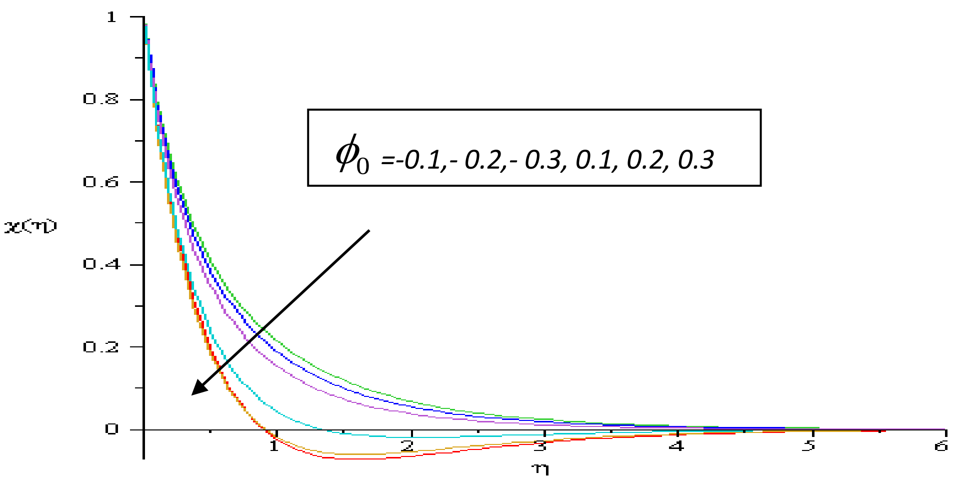

Figure 8: Effect of Generation and Absorption ϕ_0 on the Concentration Profile over a Stretching Sheet.

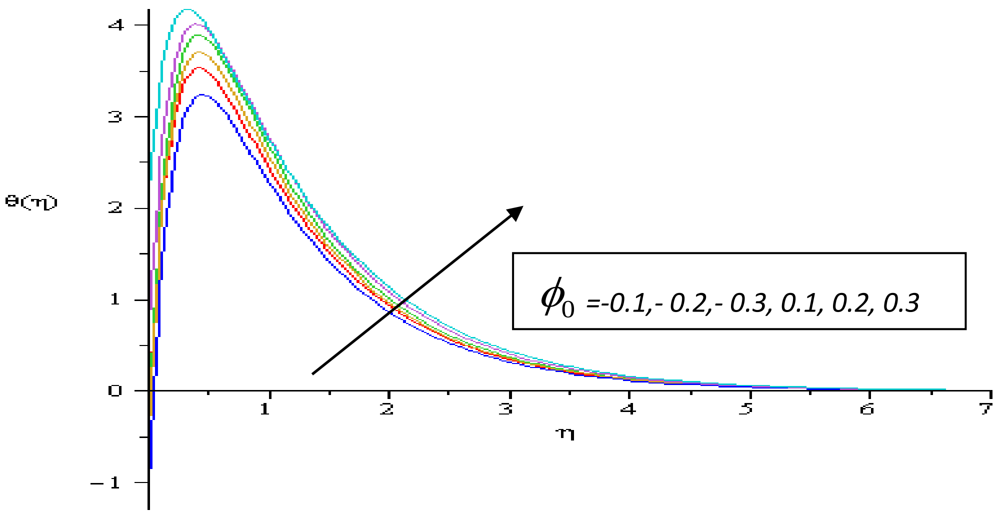

Figure 10: Effect of Absorption ϕ_0 on the Temperature Profile over a Shrinking Sheet.

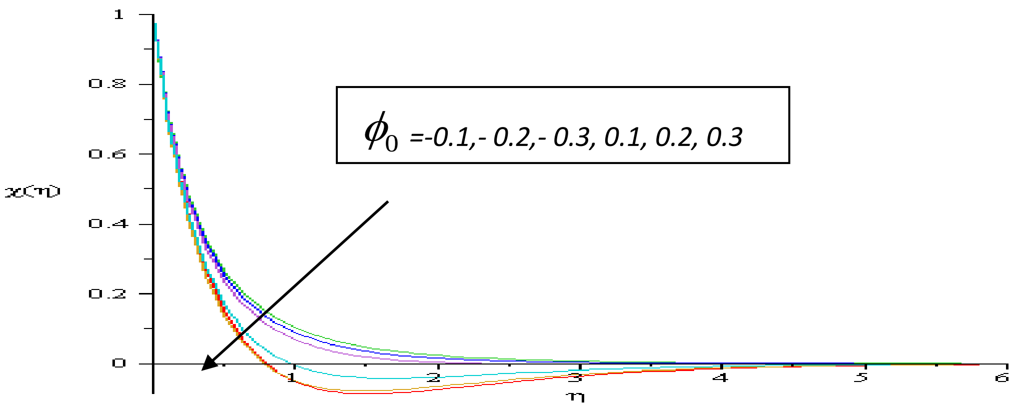

Figure 11: Effect of Absorption ϕ_0 on the Concentration Profile over a Shrinking Sheet.

the stretching sheet at the same point of magnetic parameter (Figures 2a, 2b and Figure 3). This is as a result of application of a transverse magnetic field normal to the flow direction, gives rise to a resistive drag-like force acting in a direction opposite to the flow; this has a tendency to reduce the fluid velocity and temperature while the nanofraction concentration profile boundary thickness is enhanced.

Figures 4 to 5 depict the effect of Eckert number on the temperature and concentration profile. It was observed that increase in the value of Ecket number enhances the thermal boundary thicknesses on both the stretching and shrinking sheet while the nanofraction concentration decreases on both sheets. This shows that at lower values of Eckert number, heat is able to diffuse out of the system faster. Figures 6a and 6b presents the effect of Biot number on the temperature distribution on stretching and shrinking sheets respectively. It is observed that as Biot number increases the thermal boundary layer thicknesses increases on both sheets. But the thermal boundary thickness is higher on the shrinking sheet at the same Biot number than the stretching sheet. This shows that heat is able to diffuse away from the system for lower values of Biot number on the stretching sheet than the shrinking sheet.

Figures 7 to 11 shows the effect of heat generation and absorption parameter on the fluid temperature and nanofraction concertration on both stretching and shrinking sheet. It is observed that as the heat generation parameter increases from negative to positive, the thermal boundary thickness is enhanced while the nanofraction boundary thickness is reduced on both sheets. When the parameter assume a negative value, it signifies heat absorption and generation when either.

4. Conclusion

The problem of laminar fluid flow which results from the stretching/shrinking of a flat surface in a nanofluid has been obtained using the Adomian Decomposition Method. The model used for the nanofluid was presented in its rectangular form. The model is considered in the presence of viscous dissipation, heat generation/absorption with convective boundary condition and the effect of Brownian motion and thermophoresis. A similarity solution is presented which depends on magnetic parameter (M), Eckert number (E_c), heat generation and absorption, Biot number (b_i), Lewis number (L_e), Brownian motion (N_b) number and thermophoresis number (N_t). It was found that:-

1. The velocity profile reduces due to increase in the magnetic parameter; which leads to reduction in the fluid temperature on both stretching and shrinking sheets.

2. Increase in Eckert number enhances the fluid temperature while nanofraction concentration is reduced on both sheets.

3. It was observed generally that the boundary thickness is always higher on the shrinking sheet for the same value of quantity than the stretching sheet. In view of this, stretching sheet is recommended for faster diffusion of heat energy.

5. The result for the skin friction coefficient was in good agreement with that of the numerical method.

6. All the graphs presented in this work satisfy the boundary conditions, which further proved the efficiency of this method of ADM.

7. The results presented in this work is important in applications such as extrusion, wire drawing, metal spinning, and hot rolling

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