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Impacts of Ionospheric Scintillations on Dilution of Precision and Positioning Errors

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Abstract

Equatorial scintillation impacts negatively on GPS navigation measurements by reducing the number of satellites that are readily available for a GPS receiver to calculate a navigation solution. Ideally, a minimum of four satellites is required to calculate a valid navigation solution. We analyzed GPS data from Ascension Island during the Air Force Research Laboratory (AFRL) campaign of the solar maximum year of 2002 (5–19 March 2002). The study showed that strong scintillations impacted the receiver-satellite geometry, with attendant poor dilution of precisions and positioning errors. On the non-scintillation nights, the GPS receiver showed good tracking capability, as it consistently maintained lock on six to eight satellites, and the GDOP and PDOP values were generally less than 2.2 with lesser positioning errors. In contrast, on the scintillation nights, between 2200 and 2300 UT, the number of satellites that the receiver maintained lock on were consistently reducing to four and in some cases fewer than four. Around this period, the GDOP or PDOP values were higher than 6.0, and significant vertical and horizontal positioning errors were recorded. Equatorial scintillation is capable of adding about 3 metres error to the overall error budget of a GNSS system.

Keywords: Ionospheric scintillations, GNSS, Dilution of precision, positioning errors

1. Introduction

Ionospheric scintillation is the rapid fluctuations that are imposed on the amplitude and/or phase of radio signals that traverse irregularities in the F region of the ionosphere. It is a post-sunset event in the equatorial region, but can occur at any time of the day at the polar region. Ionospheric scintillation is most predominant at the equatorial region, lesser at the high latitude and least at the mid-latitude (Aarons, 1982). It is also known to be seasonal and solar activity dependent (Akala *et al*., 2011a, Akala and Doherty, 2012), and it has correlation with geomagnetic activity (Ledvina *et al*., 2004, Basu *et al*., 2010). Scintillation could cause cycle slips, or stress the receiver to lose lock on the transmitted signals (Aarons, 1982, Akala *et al*., 2012), to be re-acquired at a later time (Carrano *et al*., 2005, Seo *et al*., 2009), leading to intermittent availability of service.

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Since the lost satellite signals cannot be used for position calculation, the geometry of the useable satellite constellation is degraded during periods of outages. Significant impairments on this geometry could lead to positioning errors. Dilution of precision (DOP) is the parameter that quantifies the influence of receiver-satellite geometry on GPS positioning accuracy. The best DOP is obtained when the satellites in view of the receiver are evenly distributed in the sky. For instance, in a case of four satellites in view of the receiver, the best DOP will be obtained if three of the satellites are equilaterally distributed over the sky, and the fourth satellite is located directly overhead in the centroid of the equilateral triangle, collectively leading to tetrahedron geometry by their line of sights (LOS) to the receiver (Kaplan, 1996; Misra and Enge, 2001; Akala *et al*., 2012).

There are six components of DOP: the geometric dilution of precision (GDOP); the position dilution of precision (PDOP); the horizontal dilution of precision (HDOP); the vertical dilution of precision (VDOP); the time dilution of precision (TDOP) (Leva, 1994, Kaplan, 1996); and the ambiguity dilution of precision (ADOP), specifically for high accuracy positioning (Teunissen, 1997; Mocron *et al*., 2014). VDOP values are often slightly larger than HDOP values, indicating that vertical position errors are larger than the horizontal position errors. We suffer this effect because all the satellites from which we obtain signals are above the receiver. The horizontal coordinates do not suffer a similar fate as we usually receive signals from all sides. The main goal of this work is to study the impacts of scintillation on DOP and positioning error, and the relationship between DOP and positioning error.

2. Theory and Derivations

For *n* satellites in view of a GPS receiver, following Akala *et al*. (2011), we defined,

$$\mathbf{HX} = \rho \, , \tag{1}$$

where **H** is *n*x*4* matrix of the satellites' pointing vectors, $\boldsymbol{X}$ = (x, y, z, t)^k the user's position and time, relative to local reference (the ground truth), and $\rho = (\rho_1, \ldots\ldots\ldots, \rho_n)^k$ is the vector of pseudo-range measurements relative to the reference.

For $n > 4$, eq. (1) becomes over-determined and hence, nonlinear (Akala *et al*., 2011b); hence, a least square technique is usually employed in linearizing such nonlinear systems, in conjunction with iterative processes. Therefore, defining $Q = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}$, so that the elements of Q becomes:

$$Q = \begin{bmatrix} d_x^2 & d_{xy}^2 & d_{xz}^2 & d_{xt}^2 \\ d_{xy}^2 & d_y^2 & d_{yz}^2 & d_{yt}^2 \\ d_{xz}^2 & d_{yz}^2 & d_z^2 & d_{zt}^2 \\ d_{xt}^2 & d_{yt}^2 & d_{zt}^2 & d_t^2 \end{bmatrix} . \tag{2}$$

Langley (1999) pointed out that a weight matrix can be used to characterize the differences in the errors of simultaneous pseudo-range measurements. From (2), the components of the dilution of precision (DOP) can be represented by:

$$HDOP = \sqrt{d_x^2 + d_y^2}\ ,\ VDOP = \sqrt{d_z^2}\ ,\ TDOP = \sqrt{d_t^2}\ ,\ PDOP = \sqrt{d_x^2 + d_y^2 + d_z^2}\ \ GDOP = \sqrt{PDOP^2 + TDOP^2}\ .$$

Further, defining the position error vector, **e** as the vector from the intersection of the four sphere surfaces, which correspond to the pseudo-ranges to the true position of the receiver, $\mathbf{e} = e_x\hat{x} + e_y\hat{y} + e_z\hat{z}$, and e_t denotes the time error, i.e. the true time minus the receiver indicated time. Hence

$$\mathbf{H}\begin{pmatrix} \mathbf{e} \\ e_t \end{pmatrix} = \begin{pmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{pmatrix}, \tag{3}$$

where e_i; i = 1,………*n* are the errors in pseudo-ranges of individual satellites. We could multiply (3) by $\mathbf{H}^{-1}$, transpose the resulting matrix, and then multiply both sides of it by the product of (3) and $\mathbf{H}^{-1}$. Taking the expectation value of both sides of the overall result, and then taking the non-random matrices outside the expectation operator, E, such that:

$$E\left(\begin{pmatrix} \mathbf{e} \\ e_t \end{pmatrix}\begin{pmatrix} \mathbf{e} & e_t \end{pmatrix}\right) = \mathbf{H}^{-1}E\left(\begin{pmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{pmatrix}\begin{pmatrix} e_1 & \cdot & \cdot & \cdot & e_n \end{pmatrix}\right)\left(\mathbf{H}^{-1}\right)^T . \tag{4}$$

Assuming the pseudo-range errors are uncorrelated and that they have the same variance. The covariance matrix on the right side of (4) can then be expressed as the product of a scalar and an identity matrix. Hence

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & \sigma_{xz}^2 & \sigma_{xt}^2 \\ \sigma_{xy}^2 & \sigma_y^2 & \sigma_{yz}^2 & \sigma_{yt}^2 \\ \sigma_{xz}^2 & \sigma_{yz}^2 & \sigma_z^2 & \sigma_{zt}^2 \\ \sigma_{xt}^2 & \sigma_{yt}^2 & \sigma_{zt}^2 & \sigma_t^2 \end{bmatrix} = \sigma_{UERE}^2\left(\mathbf{H}^T\mathbf{H}\right)^{-1} = \sigma_{UERE}^2 Q . \tag{5}$$

Comparing (2) and (5), we have:

$\sigma_{HPE} = HDOP \cdot \sigma_{UERE}$, $\sigma_{VPE} = VDOP \cdot \sigma_{UERE}$, where σ_{HPE} is the horizontal positioning error, σ_{VPE} is the vertical positioning error, and σ_{UERE} is the user equivalent range error (Langley, 1999, Van Dyke, 2001).

3. Data and Methods of Analysis

The U. S. Air Force Research Laboratory (ARFL) conducted a 15-day (5–19 March 2002) campaign to monitor GPS scintillations at Ascension Island (7.96^{o}S, 14.41^{o}W, dip lat 16.0^{o}S) during the solar maximum year of 2002. The data were acquired by an Ashtech Z-XII GPS receiver at a sampling rate of 20Hz. Akala *et al*. (2012) provided details on how Ashtech Z-XII GPS receiver compute S4 index. Intense scintillation activity (S4 ~ 1.0) was encountered on almost all the nights of the campaign and severe impacts on GPS tracking performance in terms of signal availability and positioning accuracy were consistently observed. During this campaign, a number of navigation outages were observed.

From the receiver reported position dilution of precision (PDOP) and time dilution of precision (TDOP), we calculated the geometric dilution of precision (GDOP) by the conventional method: GDOP = $[PDOP^2 + TDOP^2]^{1/2}$. The characteristic strong scintillations during most nights of the campaign caused up to five or more satellites signals to scintillate at times. Generally, navigation outages were observed during all the nights of the campaign except five; 5th–6th and 9th–11th. The durations of these outages range from 1 to 50 s, although less than 10 s most of the time, with the impacts most severe on the 13th. For this reason, the data of the 13th were used as the representative of a scintillation day, while those of the 9th were used as the representative of non-scintillation day. At Ascension Island, UT = LT.

The vertical and horizontal position errors were evaluated from the receiver reported position data. Using standard technique by Hofmann-Wellenhof *et al*. (1997) and Leick (2004), the reported position data in world geodetic system (WGS-84) reference frame were converted to the Earth Centred Earth Fixed (ECEF) Cartesian coordinates. Thereafter, the median value of all the position values during the campaign was calculated. This median value was assumed to be the receiver true position in the absence of scintillations. Subsequently, we defined a topocentric coordinate (north, east, up) centered at the median value, and we rotated all the receiver position samples into the topocentric coordinate system. Since the origin of the topocentric system represents the receiver position in the absence of scintillation, the receiver reported position samples in this coordinate system represents the “errors” due to scintillation. The vertical error is the “up” component of each position sample, while the horizontal error is equal to the square root of the addition of the square of the “north” component and the square of the “east” component. Finally, the statistical distribution of DOPs (HDOPs) was studied, and the relationships between the positioning errors and the corresponding DOP values were also investigated for scintillation and non-scintillation days.

4. Results and Discussion

Figure 1 shows the GPS amplitude scintillations between 2000–0200 LT during the days of the campaign. Five nights of the campaign (5–6th; 9–11th) were characterized with little or no scintillations, while the remaining nights were characterized with strong scintillations. Typically, no scintillations were recorded on the 6th and 9th. During the scintillation nights, most of the satellites’ signals experienced scintillation. The presence of ionospheric irregularities modulated the C/No of the signals (Figure 2). These modulations led to incursions of about 20dB–Hz or more in C/No values (Akala *et al*., 2012). Figures 3 and 4 of Akala *et al*. (2012) had earlier shown the number of visible satellites, components of DOP values, vertical and horizontal positioning errors within 2000–2400 LT on the 9th (non-scintillations day) and 13th (scintillations day), respectively.

On the non-scintillation night, the receiver showed good tracking capability, as it consistently maintained lock on six to eight satellites, and the DOP values were generally less than 2.2 with no single record of an outage. In contrast, on the scintillation night, between 2200 and 2300 UT, the number of satellites that the GPS receiver maintained lock on often reduced to four and in some cases fewer than four, leading to navigation outages.

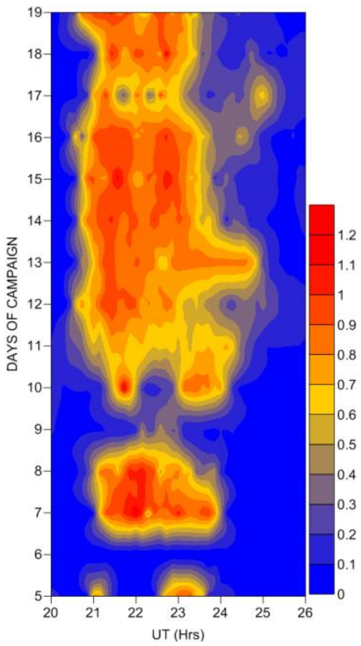


Figure 1: GPS amplitude scintillations between 2000–0200 LT during the days (5–19 March, 2002) of the campaign

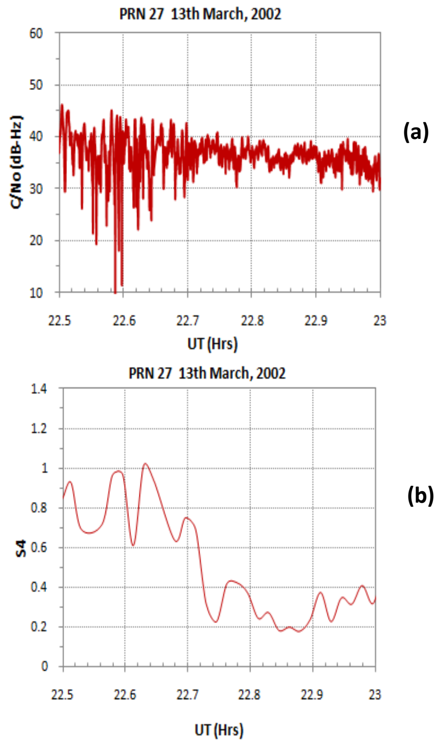


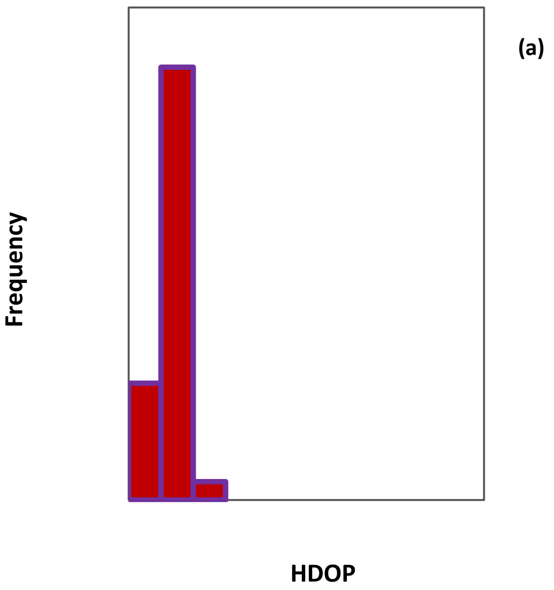
Figure 2: (a) Carrier-to-noise density ratio (C/No) of a scintillating satellite on the 13 March, 2002, **(b)** The corresponding S4 index

The vertical position error (VPE) and horizontal position error (HPE) during the scintillation night were characterized with spikes between 22.00 and 23.00 LT. In other words, very large errors and DOPs were observed between 2200–2300 LT when scintillations were most intense. Outside this time window, VPE and HPE generally vary between 0–30 m and 4–30 m respectively.

Figure 3 shows the HDOP distributions on histograms during non-scintillation and scintillation nights respectively. During the non-scintillation nights, HDOP recorded maximum distribution of one, and the maximum HDOP value was two. However, during the scintillation nights, HDOP still recorded maximum distribution of one, and the maximum HDOP value was nine. Although, the distributions of the higher HDOP values (>5.0) were infinitesimally low. For this statistical analysis, three nights: 5, 6, and 9th represent the non-scintillation nights, while 13, 16 and 18th represent the scintillation nights.

Three days scatter plots of horizontal position errors against HDOP for the non-scintillation (5, 6, and 9th) and scintillation (13, 16, and 18th) days are shown in Figure 4a-b respectively. It is noteworthy to mention that the higher the HDOP value, the bigger the scatter in the horizontal position error. However, a lower DOP value does not automatically means a lower position error. In fact, at lower DOP values, there is likelihood of obtaining low error in horizontal position estimate as with higher error in horizontal position estimate. Large position errors that occur despite a favourable viewing geometry (low DOP) are the result of the ranging errors on the individual links, rather than intermittent availability (Carrano *et al*., 2005).

Grouping the data by HDOP ranges and then computing the root-mean-square (rms) of the horizontal position error for each HDOP range bin (black filled diamonds), as shown in Figure 4, a linear relationship was observed between the rsm horizontal position error and the HDOP (Misra and Enge, 2001). This further validates eq. (5). The slope of the linear fitted to the computed (rms) horizontal position error in the different bins defines the user equivalent range error (σ_{UERE}). From Fig. 6a and b, the slopes were evaluated as 5.2 m and 8.3 m respectively.



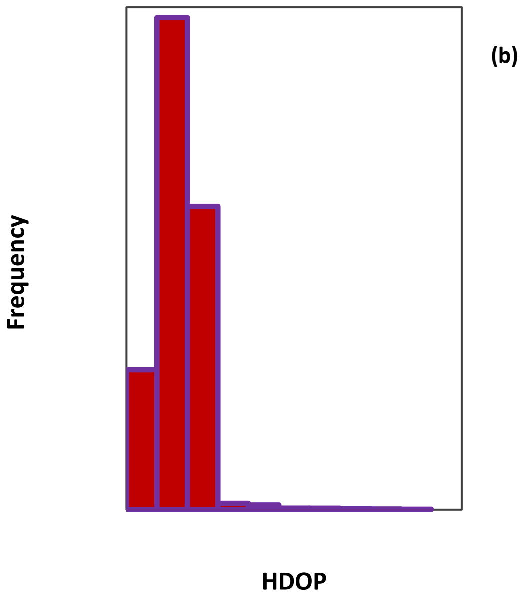


Figure 3: Histograms of HDOP for **(a)** non-scintillation nights **(b)** scintillation nights

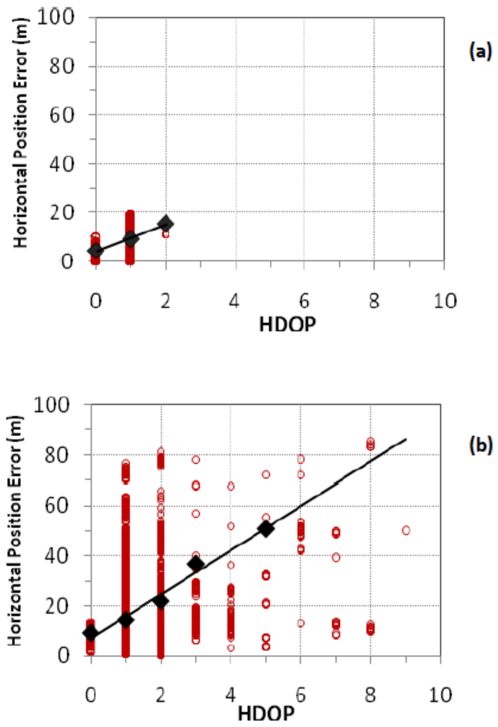


FIGURE 4: Horizontal position error against horizontal dilution of precision (HDOP) for **(a)** non-scintillation nights **(b)** scintillation nights

5. Conclusions

Equatorial scintillation impacts negatively on GPS navigation measurements by reducing the number of satellites that are readily available for a GPS receiver to calculate a navigation solution. Strong scintillations impacted the receiver-satellite geometry, with attendant poor dilution of precisions and positioning errors. On non-scintillation nights, receiver shows good tracking capability, as it consistently maintained lock on six to eight satellites, and the GDOP and PDOP values were generally lesser than 2.2, with lesser positioning errors. In contrast, on the active nights, between 2200 and 2300 UT, the number of satellites that the receiver maintained lock on were consistently reducing to four and in some cases fewer than four. Within this period, the GDOP or PDOP values were higher than 6.0, and significant vertical and horizontal positioning errors were recorded. Overall, GPS amplitude scintillation is capable of adding error in the order of 3 metres to the overall error budget of a GNSS system.

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