Ilorin Journal of Science
Volume 2, Number 2, 2015, pp. 455 – 463 (Printed in Nigeria)
ISSN: 2408 – 4840 © 2015 Faculty of Physical Sciences, University of Ilorin
https://doi.org/10.54908/iljs.2015.02.02.014

ILORIN
JOURNAL OF SCEINCE

ILJS-15-047

A Modified Adomian Decomposition Method for A MHD Third Grade Flow with Ohmic Heating

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Abstract

In this research work we studied the flow and heat transfer problem of a third grade fluid. It is assumed that the fluid is incompressible and electrically conducting in the presence of a uniform magnetic field. The equations governing the flow and heat transfer are solved using the modified Adomian decomposition method (MADM) for the velocity and temperature and the results are presented graphically. The effects of magnetic parameter are analyzed for velocity and temperature profile. It is noticed that increase in magnetic parameter reduced the velocity of the fluid and increases the temperature profile.

Keywords: Third grade fluid, Magneto-hydrodynamics, modified Adomian decomposition method.

1. Introduction

Considerable interest has been developed in the study of the flow of third grade fluid in parallel channel due to its varied applications in science, engineering and technology. Some of these applications include extraction processes, especially in polymer industry, micro fluids, geological flows within the earth's mantle, the flow of synovial fluid in human joints as well as in the drilling of oil and gas wells. For example Nayak *et al.* (2012) analysed the unsteady convective flow of a third grade fluid past an infinite vertical porous plate with uniform suction applied at the plate using an implicit finite difference scheme. While, Erdogan (1995) studied the flow of a third grade fluid in the vicinity of a plane wall suddenly set in motion.

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He observed that for short time a strong non-Newtonian effect is present in the velocity field. Hayat *et al*. (2009) carried out an analytical study of a Magnetohydrodynamic flow of a third grade fluid in a porous medium and Roohi *et al*. (2009) applied the homotopy method to obtain analytical solution of non-Newtonian channel flows. In the same way, Siddiqui *et al*. (2008) studied hydrodynamics third grade fluid between two parallel plates with heat transfer. They considered and treated three different problems, the Poiseuille flow, the Couette flow and the Poiseuille-Couette flow. Aiyesimi *et al.* (2013a) investigated the viscous dissipation effect on the MHD flow of a third grade fluid down an inclined plane with Ohmic heating.

In addition, Aiyesimi *et al*. (2013b) also studied the thin film flow of a third grade fluid down an inclined plane using homotopy and regular perturbation method. The ADM has been successfully applied to solve non-linear equations arising in applied sciences and engineering Siddiqui *et al*. (2010). Iyoko *et al.* (2017) used a the ADM to study MHD flow of a third grade fluid in a cylindrical pipe in the presence of reynolds' model viscosity and joule heating , and is usually characterized by its higher degree of accuracy.

It provides analytical solution in the form of an infinite series in which each term can be easily determined. The ADM has been proved to be a reliable method solving both linear or non-linear differential equations, it does not have the short comings of the traditional methods of approximations such as finite difference methods, homotopy and the differential transform methods, the MADM needs no discretization, linearization, spatial transformation or perturbation and the Adomian polynomials are easily computed, therefore it possess some significant advantages over other approximate methods.

In this work, we derived the solutions of the thin film flow of a third grade fluid in a parallel channel using the modified Adomian decomposition method.

2. Problem Formulation

The fundamental equations governing the MHD flow of an incompressible electrically conducting fluid are the field equation:

$$\nabla \cdot v = 0, \tag{1}$$

$$\rho \frac{Dv}{Dt} = -\nabla p + divT + J \times B + \rho f \ , \tag{2}$$

where ρ is the density of the fluid, v is the fluid velocity, B is the magnetic induction so that

$$B = B_0 + b \tag{3}$$

and

$$J = \sigma(E + v \times B), \tag{4}$$

is the current density. σ is the electrical conductivity, E is the electrical field which is not considered (i.e $E = 0$), B_0 and b are applied and induced magnetic field respectively, D/Dt denote the material derivative, p is the pressure, f is the external body force and T is the Cauchy stress tensor which for a third grade fluid satisfies the constitutive equation:

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2\left(A_1 A_2 + A_2 A_1\right) + \beta_3 (trA_1^2) A_1 , \tag{5}$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1}\nabla v + (\nabla v)^T A_{n-1}, \qquad n \geq 1, \tag{6}$$

where pI is the isotropic stress due to constraint incompressibility, μ is the dynamics viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants; T indicate the matrix transpose, A_1, A_2, A_3 are the first three Rivlin-Ericken tensors and $A_0 = I$ is the identity tensor.

We consider a thin film of an incompressible MHD fluid of a third grade flowing in a parallel channel. By neglecting the surface tension of the fluid and the film is of uniform thickness d, we seek a velocity field of the form

$$v = [u(y), 0, 0,]. \tag{7}$$

In the presence of modified pressure gradient, equations (1)-(4) along with equations (5)-(7) yield

$$\frac{d^2u}{dy^2} + 6(\beta_2 + \beta_3)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} - \frac{1}{\ell}\frac{dp}{dx} - \sigma B_0^2 u = 0. \tag{8}$$

Subject to the boundary condition

$$u(y) = 0 \text{ at } y = -d, \tag{9}$$

$$u(y) = u_0 \; at \quad y = d. \tag{10}$$

Using the dimensionless variables:

$$U = \frac{u}{u_0}, Y = \frac{y}{d}.$$

From equations (8)-(10), we have:

$$\frac{d^2U}{dY^2}+6\beta\left(\frac{dU}{dY}\right)^2\frac{d^2U}{dY^2}+k-MU=0, \qquad (10b)$$

$U(-1)=0,$
$U(1)=1,$
where

$k=-\frac{d^2}{\ell}\frac{dp}{dx}$ is the pressure gradient, $M=\frac{\sigma B_0^2 d^2}{\rho\mu}$ is the magnetic parameter and

$\beta=\frac{\beta_2+\beta_3}{\mu}\left(\frac{u_0}{d}\right)^2$ .is the material property.

Heat Transfer Analysis

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and joule heating is given as

$$k\frac{d^2T}{dy^2}+\mu\left(\frac{du}{dy}\right)^2+2\beta_3\left(\frac{du}{dy}\right)^4+\sigma B_0^2u^2=0 \quad , \qquad (11)$$

with boundary condition

$$T(y)=T_w \text{ at } y=0, \qquad (12)$$

$$T(y)=T_w \text{ at } y=d, \qquad (13)$$

where k is the thermal conductivity, T is the temperature, and T_δ is the temperature of the ambient fluid.

Introducing the following dimensionless variable:

$$u=\frac{u}{u_0} \quad \text{and} \quad \overline{T}=\frac{T-T_1}{T_2-T_1}, \qquad (14)$$

where $T_1=T_w$ and $T_2=T_\delta$. The system of equations (11)-(13) and (14) after dropping the caps take the following form:

$$\frac{d^2T}{dY^2}+B_r\left(\frac{dU}{dY}\right)^2+B_r\beta\left(\frac{dU}{dY}\right)^4+B_rMU^2=0, \qquad (15)$$

$$T(y)=0 \text{ at } y=-1, \qquad (16)$$

$T(y) = 0$ at $y = 1$, (17)

where $B_r = \dfrac{\mu u'^2}{k(T_2 - T_1)}$ is the Brinkman number and $M = \dfrac{\sigma B_0^2 h^2}{\mu}$ is the magnetic parameter.

3. Method of solution:

In order to solve the set of equations (8)-(10) and (15)-(17), we apply the modified Adomian decomposition method, equation (8) can be written as follows:

$$\frac{d^2u}{dy^2} = Mu - k - 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} \tag{18}$$

and

$$u(y) = Ay + B + \int_0^y\int_0^y \left[Mu - 6\beta\left(\frac{d^2u}{dy^2}\right)^2 \frac{d^2u}{dy^2}\right] dydy. \tag{19}$$

Given that:

$$u_0 = Ay + B + \int_0^y\int_0^y k dydy, \tag{20}$$

where

$$A = u'(0) \; and \; B = u(0). \tag{21}$$

Let $u = \sum_{n=0}^{\infty} U_n$, So that:

$$U_{n+1} = \int_0^y\int_0^y \left[MU_n - 6\beta\sum_{m=0}^{n}\sum_{j=0}^{m} U'_{n-m}U'_{m-j}U''_j \right] dydy. \tag{22}$$

Similarly, equation (15) can also be transformed into:

$$T_{n+1} = -B_r\int_0^y\int_0^y \left[\sum_{m=0}^{n} U'_{n-m}U'_m + 2\beta\sum_{m=0}^{n}\sum_{j=0}^{m}\sum_{i=0}^{j} U'_{n-m}U'_{m-j}U'_{j-i}U'_i + M\sum_{m=0}^{n} U'_{n-m}U'_m \right], \tag{23}$$

where

$$\left.\begin{aligned} &T_0 = Dy + E \\ &E = T(0) \; and \; D = T'(0) \end{aligned}\right\} . \tag{24}$$

Evaluating equations (20-24), we have the following equations:

$$U(y)=\frac{ky^2}{2}+Ay+B+\frac{1}{4}\left(-\frac{1}{6}Mk+2\beta k^3\right)y^4+\frac{1}{3}\left(\frac{1}{2}MA-6\beta k^2A\right)y^3+$$

$$+\frac{1}{2}\left(MB+6\beta kA^2\right)y^2,$$

$$\frac{1}{8}\left(-\frac{4}{7}Mk^5\beta^2+\frac{1}{42}\beta M^2k^3+\frac{24}{7}\beta^3k^7\right)y^8+\frac{1}{7}\left(-24\beta^3k^6A+4MA\beta^2k^4-\frac{1}{6}\beta M^2Ak^2\right)y^7$$

$$+\frac{1}{6}\begin{pmatrix}-\frac{36}{5}k^5\beta^2+\frac{24}{5}MB\beta^2k^4+\frac{3}{10}\beta kM^2A^2-\frac{2}{5}\beta M^2Bk^2\\+\frac{7}{10}\beta k^3M-\frac{1}{120}M^2k-\frac{48}{5}\beta^2k^3A^2M+72\beta^3k^5A^2\end{pmatrix}y^6$$

$$+\frac{1}{5}\left(\frac{3}{2}\beta kM^2BA+9\beta^2k^2A^3M-\frac{7}{2}\beta Ak^2M-108\beta^3k^4A^3+\frac{1}{24}M^2A+36Ak^4\beta^2-18MBk^3\beta^2A\right)y^5$$

$$+\frac{1}{4}\left(2\beta k^3+6\beta A^2Mk+2\beta kM^2B^2+24BM\beta^2k^2A^2-2\beta k^2M-72A^2\beta^2k^3+\frac{1}{6}M^2B+72\beta^3k^3A^4\right)y^4$$

$$\frac{1}{3}\left(72\beta^2k^2A^3+6\beta kAMB-3\beta A^3M-6\beta k^2A\right)y^3+\frac{1}{2}\left(-36A^4\beta^2k+6\beta kA^2-6\beta A^2MB\right)y^2 \qquad (25)$$

Similarly, we obtain

$$T(y)=E+Dy-\frac{1}{15}Br\beta k^4y^6+\frac{2}{5}Br\beta Ak^3y^5-\frac{1}{12}Br(k^2+12\beta A^2k^2)y^4-\frac{1}{6}Br(-2Ak-8\beta A^3k)y^3$$

$$-\frac{1}{2}Br(A^2+2\beta A^4)y^2-\frac{1}{56}Br((-\frac{1}{6}Mk+2Gk^3)^2-8\beta(-\frac{1}{6}Mk+2Gk^3)k^3)y^8-\frac{1}{42}Br(-8\beta(\frac{1}{2}MA-6Gk^2A)k^3$$

$$+24\beta(-\frac{1}{6}Mk+2Gk^3)Ak^2+2(\frac{1}{2}Mk-6Gk^2A)(-\frac{1}{6}Mk+2Gk^3))y^7-\frac{1}{30}Br(-8\beta(MB+6GkA^2)k^3$$

$$+24\beta(\frac{1}{2}MA-6Gk^2A)Ak^2-24\beta(-\frac{1}{6}Mk+2Gk^3)A^2k+2(MB+6GkA^2)(-\frac{1}{6}Mk+2Gk^3)$$

$$+(\frac{1}{2}MA-6Gk^2A)^2)y^6-\frac{1}{20}Br(24\beta(MB+6GkA^2)Ak^2-24\beta(\frac{1}{2}MA-6Gk^2A)A^2k+8\beta(-\frac{1}{6}Mk+2Gk^3)A^3$$

$$+2(MB+6GkA^2)(\frac{1}{2}MA-6Gk^2A))y^5-\frac{1}{12}Br(k^2+(MB+6GkA^2)^2-24\beta(MB+6GkA^2)A^2k+8\beta(\frac{1}{2}MA$$

$$-6Gk^2A)A^3)y^4-\frac{1}{6}Br(8\beta(MB+6GkA^2)A^3-2Ak)y^3-\frac{1}{2}BrA^2y^2. \qquad (26)$$

Results and Discussion

In order to verify the efficiency of the method equations (25) and (26) were evaluated using various values of the thermo-physical parameters to visualize the flow and heat transfer pattern for example using M = 15, k = 1, Br = 5 and $\beta = 0.001$ the values of the constants A = 0.08306296, B = 0.08936288, D = -0.3716754670 and E = 9.136933409 which represents the velocity, skin friction, wall temperature and the heat transfer rate at the wall. Fig.1 represents the effect of the magnetic parameter on the velocity profile, it is observed that as the magnetic parameter increases the velocity profile decreases which shows that the magnetic field parameter reduces fluid motion. While in Fig.2 the effect of the magnetic parameter on the temperature profile is studied. It is observed that the temperature profile decreases with the magnetic parameter. And in Fig.3 we study the effect of the Brinkman number on the temperature profile it is observed that the increase on the Brinkman number leads to a commensurate increase in the temperature profile indicate the parameter serves as a heat source to the system.

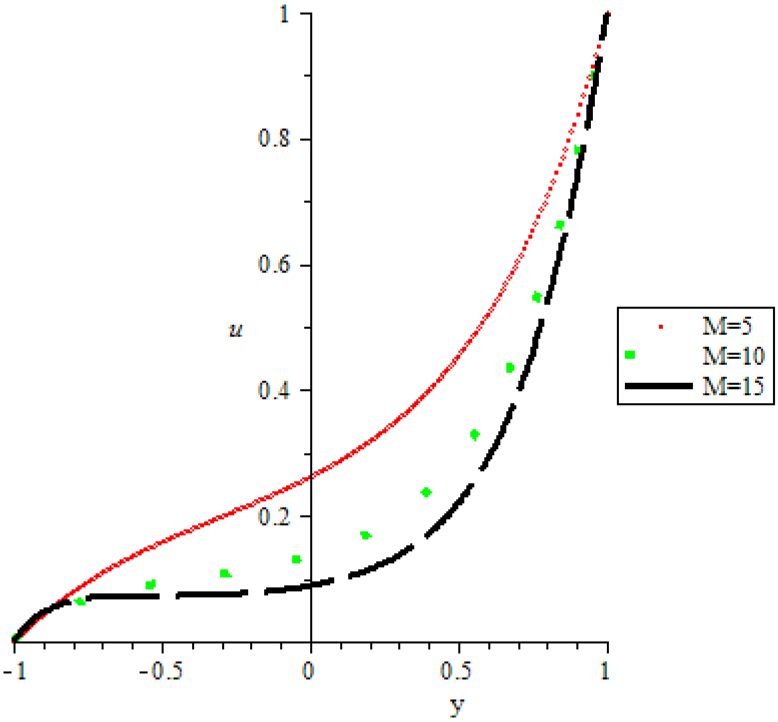

Figure.1: Effect of magnetic parameter M on velocity profile for k=1 and β=0.001

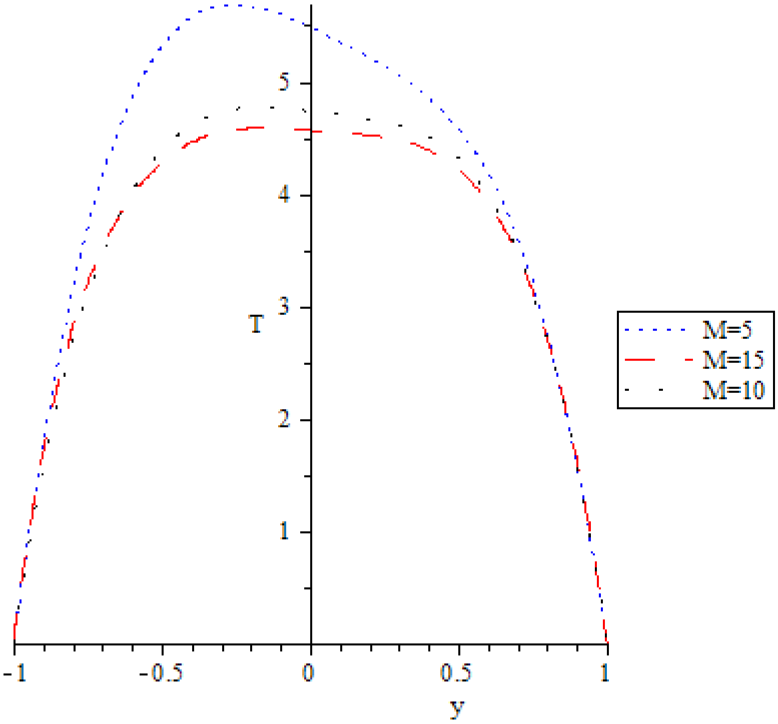

Fig.2 Effect of magnetic parameter on the temperature profile for $Br = 10,\ k = 1,\ \beta = 0001$

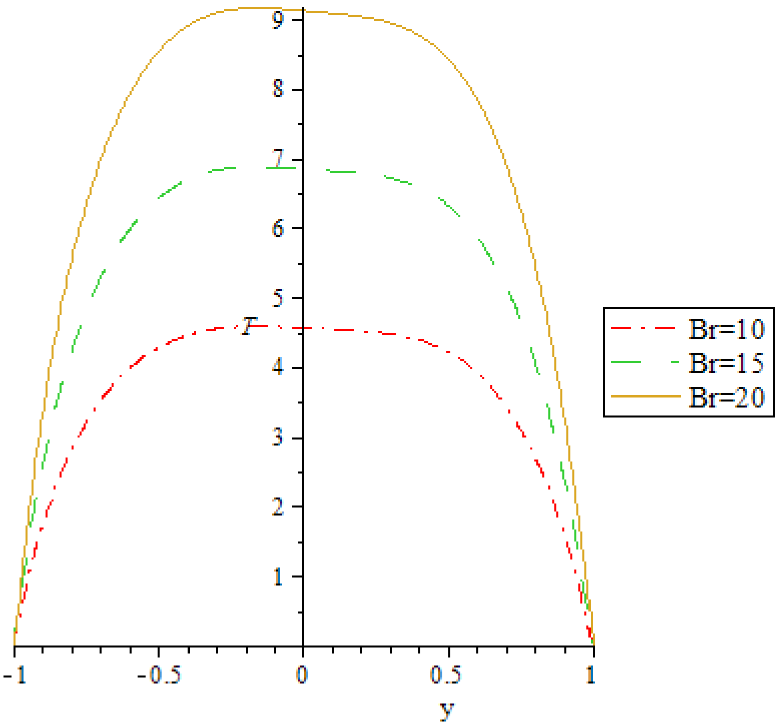

Fig.3 Effect of Brinkman number on the temperature profile for $M = 15,\ k = 1,\ \beta = 0.001$

4. Conclusion

In this research work we provide a modified Adomian decomposition analysis to the problem of flow and heat transfer in a channel with isothermal temperature. Effects of the constitutive parameters are effectively studied and the following conclusions are made:

(i) Increase in the magnetic field parameter leads to a decrease in the fluid velocity and a decrease in the temperature profile showing that the magnetic field opposes the fluid motion.

(ii) Increase in the Brinkman number leads to an increase in the temperature profile.

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