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Voltage Effects on Electric Power Systems Generators via Iterative Methods

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ABSTRACT

The determination or control of Voltages present at different points within the electric power system generators is very important for the smooth running of the power system operation. In this work, we examine the effect of high voltage on electric power generators with Y admittance bus matrix via iterative techniques, precisely Gauss- Seidel and Newton-Raphson method. Practical problems were solved for better understanding of the system. The results obtained shows that voltage had a great effect on the performance of the system as an increase in voltage increases the rate of convergence and make the slack bus to supply or generate more power than its limit capacity which may amount to breakdown of the system.

Keyword: Load Flow, Power Flow, voltage, phase angle, real power, reactive powers.

1. INTRODUCTION

Energy probably was the original creation because of its ability to produce a dynamic vital effect and its association with physical substance. Energy exists in various forms and one form can be converted into another by the use of suitable arrangement. Electrical energy, according to Rajput (2003), is preferred out of all forms of energy that we have. Electric power is the key energy source for industrial, commercial and domestic activities in the modern world. Its availability, in the right quantity is essential to the advancement of modern society because it serves as a fundamental resource for heating, cooling, and powering of machines and equipment (Aderinto and Bamigbola, 2010). Electric power system is a network of electrical components used to generate. Supply, transmit and consumption of real electric power.

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This is known as the Grid and is broadly divided into n generators that supply electric power, the transmission system that carries electric power from the generating centers to the load centers and the distribution system that feeds the electric power to nearby homes, commercial and industries. Electric power is the product of two quantities, namely current and voltage which can be varies with time (Olle, 1987).

The load flow, or power flow, computation is the most important network computation in Electric power systems. It calculates the voltage magnitude and angle in each bus of a power system, under specified system operation conditions. Other quantities, such as current values, power values, and power losses, can be calculated easily when the bus voltages are known. Load-flow study usually uses line notation and per-unit system, and focuses on various aspects of AC power parameters, such as voltages, voltage angles, real power and reactive power. Load flow computations bring insight in the steady-state behaviour of a power system. This is needed in many controls and planning applications (Grainger and Stevenson, 1994). The principal information obtained from the power flow study is the magnitude and phase angle of the voltage at each bus, and the real and reactive power flowing in each line.

Several researches have been carried out on load flow analysis. Aderinto and Bamigbola (2012) quantitatively studied electrical power generating system model via optimal control theory. Bouktir and Sliman (2005) studied optimal power flow of electrical network. Ian (2003) worked on power flow analysis using 3-bus electric power system as a case study. Beaty (2006) worked on 3-bus systems using both Gauss-Seidel and Newton Raphson methods. Adejumobi *et al*. (2013) solved non-linear algebraic power system model using iterative methods and Komolafe *et al*. (2009) worked on the anatomy of voltage collapse in the Nigeria power system. The goal of this work is to examine the effect of high voltage on electric power generators with bus admittance matrix, via iterative techniques. Precisely, Gauss-Seidel and Newton Raphson methods have been used in an attempt to prevent the generators from generating more power than its limit capacity, which may amount to breakdown of the system. Numerical problems are presented using 5-bus and 9-bus electric power systems.

2. Materials and Methods

Mathematical Formulation of the Power Flow Equations.

Bus power system is important in the formulation of power flow equation. For instance Figure 1 is a two bus electric power system connected by a transmission line and to each bus is associated with six electrical quantities (i.e, P_G , P_D , Q_G , Q_D , |V|, and δ) (Olle, 1987). A power-flow study (load-flow study) is an analysis of the voltages, currents, and power flows in a power system under steady-state conditions. In such a study, we make an assumption about either a voltage at a bus or the power being supplied to the bus for each bus in the power system and then determine the magnitude and phase angles of the bus voltages, line currents, etc. that would result from the assumed combination of voltages and power flows (Overbye *et al*., 1995; Weber *et al*., 1996).

Figure 1: Two-bus electric power system

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system. The node equation at bus i can be written as:

$$
I_i = \sum_{k=1}^{n} Y_{ik} V_k, \tag{2.1}
$$

where Y_{ik} 's are the elements of the bus admittance matrix, V_k are the bus voltages and Ii are the currents injected at each node

$$
S_i = V_i I_i^* = P_i + jQ_i, \tag{2.2}
$$

where S_i is the bus power.

Also, relationship between per-unit real and reactive power supplied to the system at bus i and the per unit current injected into the system at that bus can be express as:

$$
I_i^* = \frac{(P_i + jQ_i)}{V_i},
$$
\n(2.3)

$$
I_i = \frac{(P_i + jQ_i)}{V_i^{*}} \tag{2.4}
$$

$$
P_i - jQ_i = V_i^* \sum_{k=1}^n Y_{ik} \gamma_{ik} = \sum_{k=1}^n Y_{ik} V_k V_i^*.
$$
 (2.5)

Let $Y_{ik} \angle Y_{ik}$ and $V_i = |V_i| \angle \delta_i$,

then

$$
P_i - jQ_i = \sum_{k=1}^{n} |V_i| |V_k| |Y_{ik}| \angle (\gamma_{ik} - \delta_i + \delta_k).
$$
 (2.6)

Hence

$$
P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\cos(\gamma_{ik} - \delta_i + \delta_k)
$$
\n(2.7)

and

$$
Q_i = -\sum_{k=1}^n |V_i||V_k||Y_{ik}|\sin(\gamma_{ik} - \delta_i + \delta_k),\tag{2.8}
$$

where Y_{ik} is the Y- bus admittance matrix formulated as (denoting the element in row i,

column k, as Y_{ik})
\n
$$
\underline{Y_{ik}} = \begin{bmatrix} Y_{i1} & \cdots & Y_{in} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ Y_{k1} & \cdots & \cdots & Y_{kn} \end{bmatrix}, i = 1,...n, k = 1,...n
$$
\n(2.9)

where the terms Y_{ik} are not admittances but rather elements of the admittance matrix. Therefore, if $i = 4$ and $k=4$ equation. (2.9) becomes:

$$
\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} .
$$
 (2.10)

3. Results and Discussion

In this Section, we will present two problems by Ian (2003) and Beaty (2006), respectively:

Problem1

Using Gauss-Seidel method, solve the load flow problem in Figure 2.

Figure 2: 3-Bus system without reactive power

Solution:

$$
Y_{bus} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix},
$$

$$
V_i^{(k+1)} = \frac{1}{Y_{ii}} \begin{bmatrix} (P_i - jQ_i) \\ & \sqrt{\lambda_{k}(k)} - \sum_{j=1, j \neq i}^{n} Y_{ij} \frac{\lambda_{k}(k)}{V_j} \end{bmatrix}.
$$

All the buses will be considered as a PQ bus, the result is as in the Table 1.

Table 1: Results table

 $V_2^* = 1.0781$ p.u., $V_3^* = 0.9171$ p.u.

Power supply by swing bus: $S_1 = V_1^*(Y_{11}V_1^* + Y_{12}V_2^* + Y_{13}V_3^*) = 0.518$

Power loss on transmission line: $S_{loss} = S₁ + S₂ + S₃ = P_{loss}$

 $= 0.518 + 1.2041 - 1.4993 = 0.223$

Problem 2:

Solve the three-bus system power flow in Figure 3 by Newton-Raphson method. Given that:

 $V_1 = 1.0 \angle 0$ p.u., $|V_2| = 1.0$ p.u., $P_2 = 0.6$ p.u., $P_3 = -0.8$ p.u., $Q_3 = -0.6$ p.u.

Figure 3: A three-bus example power system

Solution:

Newton-Raphson method Approach

$$
Y_{BUS} = \begin{bmatrix} -j7 & j2 & j5 \\ j2 & -j6 & j4 \\ j5 & j4 & -j9 \end{bmatrix},
$$

\n
$$
P_i = \sum_{k=1}^n |V_i||V_k||Y_{ik}|\cos(\gamma_{ik} - \delta_i + \delta_k),
$$

\n
$$
Q_i = \sum_{k=1}^n |V_i||V_k||Y_{ik}|\sin(\gamma_{ik} - \delta_i + \delta_k).
$$

It converges at the $4th$ iteration:

$$
\begin{bmatrix} \delta_2^{(4)} \\ \delta_3^{(4)} \\ |V_3^{(4)}| \end{bmatrix} = \begin{bmatrix} 0.23219 \\ -0.28397 \\ 0.78611 \end{bmatrix}
$$

After accuracy have been achieved, we need to find Q_3, Q_1 and P_1 , obtained as Q_3 = 1.4617, P_1 = 2.1842, and Q_1 = 1.4085

Voltage Effects on Generators

In this Section, we intend to increase the voltage of the generator of the work done by Beaty (2006) by 10 percent. Finding approximations for the Real and Reactive Power that is given, using the assumed and given values for voltage/angles/admittance. Effect on the generator is examined.

Problem 3

Increase the generator voltage in problem 1 by 10% and solve the load flow problem.

Solution:

$$
Y_{bus} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix},
$$

$$
V_i^{(k+1)} = \frac{1}{Y_{ii}} \begin{bmatrix} (P_i - jQ_i) \\ V_i^{*(k)} - \sum_{j=1, j \neq i}^n Y_{ij} V_j^{(k)} \end{bmatrix}.
$$

Let assume a flat start of 1.0 p.u. If the voltage is increase by 10%, $V_1 = 1.1$, $V_2 = 1.1$, $V_3 =$ 1.1. All the buses will be considered as a PQ bus, the result is as in the Table 3.

Table 3: Results table

 $V_2^* = 1.1720p.u.$ and $V_3^* = 1.0263p.u.$

$$
S_{i} = V_{i}^{*} \sum_{k=1}^{n} Y_{ik} V_{k}^{*},
$$

\n
$$
S_{2} = V_{2}^{*} (Y_{21} V_{1}^{*} + Y_{22} V_{2}^{*} + Y_{23} V_{3}^{*}),
$$

\n
$$
= -0.0338,
$$

\n
$$
S_{3} = V_{3}^{*} (Y_{31} V_{1}^{*} + Y_{32} V_{2}^{*} + Y_{33} V_{3}^{*}),
$$

\n
$$
= -1.5012.
$$

Power mismatch at Bus 2:

$$
\Delta S_2 = S_2^{\text{cal}} - S_2^{\text{sch}},
$$

= -0.0338 - 1.2 = -1.2338,

$$
\Delta P_2 = P_2^{\text{cal}} - P_2^{\text{sch}},
$$

= -0.0338 - 1.2 = -1.2338,

$$
\Delta Q_2 = Q_2^{\text{cal}} - Q_2^{\text{sch}} = 0 - 0 = 0.
$$

Power mismatch at Bus 3:

$$
\Delta S_3 = S_3^{\text{ cal}} - S_3^{\text{ sch}},
$$

= -1.5012 - (-1.5) = 0.00012,

$$
\Delta P_3 = P_3^{\text{ cal}} - P_3^{\text{ sch}},
$$

= -1.5012 - (-1.5) = 0.00012,

$$
\Delta Q_3 = Q_3^{\text{ cal}} - Q_3^{\text{ sch}},
$$

$$
0 - 0 = 0.
$$

Power supply by swing bus:

$$
S_1 = V_1^*(Y_{11}V_1^* + Y_{12}V_2^* + Y_{13}V_3^*)
$$

$$
S_1 = 0.4849
$$

Power loss on transmission line:

$$
S_{loss} = S_1 + S_2 + S_3 = P_{loss}.
$$

= 0.4849 - 0.0338 - 1.5012 = -1.0501.

Problem 4:

Consider the three-bus system in problem 2, solve the load flow problem Given: $V_1 = 1.0 \times 0$ p.u., $|V_2| = 1.0$ p.u., $P_2 = 0.6$ p.u., $P_3 = -0.8$ p.u., $Q_3 = -0.6$ p.u. **Solution:**

$$
Y_{BUS} = \begin{bmatrix} -j7 & j2 & j5\\ j2 & -j6 & j4\\ j5 & j4 & -j9 \end{bmatrix}
$$

Initially, $V_1 = 1$ p.u. and $V_2 = 1$ p.u New voltage is; $V_1 = 1.1$ p.u. and $V_2 = 1.1$ p.u The mismatch matrix:

$$
P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\cos(\gamma_{ik} - \delta_i + \delta_k),
$$

\n
$$
Q_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\sin(\gamma_{ik} - \delta_i + \delta_k),
$$

\n
$$
P_2 = 0, P_3 = 0, Q_3 = 0,
$$

$$
\begin{bmatrix}\n\Delta P_2 \\
\Delta P_3 \\
\Delta Q_3\n\end{bmatrix} = \begin{bmatrix}\nP_2^{(sch)} \\
P_3^{(sch)} \\
Q_3^{(sch)}\n\end{bmatrix} - \begin{bmatrix}\nP_2^{(0)} \\
P_3^{(0)} \\
Q_3^{(0)}\n\end{bmatrix} = \begin{bmatrix}\n0.6 \\
-0.8 \\
-0.6\n\end{bmatrix} - \begin{bmatrix}\n0 \\
0 \\
0\n\end{bmatrix} = \begin{bmatrix}\n0.6 \\
-0.8 \\
-0.6\n\end{bmatrix}.
$$

The Jacobian matrix is:

$$
\begin{bmatrix}\n\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\
\frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\
\frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|}\n\end{bmatrix},
$$
\n
$$
J^{(0)} = \begin{bmatrix}\n6.82 & -4.4 & 0 \\
-4.4 & 10.45 & 0 \\
0 & 0 & 9.9\n\end{bmatrix},
$$
\n
$$
\Delta U^{(0)} = J^{(0)} \Delta X^{(0)},
$$

$$
\begin{bmatrix} 0.6 \\ -0.8 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 6.82 & -4.4 & 0 \\ -4.4 & 10.45 & 0 \\ 0 & 0 & 9.9 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(0)} \\ \Delta \delta_3^{(0)} \\ \Delta | V_2^{(0)} | \end{bmatrix},
$$

$$
\begin{bmatrix} \Delta \delta_2^{(0)} \\ \Delta \delta_3^{(0)} \\ \Delta | V_3^{(0)} | \end{bmatrix} = \begin{bmatrix} 0.052998 \\ -0.054249 \\ -0.060606 \end{bmatrix},
$$

$$
X^{(K+1)} = X^{(K)} + \Delta X^{(K)},
$$

$$
\begin{bmatrix} \delta_2^{(1)} \\ \delta_3^{(1)} \\ |V_3^{(1)}| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.052998 \\ -0.054249 \\ -0.060606 \end{bmatrix} = \begin{bmatrix} 0.052998 \\ -0.054249 \\ 0.9393 \end{bmatrix}.
$$

2 nd iteration:

$$
P_2^{(1)} = 0.009974,
$$
\n
$$
P_3^{(1)} = -0.012628,
$$
\n
$$
Q_3^{(1)} = -1.3585,
$$
\n
$$
\left[\begin{matrix}\Delta P_2\\ \Delta P_3\\ \Delta Q_3\end{matrix}\right] = \left[\begin{matrix}\nP_2^{(sch)}\\ P_3^{(sch)}\\ Q_3^{(sch)}\end{matrix}\right] - \left[\begin{matrix}\nP_2^{(1)}\\ P_3^{(1)}\\ Q_3^{(1)}\end{matrix}\right] = \left[\begin{matrix}\n0.6\\ -0.8\\ -0.012628\\ -1.3585\end{matrix}\right] = \left[\begin{matrix}\n0.590026\\ -0.787372\\ 0.758501\end{matrix}\right],
$$
\n
$$
J^{(1)} = \left[\begin{matrix}\n6.55291 & -4.13291 & 0.00823578\\ -4.13291 & 9.29906 & -0.00703294\\ 0.007736 & -0.0126279 & 7.00741\n\end{matrix}\right],
$$
\n
$$
\left[\begin{matrix}\n0.590026\\ -0.787372\\ 0.758501\n\end{matrix}\right] = \left[\begin{matrix}\n6.55291 & -4.13291 & 0.00823578\\ -4.13291 & 9.29906 & -0.00703294\\ 0.007736 & -0.0126279 & 7.00741\n\end{matrix}\right] \left[\begin{matrix}\n\Delta \delta_2^{(1)}\\ \Delta \delta_3^{(1)}\\ \Delta \delta_4^{(1)}\\ \Delta |\gamma_2^{(1)}|\\ \end{matrix}\right] = \left[\begin{matrix}\n0.05079\\ -0.06202\\ 0.10875\n\end{matrix}\right],
$$
\n
$$
\left[\begin{matrix}\n\delta_2^{(2)}\\ \delta_3^2\\ |\gamma_3^{(2)}\\ \end{matrix}\right] = \left[\begin{matrix}\n0.052998\\ -0.054249\\ 0.10875\n\end{matrix}\right] + \left[\begin{matrix}\n0.0507
$$

3 rd iteration:

$$
P_2^{(2)} = 0.0220945,
$$

$$
P_3^{(2)} = -0.0294093,
$$

$$
Q_3^{(2)} = -0.489969,
$$
\n
$$
\begin{bmatrix}\n\Delta P_2 \\
\Delta P_3 \\
\Delta Q_3\n\end{bmatrix} = \begin{bmatrix}\nP_2^{(sch)} \\
P_3^{(sch)}\n\end{bmatrix} - \begin{bmatrix}\nP_2^{(1)} \\
P_3^{(1)}\n\end{bmatrix} = \begin{bmatrix}\n0.6 \\
-0.8 \\
-0.6\n\end{bmatrix} - \begin{bmatrix}\n0.0220945 \\
-0.0294093 \\
-0.489969\n\end{bmatrix} = \begin{bmatrix}\n0.577906 \\
-0.770591 \\
-0.110031\n\end{bmatrix},
$$
\n
$$
J^{(2)} = \begin{bmatrix}\n7.03138 & -4.61139 & 0.0168989 \\
-4.61139 & 10.3756 & -0.0161012 \\
0.0177113 & -0.0294089 & 8.96494\n\end{bmatrix},
$$
\n
$$
\Delta U^{(2)} = J^{(2)}\Delta X^{(2)},
$$
\n
$$
\begin{bmatrix}\n0.577906 \\
-0.770591 \\
-0.110031\n\end{bmatrix} = \begin{bmatrix}\n7.03138 & -4.61139 & 0.0168989 \\
-4.61139 & 10.3756 & -0.0161012 \\
-0.01294089 & 8.96494\n\end{bmatrix} \begin{bmatrix}\n\Delta \delta_2^{(2)} \\
\Delta \delta_3^{(2)} \\
\Delta \delta_4^{(2)}\n\end{bmatrix},
$$
\n
$$
\begin{bmatrix}\n\Delta \delta_2^{(2)} \\
\Delta \delta_3^{(2)} \\
\Delta |\nu_2^{(2)}\n\end{bmatrix} = \begin{bmatrix}\n0.04728 \\
-0.01254\n\end{bmatrix},
$$
\n
$$
\begin{bmatrix}\n\delta_2^{(3)} \\
\delta_3^{(3)} \\
|\nu_3^{(3)}\n\end{bmatrix} = \begin{bmatrix}\n0.103788 \\
-0.116269 \\
1.04805\n\end{bmatrix} + \begin{bmatrix}\n0.047
$$

4 th iteration:

$$
P_2^{(3)} = 0.03187,
$$

\n
$$
P_3^{(3)} = -0.04233,
$$

\n
$$
Q_3^{(3)} = -0.60424,
$$

\n
$$
\left[\Delta P_2^{(3)}\right] = \left[\begin{matrix} P_2^{(sch)} \\ P_3^{(sch)} \end{matrix}\right] - \left[\begin{matrix} P_2^{(3)} \\ P_3^{(3)} \end{matrix}\right] = \left[\begin{matrix} 0.6 \\ -0.8 \\ -0.6 \end{matrix}\right] - \left[\begin{matrix} 0.03187 \\ -0.04233 \\ -0.60424 \end{matrix}\right] = \left[\begin{matrix} 0.56813 \\ -0.75767 \\ 0.00423 \end{matrix}\right],
$$

\n
$$
J^{(3)} = \left[\begin{matrix} 6.9745 & -4.5545 & 0.02462 \\ -4.5545 & 10.2477 & -0.02317 \\ 0.02549 & -0.04233 & 8.7324 \end{matrix}\right],
$$

\n
$$
\left[\begin{matrix} 0.56813 \\ -0.75767 \\ 0.00423 \end{matrix}\right] = \left[\begin{matrix} 6.9745 & -4.5545 & 0.02462 \\ -4.5545 & 10.2477 & -0.02317 \\ 0.02549 & -0.04233 & 8.7324 \end{matrix}\right] \left[\begin{matrix} \Delta \delta_2^{(3)} \\ \Delta \delta_3^{(3)} \\ \Delta |V_3^{(3)} \end{matrix}\right],
$$

\n
$$
\Delta U^{(3)} = J^{(3)}\Delta X^{(3)},
$$

$$
\begin{bmatrix}\n\Delta \delta_2^{(3)} \\
\Delta \delta_3^{(3)}\n\end{bmatrix} =\n\begin{bmatrix}\n0.04674 \\
-0.05316\n\end{bmatrix},
$$
\n
$$
X^{(K+1)} = X^{(K)} + \Delta X^{(K)},
$$
\n
$$
\begin{bmatrix}\n\delta_2^{(4)} \\
\delta_3^{(4)}\n\end{bmatrix} =\n\begin{bmatrix}\n0.151068 \\
-0.169549 \\
1.03513\n\end{bmatrix} +\n\begin{bmatrix}\n0.04674 \\
-0.05316\n\end{bmatrix},
$$
\n
$$
\begin{bmatrix}\n\delta_2^{(4)} \\
\delta_3^{(4)}\n\end{bmatrix} =\n\begin{bmatrix}\n0.1978 \\
-0.2227 \\
1.0343\n\end{bmatrix},
$$
\n
$$
P_1 = 2.9377, Q_1 = 6.7003.
$$

Discussion of the Results

From the results obtained from Problem 3, we observed that the active power was increased due to increase in voltage and cause the slack bus to supply more power than its capacity which causes overload of the system. Also, we observed from solution to Problem 4 that the power losses on the transmission line become a reactive power which may cause over heat of the system which may lead to fire outbreak. Hence, the breakdown of the power systems.

4. CONCLUSION

The research work reported the load flow problem using iterative methods, Newton-Raphson and Gauss-Seidel methods were considered. 3-bus electric power systems were used to check the effect of voltage on the electric power generator. And, from the result obtained, it was realized that voltage had a great effect on the performance of the electric power system as an increase in voltage increases the rate of convergence and make the slack bus to supply or generate more power than its limit capacity which may amount to breakdown of the system. Therefore, there is a need to monitor the voltage with time to avoid breakdown of the power systems.

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