Ilorin Journal of Science
Volume 2, Number 2, 2015, pp. 393–411 (Printed in Nigeria)
ISSN: 2408 – 4840 © 2015 Faculty of Physical Sciences, University of Ilorin
https://doi.org/10.54908/iljs.2015.02.02.010

ILORIN
JOURNAL OF SCEINCE

ILJS-15-061

The Effect of Serial Correlation in Estimating Dynamic Panel Data Models

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Abstract

There are several methods of estimating dynamic panel data models in the context of both micro-economic and macro-economic data. This paper investigates the performance of five different estimators of dynamic panel data models (the random effect model) when the disturbance term is serially correlated. A Monte Carlo experiment was conducted when individual, N is large and time dimension, T is finite and the error component model is assumed to be serially correlated. The bias and Root Mean Square Error criterion were used to access the performance of different estimators under consideration. We found that the Anderson-Hsiao using lagged differences as instrument (AH(d)) performs better when the time dimension is small (T=5), Anderson-Hsiao using lagged levels as instrument (AH(l)) performs better when T is moderate(T=10) and the first step Arellano-Bond estimator (ABGMM1) outperforms all other estimators when T increases to 20. For a dynamic panel data with large time dimension, we suggest that the first step Arellano-Bond Estimator (ABGMM1) Estimator is appropriate. The result shows that the bias of the first step Arellano-Bond estimator (ABGMM1) estimate is severe with small time dimension and the ordinary Least Square (OLS) and Least Square Dummy Variable (LSDV) are also bias when T is small. It was discovered that the effect of serial correlation is negligible irrespective of the order.

Keywords: Autocorrelation, Dynamic panel data, Econometric models, Generalized method of moment (GMM), Moving average.

1. Introduction

A panel data is a cross-section or group of people who are surveyed periodically over a given time span. Panel data models are used extensively both in micro and macro-economic empirical research. Dynamic models include a lagged dependent variable on the right-hand side of the equation. Application of dynamic panel data model is widely of interest in the field of science, economics and social sciences which includes Euler equations for household consumption, empirical model of economic growth etc. The dynamic specification has two basic problems associated with it; autocorrelation due to the presence of lagged dependent

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variable among the regressors and individual effects characterizing the heterogeneity among individuals (Baltagi, 2008). These problems lead to certain estimation issues which are dealt with by different estimation techniques. The discussion of dynamic panel data was opened by Balestra and Nerlove, 1966. In that paper, the authors proposed to estimate the model with unobserved component using the Generalized Least Squares (GLS) estimator.

However, GLS or ML-Random Effects (RE) estimators are not consistent if the unobserved individual effects are correlated with the exogenous variables. In the latter case the Fixed Effects (FE) specification is preferred. There are many studies on the properties of dynamic panel data estimators, most are geared towards the performance of the estimators using the conventional OLS, LSDV and some GMM estimators with micro-economic data sets with large cross-section but small time dimension this includes Arellano and Bond (1991), Kiviet, (1995), Judson and Owen (1996), Behr (2003), Haris and Matyas (2010), Flannery and Hankins (2013), Zhou and Alpert (2014) to mention but a few.

A number of works on the testing for serial correlation in the disturbances terms in dynamic panel data models are Baltagi and Li (1997), Hosung (2005), Hujer *et al*. (2005) using several test of AR(1) and MA(1). Similarly, among the notable works on the problem of serial correlation in panel data are Lillard and Willis (1978), Bhargava *et al*. (1982), Burke *et al*. (1990), Baltagi and Li (1991, 1994, 1995), Galbraith and Zinde-Wash (1992, 1995). The error component model was extended to take into account, first-order serial correlation in the remainder disturbances by Lillard and Willis (1978) for the random effects model and by Bhargava *et al*. (1982) for the fixed effects model. Both studies considered the first order Autoregressive (AR(1)) specification on the remainder disturbances. Nicholls *et al*. (1975) while considering first order moving average MA(1), find MA(1) as a viable alternative to AR(1). Baltagi and Li (1991) give a transformation which may be applied to certain autocorrelated disturbances in an error components model to yield spherical disturbances. They derive the transformations for first order Autoregressive AR(1) and second order Autoregressive [AR(2)] cases. The previous Monte Carlo studies have generally focused on panel model with fixed effect and do not allow the serial correlation of the disturbance term.

This study investigates the sensitivity of some dynamic panel data estimators in the presence of serial correlation. In this study, apart from allowing the disturbance term to be serially correlated for random individual effects; the explanatory variable is strictly exogenous. This study is not limited to a particular generating mechanism of the disturbance term rather it considered two different generating schemes namely: autoregressive and moving average processes of orders 1 and 2. In addition, the parameter values of the lagged dependent variables were varied to be mild, moderate and severe, also the values of parameters of the serial correlation (AR and MA) is assumes to take a low, moderate and value close to one. Monte Carlo experiments were performed to compare the relative efficiency of five alternative estimators, when the remainder disturbances are generated by different generating schemes. The estimators are Ordinary Least Squares (OLS), Least Square Dummy Variable (LSDV), Anderson-Hsiao estimator using lagged levels as instrument (AH(l)), Anderson-Hsiao estimator using lagged differences as instrument (AH(d)) and first step Arellano-Bond GMM estimator (ABGMM1).

2. Materials and Methods

2.1 The model

Dynamic Panel models

All panel data models are dynamic, in so far as they exploit the longitudinal nature of panel data. Dynamic models include a lagged dependent variable on the right-hand side of the equation. A widely used modeling approach is:

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + u_{it}\,; \quad i = 1, \ldots, N \;, \; t = 1, \ldots, T \;, \qquad (1)$$

with i denoting households, individuals, firms, countries, etc and t denoting time. The i subscript, therefore denotes the cross-section dimension whereas t denotes the time-series dimension. y_{it} is the dependent variable, $y_{i,t-1}$ is the lagged dependent variable , δ is a scalar, x'_{it} is the row vector of explanatory variable, dimension k , β is unknown parameter vector of k explanatory variables and u_{it} is the disturbance term. We assume that the u_{it} follow a one way error component model:

$$u_{it} = \mu_i + v_{it} \;, \qquad (2)$$

where μ_i denotes the unobserved individual specific effect and v_{it} denotes the remainder disturbance, $\mu_i \sim IID(0, \sigma^2_\mu)$ and $v_{it} \sim IID(0, \sigma^2_v)$ independent of each other and among themselves.

The fixed Effects Dynamic Panel Model

It is assumed that the variable of interest y_{it} is a linear function of the individual's previous realization of this variable, and of their contemporaneous personal characteristics x_{it} with unknown coefficient, δ and β, respectively:

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + \mu_i + v_{it}, \quad (3)$$

where: μ_i are the individual effects (constant for each i) and v_{it} are the usual white noise disturbance terms. In matrix form:

$$\underline{y}_{it} = D\underline{\mu}_i + \delta \underline{y}_{i,t-1} + X\beta + \underline{v}_{it}, \quad (4)$$

where $D = I_N \otimes \underline{l}_T$ *and* l_T *is the* $T \times 1$ *unit vector.* The usual method of estimating equation (4), i.e. when there is no lagged dependent variable, consists of estimating equation directly by OLS (the Least Squares Dummy Variable Estimator- LSDV), which also leads to the well-known within estimator. However, given the short time series component typical of panel data sets, the OLS and Within estimators are well known to be biased and inconsistent as $N \to \infty$ with finite T (see: Nickel, 1981; Sevestre and Trognon, 1985) for a theoretical approach and for a simulation based only (Nerlove, 1967 and 1971).

The Random Effects Dynamic Panel Model

Under the random effects specification, the μ_i terms of (3) are treated as independent random drawings from a particular distribution and the disturbance term becomes "composite", $u_{it} = \mu_i + v_{it}$. As with the fixed effects specification, the traditional estimators (Within and GLS) of the static random effects panel model are semi-inconsistent in the dynamic setting (Sevestre and Trognon, 1985). Again semi-consistent estimators for the dynamic random effects model rely on certain maintained hypothesis, which are violated by the inclusion of a lagged dependent variable. The assumptions concerning the equation's disturbances imply that variance-covariance matrix of the composite disturbance term will be

$$\Omega_v = V(v) = I_N \otimes E(vv') = I_N \otimes \sum\nolimits_v , \quad (5)$$

$$\sum\nolimits_v = \sigma_\mu^2 J_T + \sigma_u^2 I_T = \sigma_v^2 \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{bmatrix},$$

where ρ is the intra-class correlation coefficient and $\rho = \frac{\sigma_\mu^2}{(\sigma_\mu^2 + \sigma_u^2)}$. For the research, we assume a random effect of the dynamic panel data model.

2.2. Methodology

Here is the brief discussion on the estimators consider in the work.

Ordinary Least Square (OLS) Estimator

In the static case in which all the explanatory variables are exogenous and are uncorrelated with the effects, we can ignore the error-component structure and apply the OLS method. The OLS estimator, although less efficient, is still unbiased and consistent. But this is no longer true for dynamic error-component models. The correlation between the lagged dependent variable and individual-specific effects would seriously bias the OLS estimator.

OLS, the simplest of all estimators considered, is applied to the equation in the level form. Since the initial values of y_{it} are known, OLS can use in actual estimation all of the cross-sections.

The OLS estimator is given as:

$$\hat{\delta}^{OLS} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} y_{it}.y_{i,t-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T} y_{i,t-1}^{2}} = \delta + \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(\alpha_i + u_{it})y_{i.t-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T} y_{i,t-1}^{2}} \qquad (6)$$

Least Square Dummy Variable (LSDV)

Consider now the *least squares dummy variable* (LSDV) estimator, also known as the *fixed-effects* or *within-group* estimator. We assume that the explanatory variables in x_{it} are strictly exogenous. Estimates of $(\delta \; and \; \beta)$ are obtained by applying OLS to the model expressed in deviations from time means:

$$y_{it} - \bar{y}_i = \delta(y_{it-1} - \bar{y}_{i-1}) + (x'_{it} - \bar{x}_{i,t-1})\beta + (v_{it} - \bar{v}_i), \qquad t \in \{1, \cdots, T)$$

where $\bar{y}_i = \sum_{i=1}^{T} y_{it}/T$, $\bar{y}_{i,-1} = \sum_{i=1}^{T} y_{i,t-1}/T$, and $u_{it} = \sum_{i=1}^{T} u_{it}/T$. This transformation wipes out the unobserved individual effects, eliminating one possible source of inconsistency.

The LSDV estimators for δ is

$$\hat{\delta}^{LSDV} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - \bar{y}_i)(y_{i,t-1} - \bar{y}_{i.-1})}{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t-1} - \bar{y}_{i.-1})^2}$$

$$= \delta + \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t-1} - \bar{y}_{i.-1})(u_{it} - \bar{u}_i)\Big/NT}{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t-1} - \bar{y}_{i.-1})^2\Big/NT} \qquad (7)$$

The Anderson-Hsiao estimator

Anderson and Hsiao (1981) proposed an instrumental-variable (IV) estimator that is consistent for fixed T and $N \to\infty$. The estimator suggested by Anderson and Hsiao (1981) is based on the differenced form of the original equation (3)

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + v_{it} - v_{i,t-1} \qquad (8)$$

which cancels the individual fixed effects assumed to possibly correlate with the exogenous variables $(E(X'_{it}\mu_i \neq 0)$. When the dimension of the panel is $N \times T$, the Anderson-Hsiao we employ is

$$\hat{\gamma}^{AH} = (Z'X)^{-1} Z'Y \cdot \qquad (9)$$

We add the symbol L or D to indicate the use of levels or differences as instruments $(\hat{\gamma}^{AH,L}, \hat{\gamma}^{AH,D})$.

The Arellano-Bond estimator

The AH estimator is consistent but not efficient because it does not use all the available moment conditions. Arellano and Bond (1991) proposed a generalized method of moments (GMM) estimator that also relies on first-differencing the model. The estimator is similar to the estimator suggested by Anderson and Hsiao but exploits additional moment restrictions, which enlarges the set of instruments.

The dynamic equation to be estimated in levels is

$$y_{it} = \delta y_{i,t-1} + X'_{it}\beta + \mu_i + v_{it} \qquad (10)$$

where differencing eliminates the individual effects μ_i:

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + v_{it} - v_{i,t-1} .$$

For each year, we now look for the instruments available for instrumenting the difference equation. For $t=3$ the equation to be estimated is

$$y_{i3} - y_{i,2} = \delta(y_{i2} - y_{i1}) + (x'_{i3} - x'_{i,2})\beta + v_{i3} - v_{i2},$$

where the instruments $y_{i,1}$, x'_{i2} *and* x'_{i1} are available. Because the differencing operation introduces first order autocorrelation into the error term, the first-step estimator makes use of a covariance matrix taking this autocorrelation into account.

$$V = W'GW = \sum_{i=1}^{N} W'G_T W_i .$$

where $G = (I_N \otimes G_T')$ *and* $G_T = F_T F_T' = \begin{bmatrix} 2 & -1 & & 0 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{bmatrix}.$

The two-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by White (1980):

$$\hat{V} = \sum_{i=1}^{N} W' F_T \hat{v}_i \hat{v}_i' F_T' W_i \ .$$

The resulting estimator finally is

$$\hat{\gamma}^{ABGMM} = (XW\hat{V}^{-1}W'X)^{-1} X'W\hat{V}^{-1}W'y \,. \qquad (11)$$

2.3. Monte Carlo study

We study different estimators in the Monte Carlo experiment, the Ordinary Least Square (OLS), Least Square Dummy variable(LSDV), Anderson and Hsiao using lagged levels as instrument (AH(l), Anderson-Hsiao using differences as instrument and first step Arellano – Bond GMM (ABGMM1) and compare them under different circumstances. The data generating process closely follows Nerlove (1971). The simulation is based on the following model: $y_{it} = \delta y_{i,t-1} + X_{it}'\beta + u_{it}$, $X_{it} = \lambda x_{i,t-1} + \varepsilon_{it}$, where ε_{it} is uniformly distributed on the interval $(-0.5, 0.5)$. For the random effect specification we generate $u_{it} = \mu_i + v_{it}$ where $\mu_i \sim N(0, 1)$ and classical error term v_{it} is generated either by:

AR(1) process: $v_{it} = \rho v_{i,t-1} + \omega_{it}$, with $\omega_{it} \sim IIN(0, \sigma_\omega^2)$,

AR(2) process: $v_{it} = \rho_1 v_{i,t-1} + \rho_2 v_{i,t-2} + \omega_{it}$, with $\omega_{it} \sim IIN(0, \sigma_\omega^2)$,

MA(1) process: $v_{it} = \theta v_{i,t-1} + \omega_{it}$, with $\omega_{it} \sim IIN(0, \sigma_\omega^2)$ or

MA(2) process: $v_{it} = \theta_1 \omega_{i,t-1} + \theta_2 \omega_{i,t-2} + \omega_{it}$, with $\omega_{it} \sim IIN(0, \sigma_\omega^2)$,

where σ_ω^2 is normalized to 1.

The value of the serial correlation parameters ρ *and* θ are varied as 0.2, 0.5, *and* 0.8, the autoregressive coefficients, δ and λ alternates between 0.1, 0.5 and 0.9, $\beta = 1$ and $\gamma = (\delta, \beta)'$. In the experiment, we consider N=50, 100 and T=5, 10, 20. 500 replications are performed since GMM estimator is quite computationally intensive and time consuming. We examine the bias of different estimators under consideration to determine how their magnitudes vary with the characteristics of the dataset. Also, The Root Mean Square Error (RMSE) criterion is used to assess the efficiency of the estimators.

3. Results and Discussion

Tables 1 and 2 present the simulation results of bias and RMSE for estimate of the autoregressive coefficient, δ and the coefficient of explanatory variable, β respectively for N=50, 100 and T= 5, 10, 20. Table 3 reports results of bias and RMSE for AH(d) estimator when it follows different error component processes. Tables 4-6 in the Appendix present the bias and RMSE of the parameter of lagged dependent variable and the variable of autoregressive parameter of the explanatory variable of all possible combinations of N (50, 100) and T (5, 10, 20) when λ takes the values of 0.1, 0.5 and 0.9 for only AR(1) and MA(1) to save space. Some of the simulation results in this study are presented in the Appendix.

The results in Table 1 indicate that AH(d) outperforms other methods of estimation when T=5 and N=50 with minimum RMSE of 0.0596 on average while the ABGMM1 estimator performs worst in term of producing higher bias (in absolute) and RMSE. It is noted that OLS, LSDV and ABGMM1 estimators have a negative bias. For T=10 and N=50, AH(l) performs best with minimum RMSE of 0.04212 followed by LSDV while ABGMM1 still performs worst but the estimator seem to show serious improvement (larger percentage reduction in average RMSE and bias as T increases). When T=20, ABGMM1 shows a drastic improvement as it outperforms other estimators with minimum RMSE of 0.0208 on average, it is followed closely by AH(l), though it does not produce a superior estimate in terms of average bias.

Considering N=100 results in Table, it shows that AH(l) , LSDV and AH(d) estimators have a better performance for T=5, 10 and 20 with average RMSE of 0.0445, 0.0280 and 0.0201, respectively . Here the bias of LSDV, AH(l) and ABGMM1 are in most cases negative. It is observed that RMSE and bias (in absolute magnitude) of all the estimators decreases with T and N except ABGMM1 that behave interchangeably for small N. Regarding the estimate of β as shown in Table 2, for N=50 AH(l) perform better when T=5, ABGMM(l) has better performance when T=10 and 20 with minimum RMSE of 0.1353, 0.0665 and 0.0386, respectively while when N=100, OLS outperforms other estimators when T=5 and ABGMM1 performs better when T=10 and 20. The bias of AH(l) and AH(d) improve as T increases while the biases of other estimators perform interchangeably.

When the simulation design follows AR(1) and AR(2) processes, the bias of nearly all the estimators are negative except LSDV that is positive. The LSDV and AH(l) estimators are practically unbiased in average with 50 individuals. It was observed from our simulation that using ABGMM1 estimator with small instruments produces a smaller expected bias in most cases, but using the full set of instruments almost increases the efficiency of the estimate (Judson and Owen, 1996). Here, the LSDV have the least performance with a small reduction in terms of RMSE and bias. As the time period T increases, AH(l) performs equally well . It was also noted that AH (l) and AH(d) estimates improve in performance as ρ increases. For ABGMM1 estimator, it deteriorates in performance as the value of the parameter of serial correlation increases. The performance is the same for OLS and LSDV irrespective of the process of serial correlation of v_{it} given that the two estimators ignore the serial correlation in the remainder term.

Table 1: The RMSE and Bias of the estimate of δ at δ=0.5, ρ=θ=0.5, λ=0.5. True model is AR(1), AR(2), MA(1) and MA(2)

		OLS		LSDV		AH(l)		AH(d)		ABGMM1	
N	**T**	**RMSE**	**Bias**	**RMSE**	**bias**	**RMSE**	**Bias**	**RMSE**	**Bias**	**RMSE**	**Bias**
					AR(1)						
50	5	0.1087	-0.0076	0.06154	-0.00497	0.06271	0.0074	0.0596	0.005858	2.0668	-1.24
	10	0.0725	0.01116	0.04622	0.000344	0.04212	-0.0056	0.0475	-0.00492	0.0735	0.0089
	20	0.0501	0.0007	0.03333	0.005025	0.02978	-0.0037	0.035	-0.00334	0.0228	0.0184
100	5	0.0665	0.00118	0.04806	-1.97E-05	0.04453	-0.0058	0.0484	-0.00504	2.0763	1.4222
	10	0.0475	0.00423	0.02802	-0.00167	0.02821	-0.0013	0.0349	0.003125	0.0529	-0.002
	20	0.0332	0.00469	0.02182	-0.00147	0.02431	-0.0015	0.0201	0.003785	0.0298	-0.014
					AR(2)						
50	5	0.1087	-0.0076	0.06154	-0.00497	0.06261	0.00743	0.0596	0.005931	1.9607	-1.301
	10	0.0725	0.01116	0.04622	0.000344	0.04212	-0.0056	0.0475	-0.00492	0.0735	0.0089
	20	0.0501	0.0007	0.03283	0.004264	0.03133	-0.0041	0.0334	-0.00268	0.0235	0.0175
100	5	0.0665	0.00118	0.04806	-1.98E-05	0.04454	-0.0058	0.0484	-0.00502	2.1116	1.497
	10	0.0475	0.00423	0.02802	-0.00167	0.02821	-0.0013	0.0349	0.003124	0.0529	-0.002
	20	0.0332	0.00469	0.0245	0.003367	0.02047	0.0017	0.0222	-7.06E-05	0.0294	0.0003
					MA(1)						
50	5	0.1087	-0.0076	0.06154	-0.00497	0.06269	0.00745	0.0596	0.005909	1.5487	-0.472
	10	0.0725	0.01116	0.04622	0.000345	0.04211	-0.0056	0.0475	-0.00492	0.0735	0.0089
	20	0.0501	0.0007	0.03362	0.002711	0.03242	0.00103	0.0312	-0.00523	0.0203	0.0174
100	5	0.0671	-0.0005	0.05157	-9.89E-06	0.04072	-0.0061	0.0466	-0.00437	2.9645	1.4086
	10	0.0475	0.00423	0.02802	-0.00167	0.02821	-0.0013	0.0349	0.003126	0.0529	-0.002
	20	0.0332	0.00469	0.02312	0.001635	0.02164	0.00206	0.0221	0.001254	0.0294	0.0003
					MA(2)						
50	5	0.1087	-0.0076	0.06154	-0.00497	0.06265	0.00744	0.0596	0.005922	1.526	-0.41
	10	0.0484	0.01116	0.04623	0.000345	0.04213	-0.0056	0.0475	-0.00492	0.074	0.0089
	20	0.0501	0.0007	0.03307	0.002325	0.03301	-0.0002	0.0308	-0.005	0.0165	0.0165
100	5	0.0735	-0.0033	0.04806	-2.61E-06	0.04454	-0.0058	0.0484	-0.005	2.0369	1.3823
	10	0.0475	0.00423	0.02802	-0.00167	0.02821	-0.0013	0.0349	0.003125	0.0529	-0.002
	20	0.0332	0.00469	0.02258	0.002338	0.01941	0.00015	0.0235	-6.60E-05	0.0294	0.0003

Source: Compiled by the authors

The performance in the AR(1) process is similar to the AR(2) process (see table 1), but there is slight improvement in the performance of ABGMM1 When the serial correlation is of the higher order, that is, AR(2) compared with AR(1) in term of RMSE but the bias of the estimates in AR(1) is more than that of AR(2). AH(d) still perform better when T=5 and AH(l) perform better at T=10 and 20. As T increases ABGMM1 improves in performance in both RMSE and bias. From the simulation results, we observed that the ABGMM1 performs worst when time period, T is small, and its performance improves as T increases.

Table 3 is the simulation results of bias and RMSE of the estimate of δ for AH(d) estimator when the serial correlation follows AR(1), AR(2), MA(1) or MA(2) process. The results at different scenario show that the autoregressive of order 1 (AR(1)) is better than AR(2) though their differences are minimal. When following the moving average process, MA(2) is better than MA(1). Here, AH(d) estimator improve in performance as the serial correlation coefficients (ρ or θ) and time periods T increases.

The results in Tables 4-6 shows that for OLS estimator, as value of λ increases the bias and RMSE deteriorates but for other estimators considered, it improves as λ increases. It was also observed that the ABGMM1 has a larger bias and RMSE when the value of $\lambda = 0.1$ compared to when it is 0.5 or 0.9 especially when the time dimension, T is small. Similar results were obtained irrespective of the number of individual and pattern of serial correlation process. All the estimators improve in performance as the sample size increases and confirm the asymptotic properties.

Table 2: The RMSE and Bias of estimate of β at δ=0.5, ρ=θ=0.5, λ=0.5. True model is AR(1), AR(2), MA(1) and MA(2)

N	T	OLS		LSDV		AH(l)		AH(d)		ABGMM1	
		RMSE	**Bias**	**RMSE**	**Bias**	**RMSE**	**Bias**	**RMSE**	**Bias**	**RMSE**	**bias**
					AR(1)						
50	5	0.1581	0.01196	0.14358	-0.0012	0.13533	0.000699	0.1416	-0.01489	1.2154	-0.3514
	10	0.1094	-0.0167	0.109	0.000346	0.1074	-0.00394	0.1123	-0.01855	0.0665	-0.0059
	20	0.0737	-0.0002	0.0772	0.00911	0.0747	-0.01059	0.0729	-0.0114	0.0386	-0.0356
100	5	0.1021	0.00279	0.10626	-0.00482	0.11257	0.006348	0.1094	-0.01744	1.2251	0.77268
	10	0.0777	-0.0077	0.07379	0.007552	0.06756	-0.00305	0.0721	-0.00613	0.0537	-0.0017
	20	0.0564	-0.005	0.04806	-0.00055	0.05061	0.001552	0.0486	-0.00486	0.0247	0.00275
					AR(2)						
50	5	0.1581	0.01196	0.14483	-0.00024	0.13251	0.001392	0.1377	-0.01886	1.1274	-0.4072
	10	0.1094	-0.0167	0.10856	9.15E-05	0.10814	-0.00344	0.1128	-0.0189	0.0665	-0.0059
	20	0.0737	-0.0002	0.07972	0.000262	0.07545	0.010611	0.0719	-0.01382	0.0315	0.02671
100	5	0.1021	0.00279	0.10679	-0.0045	0.10935	0.005006	0.106	-0.01677	1.2378	0.81665
	10	0.0777	-0.0077	0.07376	0.007474	0.06699	-0.00261	0.0721	-0.00654	0.0537	-0.0017
	20	0.0564	-0.005	0.05569	0.002556	0.05127	-0.00521	0.0537	0.001167	0.029	-0.0037
					MA(1)						
50	5	0.1581	0.01196	0.14466	0.000441	0.13369	3.18E-05	0.1395	-0.01771	1.0916	0.09636
	10	0.1094	-0.0167	0.10933	0.000569	0.10694	-0.00417	0.1119	-0.01829	0.0665	-0.0059
	20	0.0737	-0.0002	0.07826	0.004367	0.07874	0.010448	0.0676	-0.00635	0.0426	0.04069
100	5	0.1064	0.0059	0.12259	0.011192	0.12768	-0.00829	0.108	-0.00421	2.038	1.26857
	10	0.0777	-0.0077	0.07381	0.007597	0.06781	-0.00325	0.0721	-0.00598	0.0537	-0.0017
	20	0.0564	-0.005	0.05328	0.003201	0.05322	-0.00372	0.0538	0.000601	0.029	-0.0037
					MA(2)						
50	5	0.1581	0.01196	0.14488	0.000571	0.13322	0.000704	0.1378	-0.01839	1.1013	0.11934
	10	0.1094	-0.0167	0.10945	0.000634	0.10689	-0.00415	0.1119	-0.01824	0.0665	-0.0059
	20	0.0737	-0.0002	0.07727	0.005254	0.07903	0.011376	0.0679	-0.0078	0.0425	0.04115
100	5	0.117	0.0117	0.12507	0.012117	0.12534	0.005075	0.1222	-0.01437	1.1991	0.74564
	10	0.0777	-0.0077	0.0738	0.007597	0.0677	-0.00327	0.0721	-0.00609	0.0537	-0.0017
	20	0.0564	-0.005	0.05473	0.002268	0.04708	0.001453	0.0589	0.00367	0.029	-0.0037

Source: Compiled by the authors

Table 3: AH(d) RMSE and Bias of estimate with respect to δ at N=50, λ=0.1. [AR(1), AR(2), MA(1) and MA(2) errors]

			AR(1)		AR(2)		MA(1)		MA(2)	
T	δ	ρ\θ	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias
5	0.1	0.2	0.059643	0.005909	0.059653	0.005938	0.059646	0.005915	0.059642	0.005919
		0.5	0.059635	0.005901	0.05965	0.005937	0.059645	0.005916	0.059641	0.00592
		0.8	0.059628	0.005894	0.059647	0.005934	0.059643	0.005918	0.05964	0.005921
	0.5	0.2	0.059643	0.005909	0.059654	0.005938	0.059646	0.005915	0.05964	0.005921
		0.5	0.059635	0.005901	0.05965	0.005937	0.059645	0.005916	0.059641	0.00592
		0.8	0.059628	0.005894	0.059647	0.005934	0.059643	0.005918	0.05964	0.005921
	0.9	0.2	0.059643	0.005909	0.059654	0.005938	0.059646	0.005915	0.059642	0.005919
		0.5	0.059635	0.005901	0.05965	0.005937	0.059645	0.005916	0.059641	0.00592
		0.8	0.059628	0.005894	0.059647	0.005934	0.059643	0.005918	0.05964	0.005921
10	0.1	0.2	0.047527	-0.00491	0.047531	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.5	0.047528	-0.00491	0.047532	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.8	0.04753	-0.00491	0.047533	-0.00491	0.047525	-0.00491	0.047525	-0.00491
	0.5	0.2	0.047527	-0.00491	0.047531	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.5	0.047528	-0.00491	0.047532	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.8	0.04753	-0.00491	0.047533	-0.00491	0.047525	-0.00491	0.047525	-0.00491
	0.9	0.2	0.047527	-0.00491	0.047531	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.5	0.047528	-0.00491	0.047532	-0.00491	0.047526	-0.00491	0.047525	-0.00491
		0.8	0.04753	-0.00491	0.047533	-0.00491	0.047525	-0.00491	0.047525	-0.00491
20	0.1	0.2	0.03293	-0.00313	0.034352	-0.00368	0.035201	-0.00045	0.033778	-0.00328
		0.5	0.032804	-0.00228	0.028295	-0.0017	0.035493	-0.00389	0.029772	-0.00322
		0.8	0.031064	-0.00366	0.032804	-0.00228	0.033616	-0.00201	0.033387	-0.00268
	0.5	0.2	0.03499	-0.00339	0.035024	-0.00403	0.033547	-0.00379	0.031766	-0.00137
		0.5	0.033387	-0.00268	0.03375	-0.00318	0.033547	-0.00379	0.030612	-0.00403
		0.8	0.033547	-0.00379	0.033617	-0.00201	0.033616	-0.00201	0.030612	-0.00403
	0.9	0.2	0.030843	-0.00501	0.035024	-0.00403	0.030612	-0.00403	0.029247	-0.00747
		0.5	0.030739	-0.0043	0.035341	-0.00352	0.031765	-0.0023	0.028563	-0.00694
		0.8	0.030613	-0.00403	0.035497	-0.00344	0.030612	-0.00403	0.033546	-0.00379

Source: Compiled by the authors

4. Conclusion

This study deals with serial correlation disturbances in the context of dynamic panel data model. This is different from the previous econometric literature wish ignore the serial correlation. The results of the Monte Carlo experiment show that AH(d) outperforms other estimator when T is small (T=5), AH(l) is better when T is moderate(T=10) and ABGMM1 perform better when T is getting larger (T=20) at various level of serial correlation under consideration . This indicates that the nature of the data determines the appropriate estimator in the dynamic panel data model that is serially correlated. Also, it was observed that as T increases, there is an improvement in the performance of ABGMM1 due to the increase in the instruments; this implies that ABGMM1 will be better when T is large. The bias and RMSE of OLS and LSDV are similar at various level of T even when the autoregressive and moving average parameters δ and θ were varied.

The bias of most of the estimators reduces as the value of T increases especially the ABGMM1 estimator. The effect of making the serial correlation, v_{it} to follow AR(1), AR(2), MA(1) or MA(2) are negligible in the performance of the estimators. Also, the result revealed that the bias and RMSE of OLS deteriorates as the value of λ increases while other estimators improve with increase in the value of λ. The GMM estimator proposed by Arellano-Bond (1991) has a larger bias and RMSE when the value of the autoregressive parameter of exogenous variable, λ is mild and when the time dimension is small (i.e. T=5). It was noted that as the sample sizes increase, the performance of all the estimators improve when the error term is assumed to be serially correlated.

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Appendix

Table 4: The RMSE and Bias of estimate of δ when δ =0.9, ρ=0.8. True Model is AR(1)

	AR(1)		OLS		LSDV		AH(l)		AH(d)		ABGMM1	
N	T	λ	RMSE	Bias	RMSE	Bias	RMSE	bias	RMSE	bias	RMSE	Bias
50	5	0.1	0.0734	-0.0031	0.0616	-0.004968	0.0627	0.0074	0.05963	0.00589	11.4453	-6.4576
		0.5	0.1087	-0.0076	0.0615	-0.004973	0.0627	0.00738	0.05957	0.00584	2.313	-1.4754
		0.9	0.1174	-0.0084	0.0615	-0.004973	0.0627	0.007385	0.05956	0.00581	1.28376	-0.8805
	10	0.1	0.0484	0.00441	0.0462	0.000345	0.0421	-0.00559	0.04753	-0.0049	0.07399	0.00784
		0.5	0.0725	0.01116	0.0462	0.000344	0.0421	-0.00561	0.04752	-0.0049	0.07349	0.00894
		0.9	0.0835	0.01252	0.0462	0.000343	0.0421	-0.00561	0.04753	-0.0049	0.07193	0.00967
	20	0.1	0.0326	0.0016	0.0336	0.002047	0.0321	-0.00096	0.03061	-0.004	0.02414	0.01984
		0.5	0.0501	0.0007	0.0336	0.003403	0.0329	0.000196	0.02932	-0.006	0.02289	0.01829
		0.9	0.0578	-0.0014	0.0327	0.001974	0.0318	-0.00064	0.03105	-0.0034	0.0203	0.01735
100	5	0.1	0.045	0.00216	0.0481	-1.43E-05	0.0445	-0.00574	0.0484	-0.00506	9.94916	6.59152
		0.5	0.0665	0.00118	0.0481	-1.97E-05	0.0445	-0.00575	0.0484	-0.005	2.07666	1.42704
		0.9	0.0783	0.00108	0.0481	-2.19E-05	0.0445	-0.00576	0.04839	-0.005	1.19475	0.84621
	10	0.1	0.0324	0.00068	0.028	-0.001669	0.0282	-0.0013	0.03487	0.00311	0.05488	-0.0028
		0.5	0.0475	0.00423	0.028	-0.001668	0.0282	-0.0013	0.03486	0.00312	0.05292	-0.002
		0.9	0.0543	0.00683	0.028	-0.001664	0.0282	-0.0013	0.03486	0.00313	0.05049	-0.0012
		0.5	0.0332	0.00469	0.0226	-0.000193	0.0235	-0.00019	0.02017	0.00328	0.02955	-0.0039
		0.9	0.0396	0.00447	0.0225	0.001528	0.021	0.001528	0.02224	-0.0019	0.02833	0.00044

Source: Compiled by the authors

Table 5: The RMSE and Bias of estimate with respect to δ when δ =0.9, θ=0.8. True Model is MA(1)

	MA(1)		OLS		LSDV		AH(l)		AH(d)		ABGMM1	
N	T	Λ	RMSE	Bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	Bias
50	5	0.1	0.0734	-0.0031	0.0616	-0.005	0.0627	0.007464	0.05964	0.00592	8.03205	-1.9221
		0.5	0.1087	-0.0076	0.0615	-0.005	0.0627	0.007443	0.05962	0.00591	1.53896	-0.4598
		0.9	0.1174	-0.0084	0.0615	-0.005	0.0627	0.007437	0.05961	0.00589	0.82317	-0.2848
	10	0.1	0.0484	0.00441	0.0462	0.00035	0.0421	-0.00559	0.04753	-0.0049	0.07399	0.00784
		0.5	0.0725	0.01116	0.0462	0.00035	0.0421	-0.00561	0.04752	-0.0049	0.07349	0.00894
		0.9	0.0835	0.01252	0.0462	0.00034	0.0421	-0.00561	0.04752	-0.0049	0.07193	0.00967
	20	0.1	0.0326	0.0016	0.0336	0.00205	0.0321	-0.00096	0.03061	-0.004	0.02028	0.0155
		0.5	0.0501	0.0007	0.0329	0.00433	0.031	-0.00349	0.03375	-0.0032	0.02063	0.01587
		0.9	0.0578	-0.0014	0.0329	0.00304	0.0317	-0.0027	0.03175	-0.0023	0.0216	0.01697
100	5	0.1	0.0433	0.00136	0.0516	3.73E-07	0.0407	-0.00609	0.04669	-0.0044	17.5865	11.5782
		0.5	0.0671	-0.0005	0.0516	-9.13E-06	0.0407	-0.0061	0.04662	-0.0044	3.01919	1.51332
		0.9	0.0833	-0.0027	0.0481	-9.03E-06	0.0445	-0.00577	0.04839	-0.005	1.19705	0.872
	10	0.1	0.0324	0.00068	0.028	-0.0017	0.0282	-0.0013	0.03487	0.00311	0.05488	-0.0028
		0.5	0.0475	0.00423	0.028	-0.0017	0.0282	-0.00131	0.03486	0.00313	0.05292	-0.002
		0.9	0.0543	0.00683	0.028	-0.0017	0.0282	-0.00131	0.03486	0.00313	0.05049	-0.0012
	20	0.1	0.022	0.00301	0.0241	0.00289	0.021	0.002734	0.02264	-0.0017	0.03009	9.99E-05
		0.5	0.0332	0.00469	0.0234	0.00234	0.0202	0.003277	0.02233	-0.0008	0.02938	0.00026
		0.9	0.0396	0.00447	0.0231	0.00163	0.0203	-1.93E-05	0.022	-0.0003	0.02833	0.00044

Source: Compiled by the authors

Table 6: The RMSE and Bias of estimate of β when δ =0.9, ρ=0.8. True Model is AR(1)

	AR(1)		OLS		LSDV		AH(l)		AH(d)		ABGMM1	
N	T	λ	RMSE	Bias	RMSE	bias	RMSE	bias	RMSE	Bias	RMSE	Bias
50	5	0.1	0.23037	0.01409	0.18214	-0.00767	0.33369	-0.0089	0.3342	-0.0801	6.65052	-1.9897
		0.5	0.15814	0.01196	0.1431	-0.00233	0.13553	0.00206	0.1414	-0.0133	1.31747	-0.4986
		0.9	0.10436	0.00785	0.10246	-0.00108	0.08539	0.0013	0.0884	-0.0031	0.72146	-0.3094
	10	0.1	0.15627	-0.0196	0.14609	0.002833	0.30888	-0.0315	0.322	-0.0471	0.0675	-0.0072
		0.5	0.10939	-0.0167	0.10893	0.000288	0.10765	-0.0038	0.1125	-0.0187	0.0665	-0.0059
		0.9	0.07556	-0.0107	0.07546	-0.00086	0.06245	-0.0016	0.0662	-0.0107	0.06455	-0.0045
	20	0.1	0.09823	-0.0088	0.10366	0.002421	0.22533	0.03191	0.2294	-0.0079	0.03233	0.02928
		0.5	0.0737	-0.0002	0.07834	0.00186	0.07864	0.00847	0.068	-0.0032	0.0361	0.03312
		0.9	0.0524	0.00217	0.0561	0.001811	0.04466	0.00605	0.0406	-0.0054	0.03891	0.03739
100	5	0.1	0.13067	0.00395	0.15403	-0.00785	0.22139	0.01649	0.2272	-0.0338	5.90899	3.55115
		0.5	0.10206	0.00279	0.10608	-0.00488	0.11284	0.00532	0.1104	-0.0173	1.22725	0.77527
		0.9	0.07314	0.00168	0.07358	-0.00295	0.07316	0.00184	0.0705	-0.0096	0.70306	0.4626
	10	0.1	0.1051	-0.0009	0.10827	-0.00397	0.17983	0.00453	0.1799	-0.0218	0.0549	-0.0017
		0.5	0.07766	-0.0077	0.07378	0.007526	0.06738	-0.0029	0.0721	-0.0063	0.05367	-0.0017
		0.9	0.05367	-0.0072	0.04925	0.008364	0.04008	-0.0029	0.0426	-0.0025	0.0518	-0.0017
	20	0.1	0.07655	-0.009	0.07764	0.000414	0.16219	-0.0231	0.148	0.01634	0.02924	-0.0037
		0.5	0.05637	-0.005	0.05077	0.000103	0.05093	-0.002	0.0508	-0.0026	0.02812	-0.0022
		0.9	0.03987	-0.0025	0.03774	0.001874	0.03024	0.00117	0.0321	0.00252	0.02833	-0.0036

Source: Compiled by the authors